The meaning of c^2 in the expression $\mathrm{E} = mc^2$ is it a just constant multiplier - Thought Experiments for Left Handed Maxwell Systems

Dedicating this new thought to my blind father Late Soumendra Kumar Das

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Key words

Negative Indexed Material (NRM), Meta-Material, Group velocity, Phase velocity, Corpuscular Momentum, Wave momentum, phase index, group index.

Abstract

In this paper simple thought experiments are described to elucidate the principle of the mechanical, and wave momentum-and need to have new wave momentum; along with an interpretation of (constant multiplier that is) c^2, as equivalent (and not always equal to) as product of group and phase velocities. The waveguide example is taken to show c^2 is product of phase and group velocity of accompanying electromagnetic radiation, then thought experiments show the results in positive as well as negative refractive indexed medium the meaning of c^2. Thereafter total energy triangles are explained to make distinction of the corpuscular and wave momentums of the EM radiation. So question is answered as what is meaning of c^2 in relation $\mathrm{E} = mc^2$, that it is equivalent to product of phase and group velocities.

1. Introduction

The question is for the multiplier as c^2 , which is numerically equal to square of velocity of light in vacuum, which is used to justify the dimensions of the Energy Mass equation that is $mc^2 = \mathrm{E}$! Well can this multiplier have different physical meaning? Here we derive from waveguide principal propagation modes, plus few thought experiments that is c^2 is a product of phase and group velocities. Then we extend the thought experiment to see the meaning of c^2 when the phase and group velocities are in opposition that is meta-material case. Thereafter we give distinction of two momentums corpuscular momentum and wave momentum, through energy triangle, describing positive as well as negative refractive media; via thought experiments.

2. Review of wave propagation inside waveguide where and c^2 of waveguide

I will now highlight a special case of propagation of radiation inside wave guide where c^2 is exactly equal to $v_p v_g$. In wave guide Electric Field $\bar{E}$ must agree with all Maxwell's equations in the free space inside the guide. Along with divergence of $\bar{E}$ must be zero in the free space inside the guide since there are no charges there. That is the same thing as saying that it must satisfy the wave equation, which is in 3-D;

$$\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}+\frac{\partial^2 E_y}{\partial^2 z^2}-\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}=0$$

The wave guide of my example guides the waves in z direction with $x-y$ plane as its cross section having y dimension ($b-\mathrm{cm}$) shorter than x dimension ($a-\mathrm{cm}$). Electric field $\bar{E}$ has only a $y-$component, and it doesn't change with y. This gives principal propagating mode with $k_x a=\pi$ as $E_y=E_0\sin(k_x x)\exp[\mathrm{i}(\omega t-k_z z)]$. In above wave equation, where E_y doesn't depend on y I can write as following

$$k_x^2 E_y+k_z^2 E_y-\frac{\omega^2}{c^2}E_y=0$$

Unless E_y is zero everywhere (which is not very interesting) this above expression is correct if

$$k_x^2+k_z^2-\frac{\omega^2}{c^2}=0$$

I have already fixed $k_x=a/\pi$, as for principal mode, so the above expression tells me that there can be waves of type of principal mode (as I have assumed) if k_z is related to the frequency ω so that same above equation gets satisfied. In other words that implies

$$k_z=\sqrt{(\omega^2/c^2)-(\pi^2/a^2)}$$

The waves I assumed and described in the wave-guide are propagated in $z-$direction with value of wave number k_z given by above expression. This wave number from above relation tells me that, for a given frequency ω the speed with which nodes (or antinodes) of waves propagate down the guide, thus giving 'phase velocity' $v_p=\omega/k_z$. The cut-off frequency of wave guide is $\omega_c=\pi c/a$ below which waves do not propagate down the guide. Using these facts and above equation I get, expression for phase velocity as the following

$$v_p=\frac{c}{\sqrt{1-(\omega_c/\omega)^2}}$$

For frequencies above cut-off where travelling waves exists the v_p in wave guide is greater than the speed of EM wave in vacuum c. Therefore, the wave guide simulates a material with refractive index less than unity. In order to know how fast the 'signals' travel, I have to calculate the speed of pulses or modulations made by the interference of waves of one frequency with one or more waves of slightly different frequencies. The speed of the envelope of such group of waves is the group velocity, it is $v_g = \mathrm{d}\omega / \mathrm{d}k$. Taking derivative of $k_z = \sqrt{(\omega^2 / c^2) - (\pi^2 / a^2)}$ and utilizing the definitions of cut-off frequency, $\omega_c = \pi c / a$ I get the following for 'group velocity'

$$v_g = c\sqrt{1 - (\omega_c / \omega)^2}$$

This is less than the speed of EM waves in vacuum c .Therefore geometric mean of v_p and v_g in this special case is just equal to c , or $v_p v_g = c^2$. This is how perhaps c^2 should be described!

3. Waveguide expression similar to quantum mechanical expression!

I take a detour here to show that relation $v_p v_g = c^2$ similarity with Quantum Mechanics. For a particle with any velocity (even relativistic) the momentum p and energy E are related by

$$\mathrm{E}^2 = p^2 c^2 + m^2 c^4$$

But in the quantum mechanics the energy is $\hbar\omega$ and the momentum is $\hbar / \lambda\!\!\!^-$, that is $\hbar k$ so I write above energy 'right triangle' expression as (refer figure-1)

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2 c^2}{\hbar^2}$$

From above I get $k = \sqrt{(\omega^2 / c^2) - (m^2 c^2 / \hbar^2)}$, which looks very similar to wave guide principal propagation equation, that is $k_z = \sqrt{(\omega^2 / c^2) - (\pi^2 / a^2)}$, an interesting observation!

The equation of total energy, $\mathrm{E}^2 = p^2 c^2 + m^2 c^4$ has two parts a corpuscular part represented by $\mathrm{E}_m = mc^2$ and the wave-energy momentum part represented by $\mathrm{E}_w = pc$. These two components are represented by right angle triangle of figure-1, free space diagram. So I get total energy as $\mathrm{E}^2 = \mathrm{E}_m^2 + \mathrm{E}_w^2$. In the next sections I shall use this relation and see how, equivalence of c^2 that is product of v_p and v_g , is utilized see have energy and momentum transport, for a photon or EM pulse!

4. What is c^2 just a multiplier or something else?

I consider a space between radiator and receiver is filled by vacuum that carrying between them electromagnetic radiation with energy E and to that I assign a linear momentum (due to wave) as $p_w = \mathrm{E}_w / c$, is also accompanying by a mass (corpuscular nature) $m = \mathrm{E}_m / c^2$, figure-2 depicts this thought experiment.

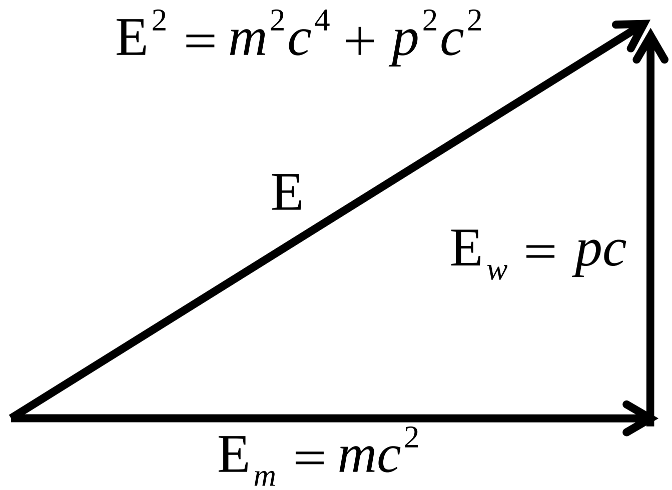

Figure-17 The Energy Diagram for corpuscular and phase (wave) energy in free space

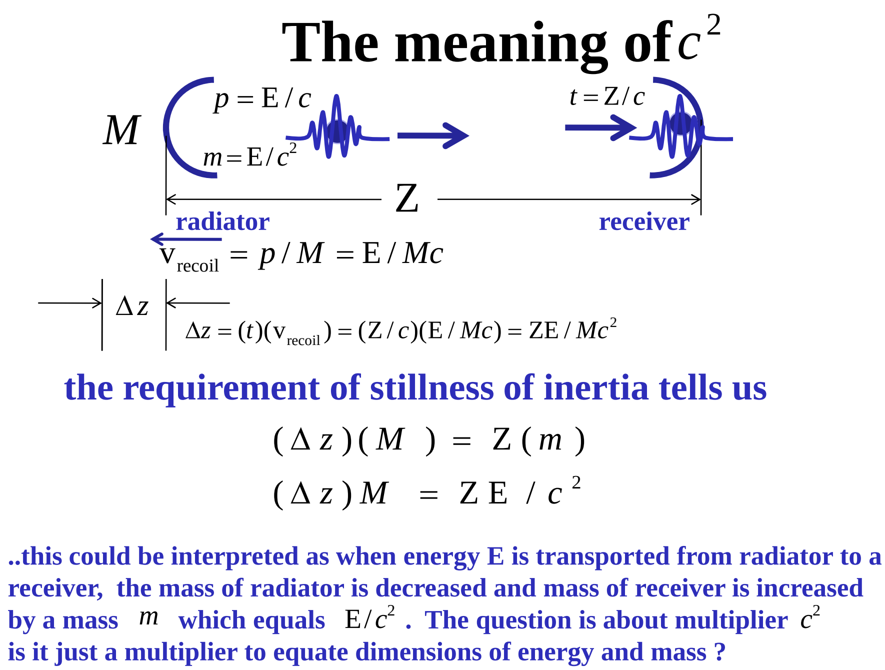

Figure-2: What is c^2 ?

The subscripts m and w distinguishes mechanical and wave energy. The wave particle duality states, that $E_m \equiv E_w \equiv E$. Really radiator after emitting wave-packet recoils with velocity $v_{recoil} = p / M = E / Mc$, where M is the mass of radiator. The wave packet reaches receiver sitting at distance Z after time $t = \mathrm{Z}/c$, and the radiator moves a distance $\Delta z = tv_{\text{recoil}} = (\mathrm{Z}/c)(\mathrm{E}/Mc) = \mathrm{ZE}/Mc^2$. The requirement of stillness of inertia of entire system gives moment balance as $(\Delta z)M = \mathrm{ZE}/c^2$. This description could be interpreted as, when energy E is transported from radiator to receiver; the mass of radiator gets decreased, but the mass of receiver gets increased by m equal to E/c^2 ! (Figure-2 summarizes this).

The question is for the multiplier as c^2 , which is numerically equal to square of velocity of light in vacuum, which is used to justify the dimensions of the Energy Mass equation that is $mc^2 = \mathrm{E}$! Well can this multiplier have different physical meaning? Let me associate c_g as group velocity of the wave-packet, and then in above paragraph the expression for time will be $t = \mathrm{Z}/c_g$.

Let the phase velocity be associated to crest and trough be identified as wave-velocity as c_p then wave momentum correlation will be $p_w = \mathrm{E}/c_p$, this makes the accompanying mass as $m = \sqrt{\mathrm{E}_m^2/(c_p c_g)^2}$. I have kept this expression as under root instead writing $\mathrm{E}_m / c_p c_g$ to state that even if $c_p c_g < 0$; I do not land to a 'negative mass'. This validates my choice of multiplier $c^2 = v_p v_g$, and this could be new physical interpretation also. Refer Figure-2.

Consider photon travelling in free space with mechanical energy $\mathrm{E}_m = mc^2$ that is energy associated with its corpuscular part, and with phase or wave- momentum as $p = \hbar\omega_0 / c$ having wave energy as $\mathrm{E}_w = pc$, thus total energy is E, having relation as (1) below, also refer figure-1.

$$\mathrm{E}^2 = p^2c^2 + m^2c^4 \tag{1}$$

Call v_p as phase velocity and v_g as group velocity of monochromatic EM signal travelling in the region $0 < z < (d/2)$, where the phase refractive index is $n_p(\omega) = +1$, with relative permeability $\mu_{r+} = 1$, and relative permittivity as $\varepsilon_{r+} = 1$. Conventionally, I can write for the dispersion less ideal region that;

$$v_p v_g = c^2 \tag{2}$$

This I am assuming that $v_p = (\omega / k) = c$; $(\mathrm{d}\omega / \mathrm{d}k) = c$ in a vacuum where EM waves are travelling is ideal condition. Now I pose a question as, how am I writing (2) that are square of velocity of EM wave equal to product of the phase velocity and group velocity? The answer to that I addressed in following description.

5. What is c^2 when the media is negatively refracting?

Now for negative indexed material NRM (lossless and ideal case, with $n_p = -1$) I can write, an approximate relation (2), for region, where I have assumed perfect condition as $\mu_{r-} = \varepsilon_{r-} = -1$; with phase refractive index as $n_p(\omega_0) = -1$, and group refractive $n_g(\omega_0) \cong +1$ this enables the propagating modes inside the LHM slab, with (3). In (3) where I assume that; $v_g \cong c$, inside LHM.

$$v_p v_g \cong -c^2 \tag{3}$$

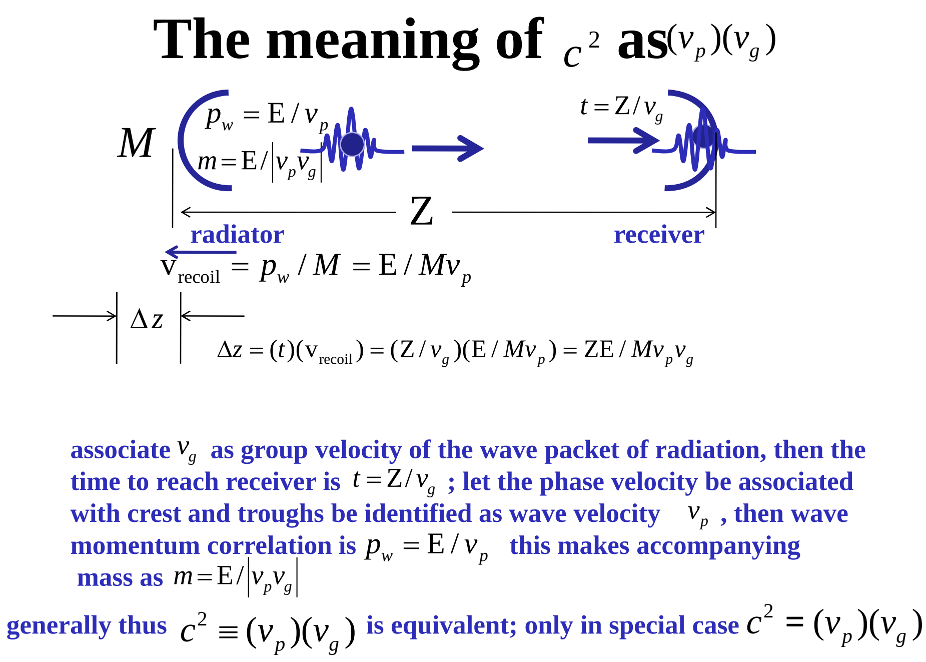

Figure 2: Can we have $c^2 = v_p v_g$

This negative sign in right hand side (3) is representing that group velocity and phase velocity are 180^{o} apart from each other, magnitude being c. Energy mass momentum expression for particle at speed of light in relativistic approach is (1), and substituting (3) I get

$$\mathrm{E}^2 = p^2c^2 + m^2c^4 = p^2(v_p v_g) + m^2(v_p v_g)^2 \tag{4}$$

This is depicted in figure-1. The corpuscular energy (momentum) is orthogonal to wave energy (momentum). Where E is total energy p is momentum of the wave which is present inside the meta-material; m is (rest) mass of the particle carrying the energy packet. Well the rest mass of photon is zero, but I can always associate a mass $m = \sqrt{\mathrm{E}_m{}^2/(c^2)^2}$, for the Electro Magnetic Energy carrying mechanical (corpuscular) energy E_m. This mechanical energy is responsible for positive radiation pressure of EM radiation. While the other part of energy I should associate to phase wave-momentum, hidden momentum, pseudo momentum energy due to the wave nature associated with photon-movement or translation of phases 'crests' and 'trough's' motion, in the media. Manipulating (4) I get as follows:

$$\mathrm{E}^2 = p^2(v_p v_g) + m^2(v_p v_g)^2$$
$$\mathrm{E}^2 = m^2 v_p v_g \left[v_p v_g + \frac{p^2}{m^2} \right] \tag{5}$$

The equation (5) is for free-space, medium with positive phase and group velocity and both equal to c. That is $v_p = v_g = c$. Now I use (5), for NRM medium and manipulate as below:

$$\begin{aligned} E^2 &= p^2c^2 + m^2c^4 \\ &= p^2(-v_pv_g) + m^2(-v_pv_g)^2 = m^2(v_pv_g)^2 - p^2(v_pv_g) \\ &= m^2\left|v_pv_g\right|\left[\left|v_pv_g\right| - \frac{p^2}{m^2}\right] \end{aligned} \tag{6}$$

Put in the equation (6) $\left|v_pv_g\right| \cong c^2$, I get

$$E^2 = m^2c^2\left[(c^2) - \left(\frac{p}{m}\right)^2\right] = (m^2c^2)\left[c^2 - \frac{p^2}{m^2}\right] = m^2c^4 + (-p^2c^2) \tag{7}$$

The expression of (7) I split into two parts, the mechanical (corpuscular) energy part $(E_m^2 = m^2c^4)$ and the energy transport by wave-momentum part $(E_w^2 = -p^2c^2)$ part. The (7) show that particle energy is retained itself by the particle, inside NRM where the phase velocity is opposite to group velocity. In this case no (mechanical-corpuscular) energy is transferred to the NRM medium. This I have derived from the part of rest mass-energy that is the first part of expression $E_m = mc^2$; meaning that corpuscular energy by photon is retained. But the intriguing question is the energy due to wave-momentum part is imaginary, inside NRM! That is equal to $E_w = -i(pc)$ (considering the positive root). Note the imaginary wave energy in free space figure-1 is $E_w = +i(pc)$. I can ascribe to this imaginary 'negative'- photon' a wave-momentum a value $-\hbar\omega_0 / c$. This is depicted in figure-3 (B). Compare the figure 3 A and B, the perpendicular of right angle triangle is opposite as one is positive indexed media and another is negative indexed media of refractive index (phase and group) as unity. The energy E_w associated with the 'wave-energy' is reversed, while mechanical energy remains the same. The 'reactive' nature of E_w opposite sign in both media gives the wave momentum opposite.

Now I retard the group velocity to $v_g = c/3$, and have phase reversal with phase velocity inside NRM (with $n_p = -1; \quad n_g = +3$) as $v_p = -c$ then $\left|v_pv_g\right| = c^2/3$, and put the same in (4) to get

$$\begin{aligned} E^2 &= m^2\left(\frac{c^2}{3}\right)\left[\frac{c^2}{3} - \frac{p^2}{m^2}\right] \\ &= \frac{1}{9}(m^2c^4 - 3p^2c^2) = \frac{1}{9}m^2c^4 + \frac{1}{3}(-p^2c^2) \end{aligned} \tag{8}$$

Here the particle inside the NRM has less total corpuscular energy; the difference of energy has been absorbed by the media itself. Expression (8) suggests one third of the corpuscular energy $E_m^{\text{NRM}} = (1/3)mc^2$ is retained by the 'photon' inside the NRM slab, and the two thirds of its corpuscular energy are given to the slab!! Well the energy due wave momentum of the photon manifests as imaginary energy in this case as $E_w^{\text{NRM}} = -i(1/\sqrt{3})pc$, (again retaining the positive root). I ascribe to this imaginary 'negative'- photon' a wave-momentum a value $p_c^{\text{NRM}} = -(1/\sqrt{3})\hbar\omega_0 / c$. This is depicted in figure-3 C. The momentum transfer cases I have discussed in earlier section also and maps correctly with the total energy argument cases as described here.

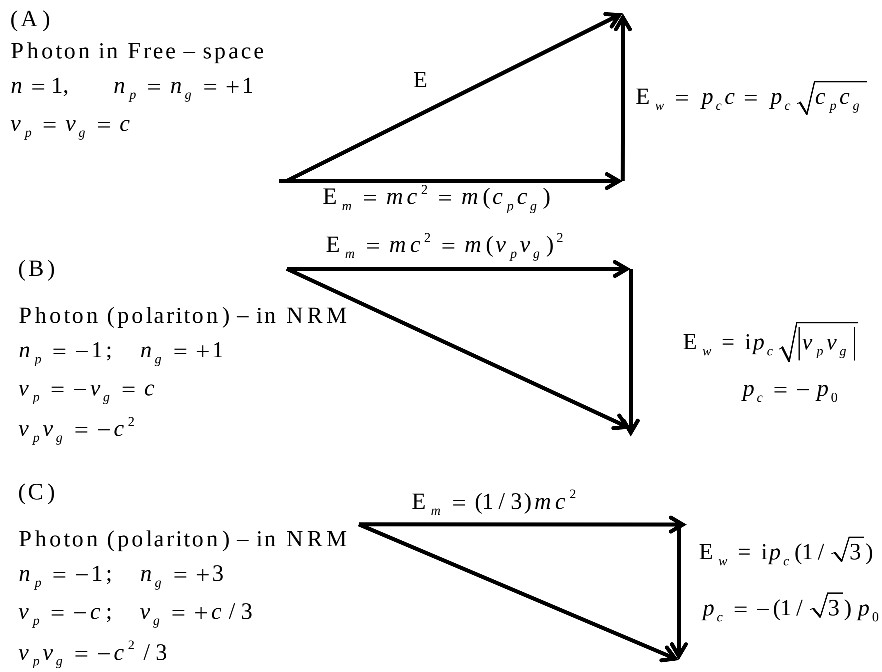

Figure-3 Energy diagrams of corpuscular and wave (phase) energy in NRM

6. A thought experiment

Refer figure-4; let me consider the length of NRM slab, as Z, with $n_p = -1$, and $n_g = 3$. The photon (polariton) is retarded in comparison to its position in absence of medium by distance z, which is

$$z = (c - v_g)\frac{Z}{v_g} = (n_g - 1)Z \tag{9}$$

The relativistic form of Newton's first law of motion requires that the centre-of-mass energy of a system not subjected to any external force should be stationary or in uniform motion. My medium is isolated from such external influence then the relevant total energy is sum of photon energy $\hbar\omega_0$ and the rest mass energy of the medium Mc^2, where M is mass of medium. Medium is on zero friction surface. The fact that photon has been retarded by the medium means the centre-of-mass-energy can only have been in uniform motion if the medium has itself moved to the right by a distance Δz, then the moments are (about vertical axis)

$$(\Delta z)(Mc^2) = (z)(\hbar\omega_0) \tag{10}$$

Substituting value of z from (9) I get

$$\Delta z = \frac{\hbar\omega_0 Z}{Mc^2}(n_g - 1) \tag{11}$$

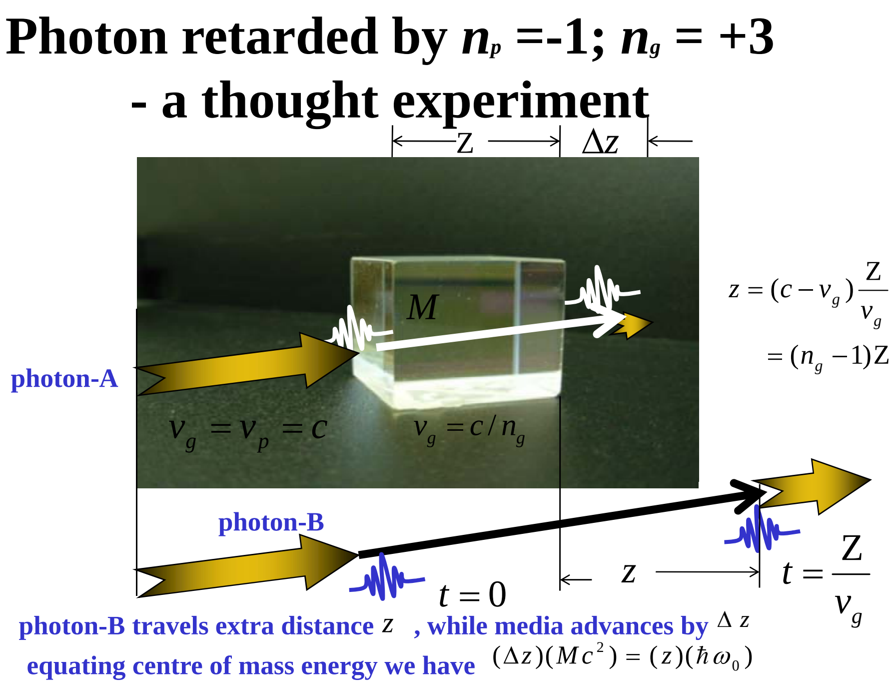

Figure-4 Thought experiment for mechanical momentum transfer to a negatively refractive media (NRM)

This motion can only take place if energy transfer takes place from photon whilst inside the medium. The required velocity of the medium is $v_g(\Delta z)/Z$, from which I can readily obtain momentum

$$p^{\text{medium}} = Mv_g\frac{\Delta z}{Z} = \frac{\hbar\omega_0}{c}\left(1 - \frac{v_g}{c}\right) = \frac{2}{3}\frac{\hbar\omega_0}{c} = \frac{2}{3}p_0 \tag{12}$$

Where $p_0 = \hbar\omega_0 / c$ is the initial momentum of the photon in free space. Momentum conservation suggests that I ascribe the difference between the initial momentum and this medium's momentum to the photon's momentum inside the medium. From [11] [12] [15] [16] [17] [18] [38] the mechanical momentum of photon in this NRM would be

$$p^{\text{NRM}}{}_{m1} = n_p^2\hbar\omega_0 / n_g c = v_g n_p^2 \hbar\omega_0 / c^2 = \frac{1}{3}\frac{\hbar\omega_0}{c} = \frac{1}{3}p_0 \tag{13}$$

$$p^{\text{NRM}}{}_{m2} = \hbar\omega_0 / n_g c = v_g \hbar\omega_0 / c^2 = \frac{1}{3}\frac{\hbar\omega_0}{c} = \frac{1}{3}p_0 \tag{14}$$

The wave momentum [11] [12] [15] [16] [17] [18] [38] of photon-polariton inside this NRM slab is

$$p^{\text{NRM}}{}_{c} = \frac{\text{sgn}(n_p)}{\sqrt{|n_p n_g|}}\frac{\hbar\omega_0}{c} = -\frac{1}{\sqrt{3}}p_0 \tag{15}$$

The (13) (14) states that; $(1/3)$ of the mechanical momentum is retained by the 'photon' inside this NRM. This is well equating as if 1/3 of 'particular' photon corpuscular energy is retained by photon inside NRM, whereas the wave-momentum retained by photon inside NRM (15) is $-(1/\sqrt{3})$ times the original wave momentum, this part of wave-momentum I have not got from (12), that is by this thought experiment; but via reflection transmission probabilities as derived earlier in [15] [16] [17] [18] [38].

This is because if for the thought experiment comprises of only wave momentum without any corpuscles part or mechanical components-the waves that is translation of phases carrying E_w energy just passes the medium without making mechanical displacement. What this E_w part does is exactly like phonons of 'sound' waves; that is while E_w part gives the atomic molecular vibrations and also (for EM signals) this part makes the medium polarized (magnetized)-without doing mechanical displacement. Therefore I am not applying the thought experiment on this wave-momentum part. Unfortunately, unlike phonons the photons require mechanical E_m as well as wave part E_w of the Electromagnetic energy; whereas for 'sound' phonons no mechanical part exist only the wave part does the translation hence for sound phonon we can associate only wave momentum concept.

Conclusion

What is the square of speed of light in vacuum i.e. c^2? It is just not a constant to make mass energy equation that is $\mathrm{E} = mc^2$, but has an interpretation of equivalence to product of phase and group velocities $c^2 \equiv v_p v_g$. Only in case of free space wave propagation or guided wave propagation is numerically equal to $v_p v_g = c^2$. Inside NRM the c^2 can be negative, and $\left|v_p v_g\right| \le c^2$, as demonstrated via thought experiments. The thought experiments also elucidated the concept of mechanical (corpuscular) and wave momentums and are distinguished for positive indexed and negative indexed material via sign; though the corpuscular part takes part in mechanical energy transport retaining its sign; the wave momentum behaves the opposite. Thought experiments is interesting way to perhaps bridge the gaps of these counterintuitive phenomena arising out of meta-material practical research!

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