Left Handed Maxwell Systems
In Optical Regime

PART-7

Negative Refractive Index Meta-material-in optical regime

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Few salient points

We have learnt how to obtain magnetic response and negative magnetic moments in nano-wire/rod/strips pairs ,this paves way to make negative refractive indexed material at optical wavelengths.

The electromagnetic response of metals in optical range is vastly different than at lower frequencies where $|\varepsilon_m|$ is extremely large and metals behave as near perfect conductors. This distinction prohibits the design of optical NIM's (Negative Indexed Material) using same structures as their uW counterparts. On the other hand at optical frequencies $|\varepsilon_m|$ of metals can be comparable to the ε_h of the host material, enabling the excitation of a surface plasmon resonance. $\varepsilon_m \sim \kappa\varepsilon_h$, $-1 \geq \kappa \geq -2$. This opens up a new method of getting negative permittivity and permeability.

The experimental verification of optical NIM is far more complicated than their uW counterparts. Refrac--tive index by definition implies the bending of the direction of Poynting vector at an interface. Unfortuna-tely, most of the optical NIM's are planer layers of sub-wavelength thickness fabricated by EBL (electron beam lithography). This limitation in fabrication prohibits experimentalists to observe the bending the beam of light, from a prism or wedge like meta-material of NIM in uW region.

To make a NIM it is essential to 'tune' the resonance property of the artificial material in such a way that the frequencies for the negative electric response and those for negative magnetic response occur in the overlapping spectral range. Having a negative permittivity background via diluted noble metal, with magnetic resonance strong enough to give negative permeability in optical region gives negative indexed meta-material (NIM)

Simultaneous negative mu and epsilon to get NIM is sufficient but not necessary

Here we reiterate that the condition of simultaneous negative ε and μ (given by Veselago 1967) is a sufficient but not necessary condition to construct NIM. A possible approach to achieve a negative refractive index in a passive medium is to design a material where the isotropic permittivity $\varepsilon = \varepsilon' + i\varepsilon''$ and the isotropic permeability $\mu = \mu' + i\mu''$ satisfies the following inequality

$$\varepsilon'\mu'' + \mu'\varepsilon'' < 0$$

Refer: http://pdfcast.org/pdf/physics-of-resonance-scaling-up-the-frequency-for-left-handed-maxwell-systems-lecture-6-by-shantanu

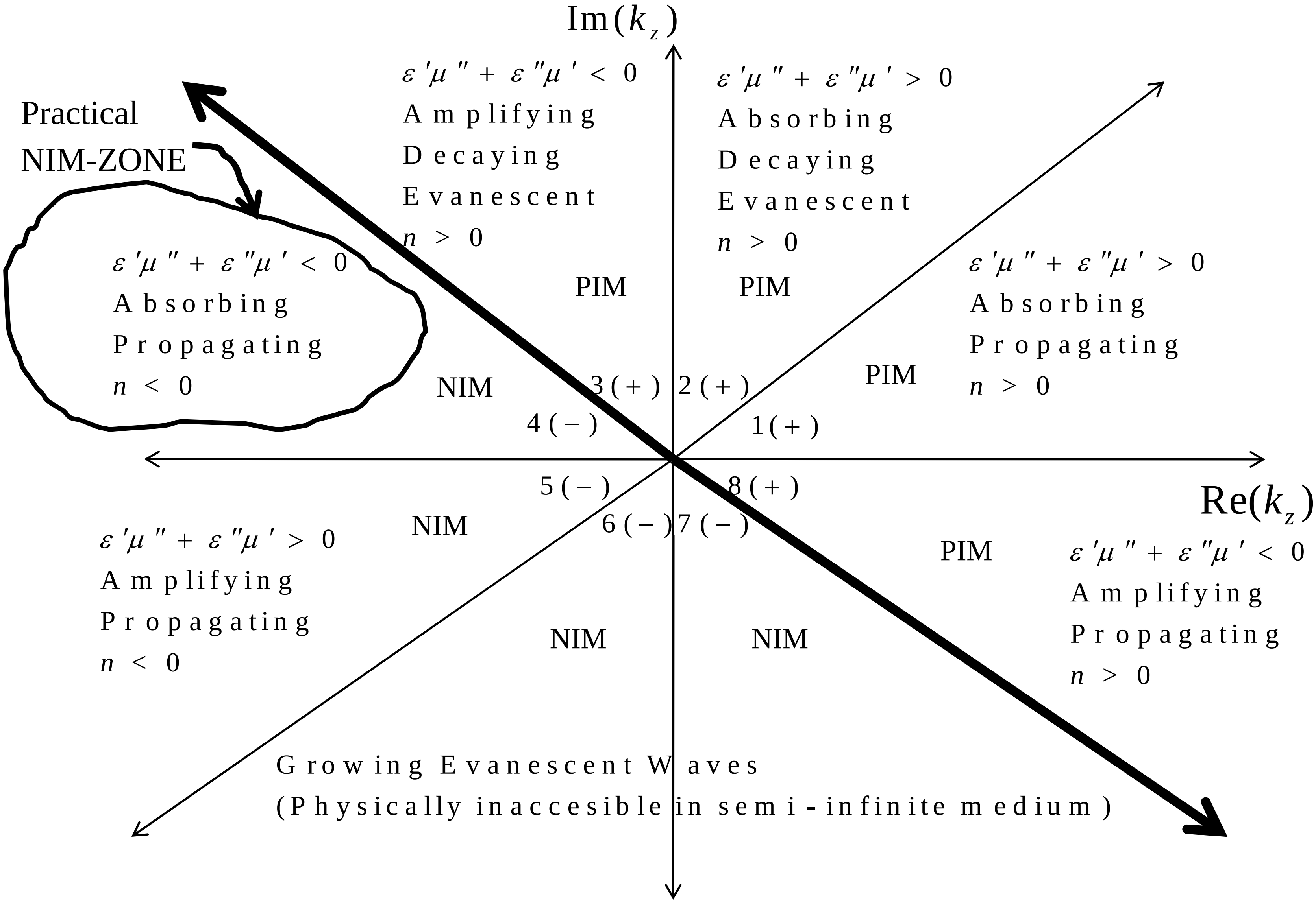
This leads to negative real part of the refractive index $n = n' + in'' = \sqrt{\varepsilon\mu}$. The above inequality is satisfied if both the $\varepsilon' < 0$ and $\mu' < 0$

However due to natural inertness of magnetic permeability at optical frequencies it is a practical challenge to obtain effective permeability very different from unity, especially at very high frequencies (visible). The above inequality thus implies that $n' < 0$ cannot occur in magnetically inactive medium with $\mu = 1 + i0$. Consequently magnetic response is essential in NIM. Instead of relying on relying on the negative μ , a NIM can be achieved via magnetically lossy medium with a negative ε and $\mu'' > 0$ in this case the inequality may too be fulfilled and in this case too negative real part of the refractive index $n' < 0$ can be obtained!! However the 'Figure of merit' FOM defined as $\mathrm{FOM} = |n'| / n''$ is low. The necessary condition can be achieved for an NIM , (i.e. the inequality) in an array of coupled nano-rods These configuration can lead to paramagnetic or dia magnetic response, and most importantly negative n' at the optical frequencies.

Lagarkov AN, Sarychev AK (1996), Electromagnetic properties of composites containing elongated conducting inclusions, Phys. Rev. B 53: 6318-6336

Podolosky, Shalaev et al, (2002), Plasmon modes in metal nano-wires and left-handed material; J of Non linear Opt. Phys Matter 11:65

Complex wave vectors in two Riemann sheets first sheet PIM second sheet NIM



(+) Primary Riemann sheet $n > 0$, $\mathrm{Re}(k_z) > 0$, PPV, for Propagating

(−) Secondary Riemann sheet $n < 0$, $\mathrm{Re}(k_z) < 0$, NPV, for Propagating

• Refer: http://pdfcast.org/pdf/physics-of-resonance-scaling-up-the-frequency-for-left-handed-maxwell-systems-lecture-6-by-shantanu

Characterization of spectral parameters and properties

When the optical plasmonic meta-material is properly designed and successfully fabricated the next step is to characterize developed nanostructure. The characterization may be grouped into two categories (1) Standard nano-characterization tools (2) desk top optical measurements.
The (1) comprises of scanning electron microscope (SEM), and atomic force microscope (AFM), and several near field optical microscopes. This (1) is integrated procedure after fabrication of these nano--structures via Electron beam lithography (EBL) etc.

What is required is a desk top equipment to characterize effective parameters of the nano-structured meta--material.

When a probe light impinges on the piece of sample, the most prominent information, which are all important from the specimen are contained in reflected and transmitted light. As with all e.m. waves the reflected and transmitted beams contains both the magnitude and phase information, which are all important and whose values are deeply rooted in the property of the sample. At the optical regime the magnitudes of reflectance R and transmittance T which can be obtained via intensity measurement using various power detectors, are much more accessible than phase information, which is attainable only by complicated interferometer.

Commonly used characterization method

The most commonly used characterization method for optical meta-material is measurement of broadband *T* and *R* spectra. Although not a complete set of information due to lack of phase properties, the spectrally dependent *T* and *R* curves help to locate the spectral positions and evaluate the relative strengths of resonances in a meta-material.

In most of the cases a layer of optical meta-material are macroscopically uniform planer slab, consequently when light is incident upon the sample only regular transmission and specular reflections are considered, while all other scattering and diffusion process are generally neglected. The principle to get *T* and *R* are simple. The probe light should come from a broadband or tunable light source, eg. Tungsten-Halogen lamp the probe light impinges upon the sample and the transmitted and reflected beam is then introduced into a detecting device as photodiode, a CCD, or a photo-multiplier. A monochromator is usually inserted in the optical path either before or after the sample, in order to analyze the spectrum.

The *T* and *R* spectra should be collected using appropriate references. In the *T* measurement, free-space is good reference for *T* = 100% if the whole specimen including the substrate is considered to be the sample, or an empty substrate area without patterns can be used as reference if only the nano-structured layer is to be evaluated. When measuring the *R*, a good mirror or a special reflectance standard should be used to calibrate the *R* = 100% reference. Polarization control of the optical system is necessary, as most optical meta-materials are highly anisotropic.

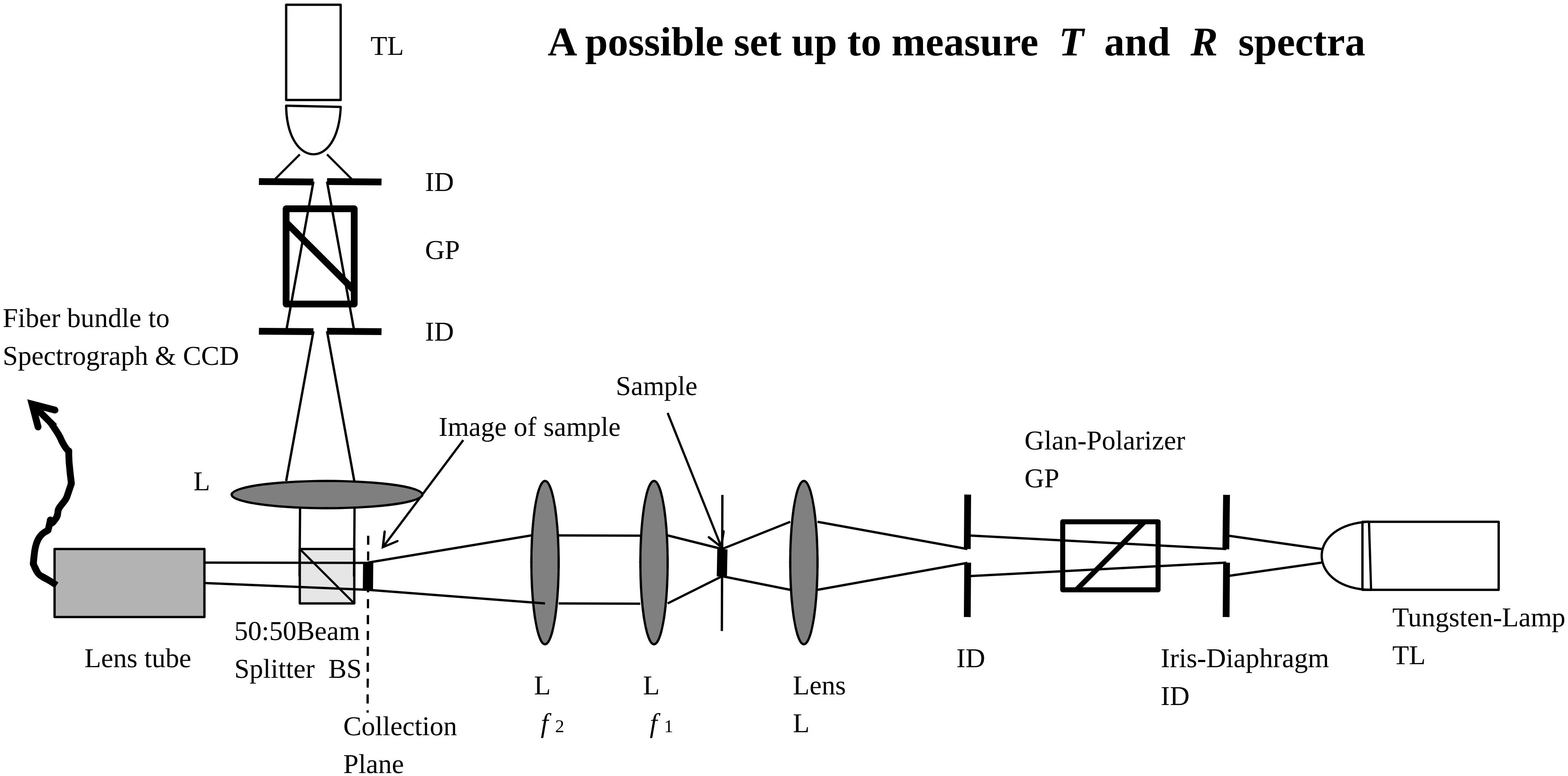
Spectrophotometers

Spectrophotometers are the standard commercial equipment for the collection of the broadband *T* and *R* spectra. A spectrophotometer can be viewed as a combination of a spectrometer, which generates light of any selected wavelength, and a photometer that measures the radiant flux. Most spectrophotometers are configured in a dual-beam manner, where one beam is used to probe the sample and the other serves as reference. In a typical spectrophotometer the angle of incidence is easily adjustable in the transmission mode, while reflection at normal incidence is usually approximated using small incidence angle in order to separate the incident and reflected beams.

In characterization of optical meta-material however, the measurement of transmission and reflection usually cannot be carried out in a commercial spectrophotometer. Samples made by EBL are very small with a typical size of the order of hundred micro-meter. Such a sample area is not large enough for most of the spectrophotometers, and appropriate sample holders and accessories are not available for holding handling and locating the invisibly small pattern region.

The spectral measurements can be carried out via a set-up, where a small sized sample is imaged onto a focal plane of the collection system, which may include a fiber bundle and a spectrograph. The magnified image size is $S.(f_1/f_2)$, which is set to be significantly larger than the collecting area of the optical system to ensure reliable data collection. A high quality polarizer like a Glan-Taylor prism is placed at the output of the broad-band lamps to select light with the desired polarization The T and R spectra can be normalized to a bare substrate and a calibrated silver mirror respectively.

A possible set up to measure *T* and *R* spectra



Cai W S, Chettiar UK, et al, 'Metamagnetics with rainbow colors,' Opt. Express 15:3333-3341

Extraction of homogenized parameters

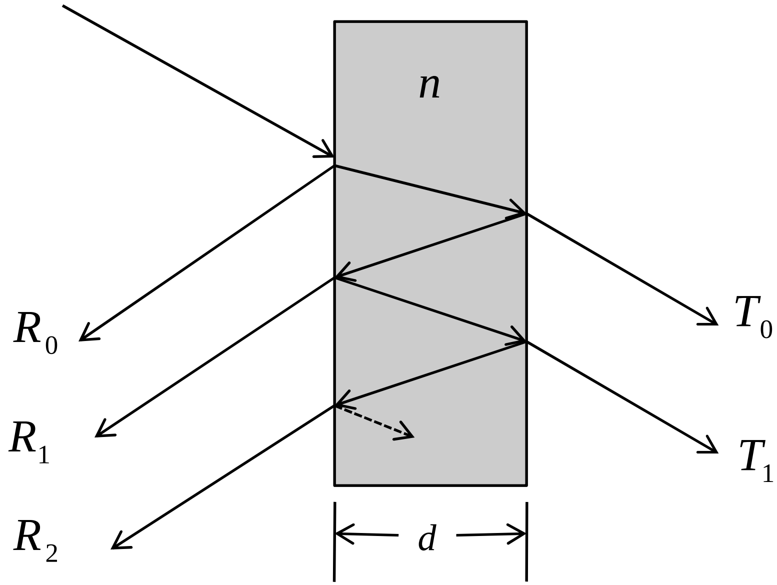
For a given meta-material with features smaller than the wavelength of interest, one of the most critical task is to extract the effective parameters ε, μ, n and impedance Z, from experiment observables. In general all these parameters are complex numbers. These four parameters are not indepen--dent, and they can be grouped into two sets. The first set is ε and μ stems from constitutive relation which enters Maxwell's equation

$$D = \varepsilon_0 E + P = \varepsilon_0 (1 + \chi_e) E = \varepsilon_0 \varepsilon_r E \qquad B = \mu_0 (H + M) = \mu_0 (1 + \chi_m) H = \mu_0 \mu_r H$$

The other two quantities n and Z are more conveniently used in the description of wave phenomena at the boundary of different material. The two sets are related as $\varepsilon = n / Z$ and $\mu = nZ$

for a uniform slab of meta-material an ideal retrieval process is expected to replace the microscopically inhomogeneous slab with a conceptually uniform medium with effective parameters (ε, μ, Z), such that far field scattering patterns are faithfully reproduced. Even though the unit structures of a meta-material can be very complicated, the local field details of e.m. responses are encapsulated by distilling the macro--scopic parameters from the composite parameters. The philosophy involved here is not much different from the use of these quantities for conventional materials-the responses from a collection of scattering atoms or molecules are conveniently described in an averaged fashion. The most accessible experimental observables from which homogeneous parameters are extracted include the complex transmission T and reflection coefficients R . In uW-experiments these are called S parameters; S_{11}, S_{12}, S_{21}, S_{22}

Transmittance and reflectance from 'multiple' interface

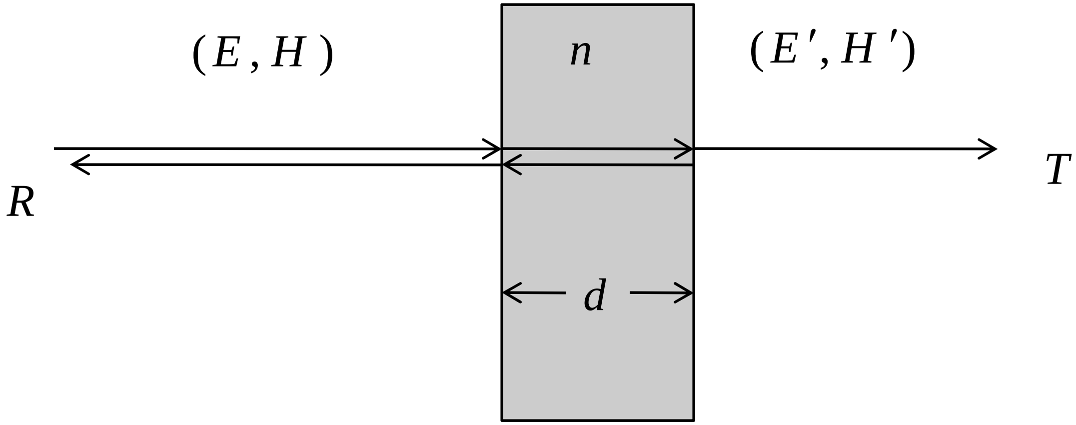


$$R = R_0 + R_1 + R_2 + \ldots \qquad T = T_0 + T_1 + T_2 + \ldots$$

The reflection of light from a single interface between two media is described by the Fresnel equations. However, when there are multiple interfaces such as in our case, the reflections themselves are also partially transmitted and then partially reflected. Depending on the exact path length, these reflections can interfere destructively or constructively. The overall reflection of a layer structure is the sum of an infinite number of reflections, which is cumbersome to calculate.

Transfer Matrix [T] for scattering studies

The transfer-matrix method is based on the fact that, according to Maxwell's equations there are simple continuity conditions for the electric field across boundaries from one medium to the next. If the field is known at the beginning of a layer, the field at the end of the layer can be derived from a simple matrix operation. A stack of layers can then be represented as a system matrix, which is the product of the individual layer matrices. The final step of the method involves converting the system matrix back into reflection and transmission coefficients.



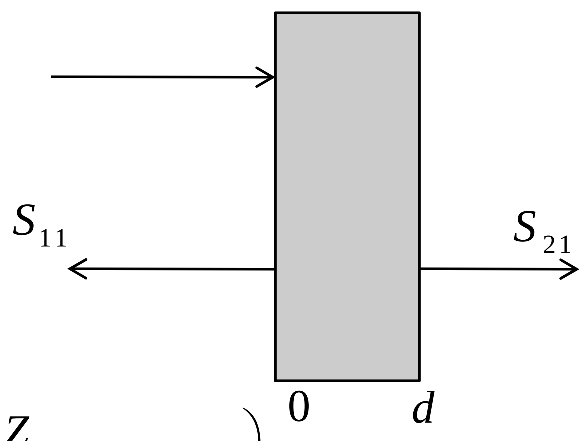
$$\begin{pmatrix} E' \\ H' \end{pmatrix} = [\mathrm{T}] \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} \cos(nkd) & -\frac{Z}{k}\sin(nkd) \\ \frac{k}{Z}\sin(nkd) & \cos(nkd) \end{pmatrix} \begin{pmatrix} E \\ H \end{pmatrix}$$

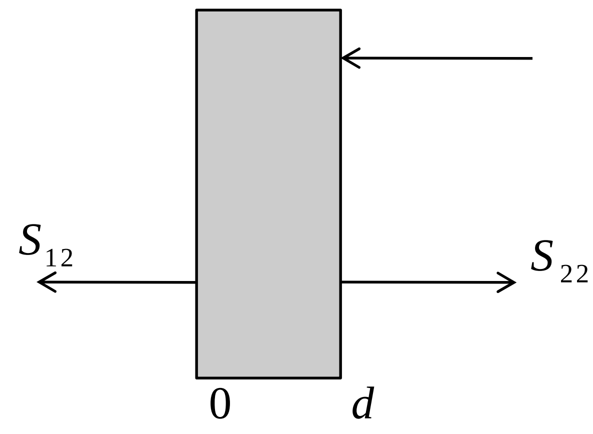
D M Pozar, Microwave Engineering, 2nd edition John Wiley & Sons, New York, 1998

D R Smith, R Dalichauch, N, Kroll, S Schultz, et al, J Opt. Soc. Am. B 10 314, 1993

D R Smith et al, Electromagnetic parameter retrieval from inhomogeneous meta-material, Phys. Rev. E 71, 036617 (2005)

Scattering coefficients *S* from the 'Transfer Matrix' [T]





$$[\mathrm{F}'] = [\mathrm{T}][\mathrm{F}]$$

$$[\mathrm{T}] = \begin{pmatrix} \cos(nkd) & -\dfrac{Z}{k}\sin(nkd) \\ \dfrac{k}{Z}\sin(nkd) & \cos(nkd) \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$$

$$S_{21} = \frac{2}{T_{11} + T_{22} + \left(ikT_{12} + \dfrac{T_{21}}{ik}\right)} \qquad S_{12} = \frac{2\det[T]}{T_{11} + T_{22} + \left(ikT_{12} + \dfrac{T_{21}}{ik}\right)}$$

$$S_{11} = \frac{T_{11} - T_{22} + \left(ikT_{12} - \dfrac{T_{21}}{ik}\right)}{T_{11} + T_{22} + \left(ikT_{12} + \dfrac{T_{21}}{ik}\right)} \qquad S_{22} = \frac{T_{22} - T_{11} + \left(ikT_{12} - \dfrac{T_{21}}{ik}\right)}{T_{11} + T_{22} + \left(ikT_{12} + \dfrac{T_{21}}{ik}\right)}$$

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D R Smith et al, Electromagnetic parameter retrieval from inhomogeneous meta-material, Phys. Rev. E 71, 036617 (2005)

Transmittance (*T*) and reflectance (*R*) from *S* parameters of T matrix

For homogeneous slab $T_{11} = T_{12} = T_s$ and $\det[\mathrm{T}] = 1$; as $[\mathrm{T}] = \begin{pmatrix} \cos(nkd) & -\frac{Z}{k}\sin(nkd) \\ \frac{k}{Z}\sin(nkd) & \cos(nkd) \end{pmatrix}$

and $[\mathrm{S}] = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$ is symmetric matrix, where

We have transmission coefficient and reflection coefficient as following:

$$S_{21} = S_{12} = T = \frac{1}{T_s + \frac{1}{2}\left(ikT_{12} + \frac{T_{21}}{ik}\right)} = \frac{1}{\cos(nkd) - \frac{i}{2}\left(Z + \frac{1}{Z}\right)\sin(nkd)}$$

$$S_{22} = S_{11} + R = \frac{\frac{1}{2}\left(\frac{T_{21}}{ik} - ikT_{12}\right)}{T_s + \frac{1}{2}\left(ikT_{12} + \frac{T_{12}}{ik}\right)} = T\,\frac{i}{2}\left(\frac{1}{Z} - Z\right)\sin(nkd)$$

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D R Smith et al, Electromagnetic parameter retrieval from inhomogeneous meta-material, Phys. Rev. E 71, 036617 (2005)

Parameter extraction from observed T and R values

Consider a homogeneous slab of thickness d under 'normal incidence' . Assuming the slab is placed in vacuum the complex T and R as

$$T = \left[\cos(nkd) - \frac{i}{2}\left(Z + \frac{1}{Z} \right) \sin(nkd) \right]^{-1} \qquad R = -T \cdot \frac{i}{2}\left(Z - \frac{1}{Z} \right) \sin(nkd)$$

The reflection coefficient R is related to impedance Z and refractive index n as obtained via inverting above

$$Z = \pm \left[\frac{(1+R)^2 - T^2}{(1-R)^2 - T^2} \right]^{\frac{1}{2}} \qquad \cos(nkd) = \frac{1 - R^2 + T^2}{2T}$$

Where $k = 2\pi / \lambda_0$ is the free space wave-vector. When n and Z are available then the other sets of material parameters are obtained by $\varepsilon = n / Z$ and $\mu = nZ$

The above lead to ambiguity in the retrieved parameters due to the multi-valued nature of the trigonometric functions and square-root function. This problem can be resolved by the combination of several considerations First, for any passive material the imaginary part of ε, μ, n and the real part of Z must be positive. With this constraint the impedance can be uniquely determined from above expression. Similarly imaginary part of n can be found by solving above

$$n'' = \pm \frac{1}{kd} \operatorname{Im}\left[\cos^{-1}\left(\frac{1 - R^2 + T^2}{2T} \right) \right]$$ along with additional requirement $n'' > 0$

Note that the inverse cosine function $\cos^{-1}(x)$ allows arbitrary complex arguments , in which case the real part of $\cos^{-1}(x)$ is bound within $[0, \pi]$, but there is no such constraint for the range of its imagi- -nary part.

The real part of the refractive index

$$n' = \pm \frac{1}{kd} \mathrm{Re}\left[\cos^{-1}\left(\frac{1 - R^2 + T^2}{2T} \right) \right] + \frac{2\pi m}{kd} \qquad m \in \mathbb{Z}$$

There are two ambiguities hidden, the sign before the first term on the RHS, and the correct branch corres--ponding to a definite value of m. The first one is easy to resolve-the same sign as for n'' should be taken since both equations stems from the same complex solution.

For the second ambiguity m, for the slabs with a very small thickness $d \ll \lambda_0$, the branches with diff--erent value of m are well separated. The correct branch can be identified by the fact that the frequency -dependent $n'(\omega)$ has to be continuous across a wide wavelength range. The retrieval process usually starts at a sufficient large wave-length away from all pertinent resonances in meta-material. In this case $m = 0$ is taken For a slab of sub-wavelength thickness. The retrieval process is then carried out towards shorter wave-length, and the m in the above expression should be adjusted to counter any discontinuity in $n(\omega)$

D M Pozar, Microwave Engineering, 2nd edition John Wiley & Sons, New York, 1998

D R Smith, R Dalichauch, N, Kroll, S Schultz, et al, J Opt. Soc. Am. B 10 314, 1993

D R Smith et al, Electromagnetic parameter retrieval from inhomogeneous meta-material, Phys. Rev. E 71, 036617 (2005)

Consideration of substrate layer while extraction

In most of the meta-material in optical regime, the one side of the meta-material is attached to a microscopically thick substrate, and the other side interfaces air or vacuum ; therefore the basic retrieval formulas (i.e. for Z and n) gets modified as:

$$\cos(nkd) = \frac{1 - R^2 + n_s T^2}{\left[(n_s + 1) + R(n_s - 1)\right]T} \qquad Z = \frac{i\left[(R+1) - T\cos(nkd)\right]}{n_s T \sin(nkd)}$$

Where, n_s represents the index of refraction for the substrate.

D M Pozar, Microwave Engineering, 2nd edition John Wiley & Sons, New York, 1998

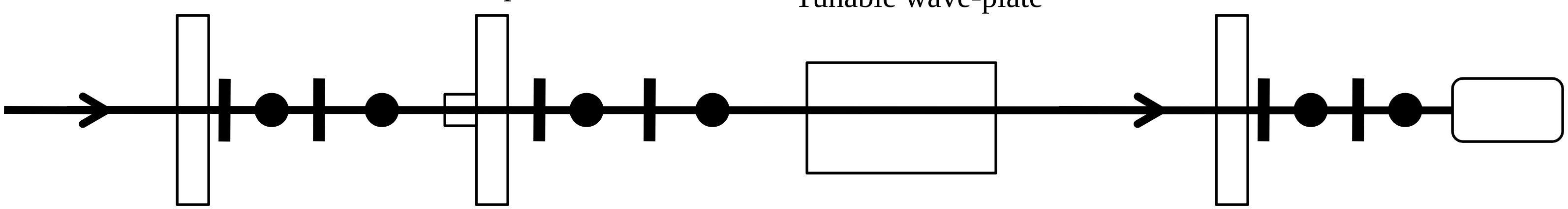
D R Smith, R Dalichauch, N, Kroll, S Schultz, et al, J Opt. Soc. Am. B 10 314, 1993

Kildishev AV, Cai WS, Chettiar UK, et al, ‘Negative refractive index in optics of metal dielectric composites’, J Opt Soc Am B 23 : 423-433 (2006)

D R Smith et al, Electromagnetic parameter retrieval from inhomogeneous meta-material, Phys. Rev. E 71, 036617 (2005)

Phase measurement in optical meta-material

In order to evaluate the phase shift of the light when passing through meta-material layer a combination of special interferometer and power measurement is usually necessary. When meta-material being studied is highly anisotropic, it is possible to estimate phase shift, by measuring the phase difference between two ortho- -gonally polarized light beams.



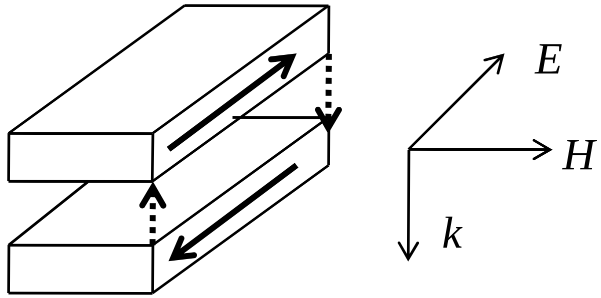
The above gives set-up of common path polarization interferometer for phase anisotropy measurement. The orthogonal polarization are shown in dot and lines. This setup employs the polarization interferometry to evaluate the difference in phase shifts between two orthogonally polarized waves $\Delta\varphi = \varphi_{\parallel} - \varphi_{\perp}$ the spectra of $\Delta\varphi$ is capable of revealing the resonance property of the phase shift in the sample.

Kim E, Shen YR, Wu W, et al, (2007), ‘ Modulation of negative indexed meta-material in the near IR range’, Appl. Phys. Lett. 91 : 173105.

Shalaev VM, Cai WS, Chettiar UK (2005), ‘ Negative index of refraction in optical meta-material’, Opt. Lett. 30: 3356-3358.

For absolute phase measurement, walk off interferometer is employed

The building block of optical meta-material for NIM-a paired nano-wire gives electric plasmon & magnetic plasmon resonance thus resonant refractive index

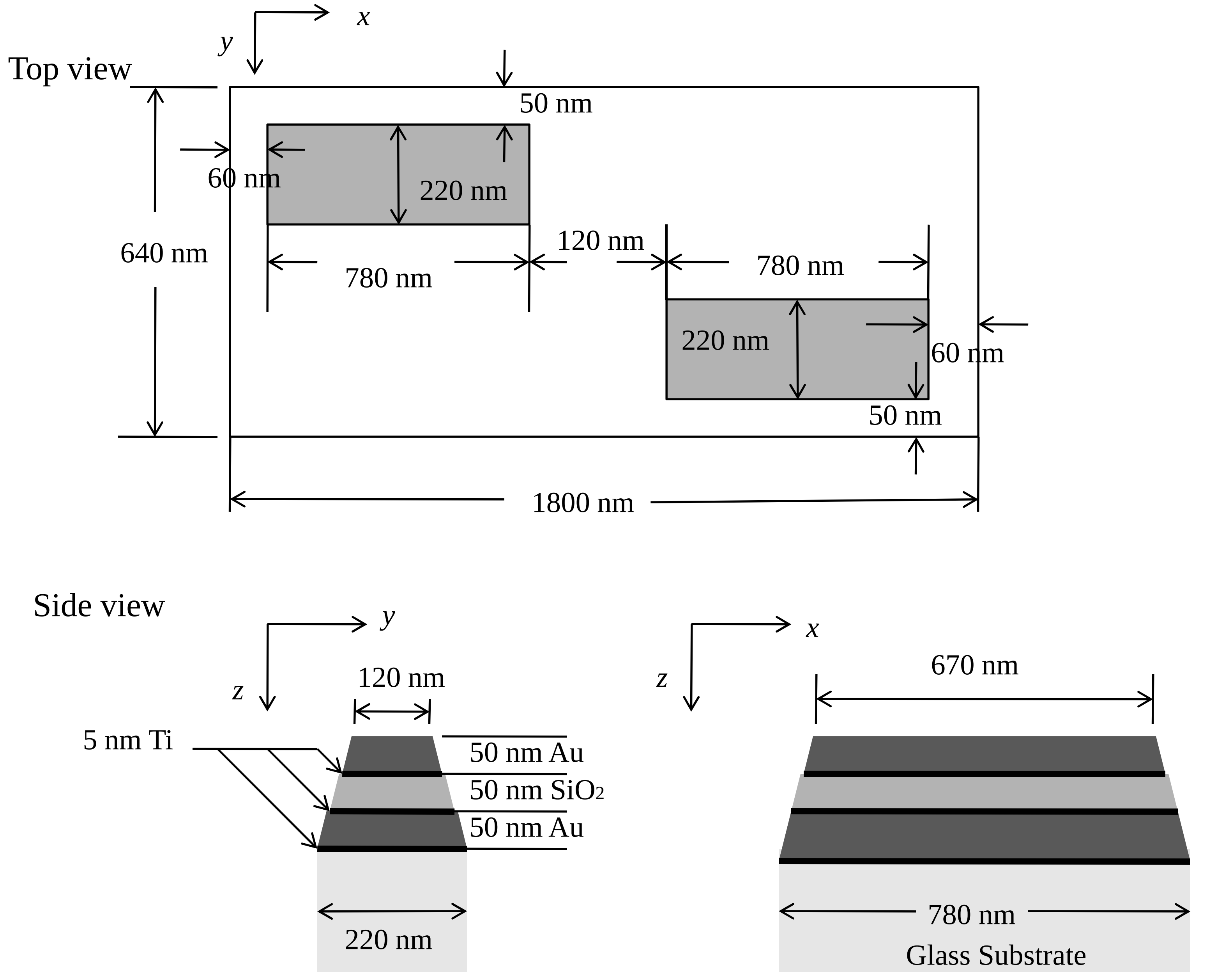


The building block of the optical meta-material is pair of nano-rods (strips), an AC electric field parallel to both rods induces parallel currents in both the rods (i.e. symmetric currents). The *H* field, which is oriented perpendicular to the plane of the rods causes anti-parallel currents in the two rods. These anti-parallel currents cause the mag--netic response of the system. The magnetic response will be dia- or paramagnetic depending on whether the wavelength of the incoming e.m field is shorter or longer than the magnetic resonance of the coupled rods. The two parallel rods form an open current loop, which acts as transmission line with current reso--nance. Such current loops are closed at bthe ends of the rod-pair through displacement currents.

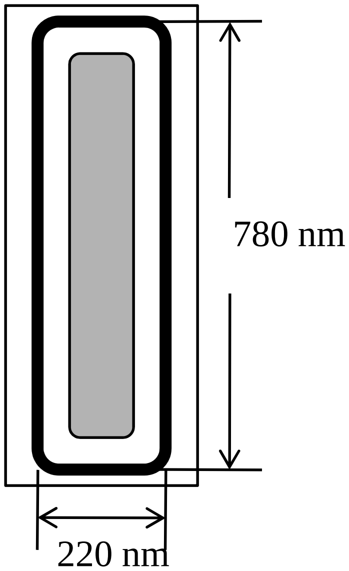
For normally incident light with *E* field polarized along the rods and the *H* field perpendicular to the pair, both electric and magnetic responses can experience resonant behavior at certain frequencies. Above the resonance frequency, the circular current in the pair of rods and the displacement currents at the end of the rods can lead to a magnetic field opposing the external *H* field of the incident light. In this design the electric component of the incident wave excites a symmetric current mode in each pair whereas the magnetic component excites an anti-symmetric mode. The excitation of plasmon resonances for both the electric and magnetic light component in an overlapping frequency results in resonant behavior for the refractive index, which can become negative, (only if these two resonances overlap!!)

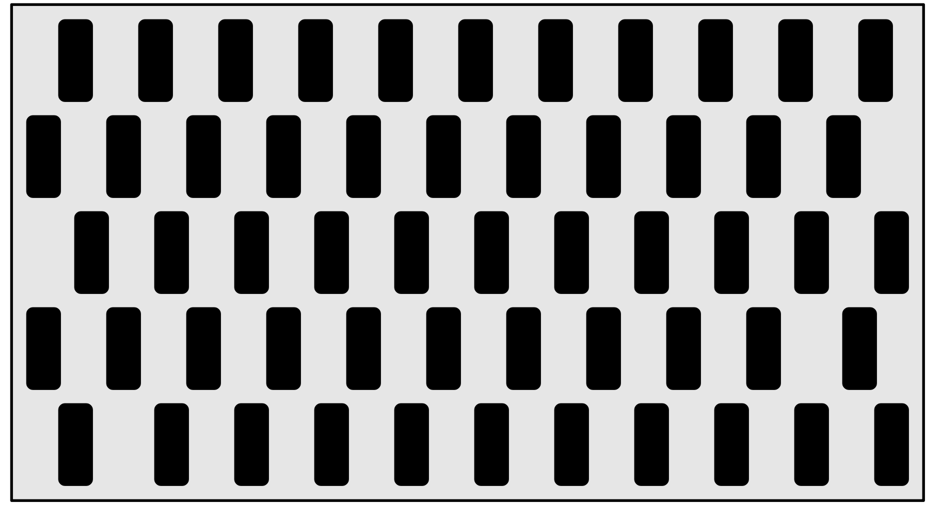
Shalaev VM, Cai WS, Chettiar UK et al, (2005) Negative index of refraction in optical meta-materias; Opt. Lett. 30: 3365-3358

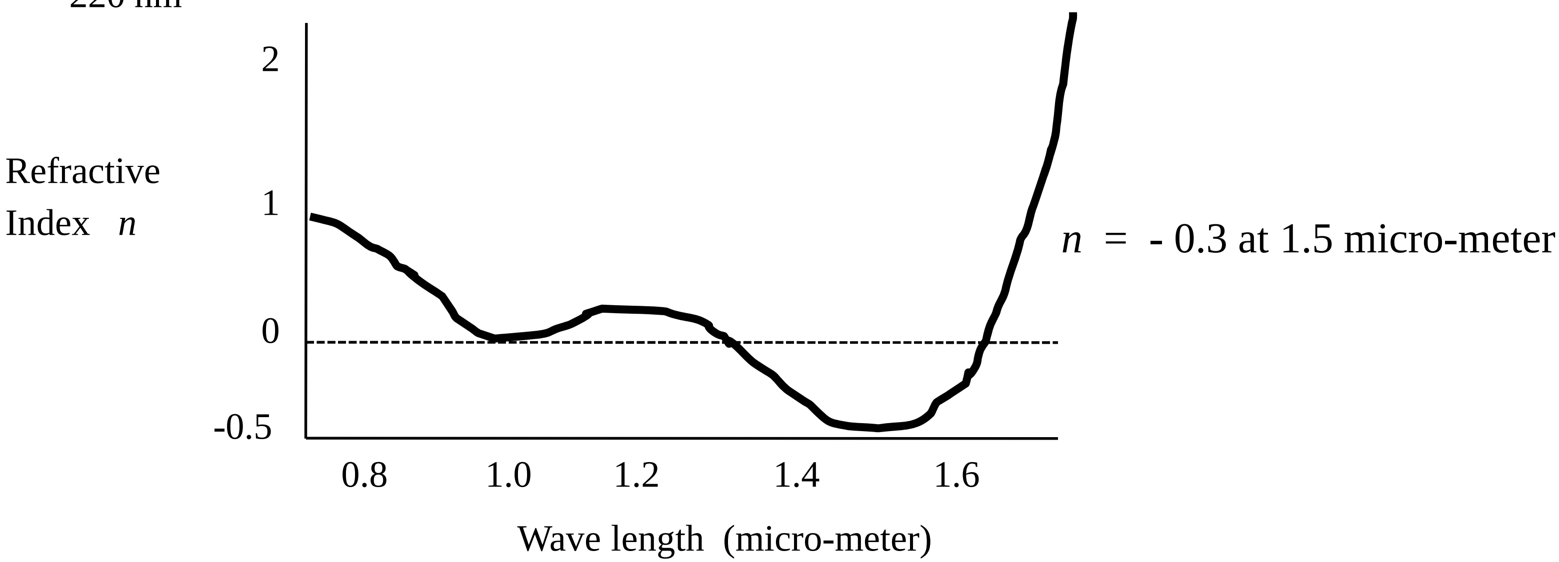
The nano-rod meta-material unit cell



Array of nano-rod pairs for optical meta-material







Shalaev VM, Cai WS, Chettiar UK et al, (2005) Negative index of refraction in optical meta-materias; Opt. Lett. 30: 3365-3358

Zhang S, Fan WJ, Osgood RM et al, (2005), Experimental demonstration of near IR negative indexed meta-material, Phys. Rev Letts. 95: 137404

FOM of negative refractive index material at optical wave-length from nano-rod pairs

These were the first optical NIM's at near IR, 1.5 micron wave-length, however were rudimentary demonstrations. This is the case because the negative indices in those demonstrations were accomplished in part, because of significant contribution of imaginary part of permeability μ'', which typically allow for a sufficient low loss factor. While these earliest optical NIM have proven that NIM are possible at optical regime, but these structures posses very large imaginary part n'' in effective refractive index. Based on our earlier discussion about the sufficient and necessary condition for the negative index of refraction, we can categorize NIM into two types, doubly negative NIM with $\mu' < 0$ & $\varepsilon' < 0$ satisfied simultaneously (DN-NIM); or single-negative NIM (SN-NIM) where the necessary condition is $\varepsilon'\mu'' + \varepsilon''\mu' < 0$ fulfilled with $\varepsilon' < 0$ and $\mu' > 0$; the NIM with nano-rods fall into this category which inevitably exhibits a low FOM.

As we stated earlier, for NIM, the ratio $-n'/n''$ is often taken as FOM because low loss NIMs are desired in most applications. The FOM of NIM can be expressed as effective μ and ε

$$\mathrm{FOM} = -\frac{|\mu|\varepsilon' + |\varepsilon|\mu'}{|\mu|\varepsilon'' + |\varepsilon|\mu''}$$

This above expression states that a DN-NIM with $\varepsilon' < 0$ and $\mu' < 0$ will have a lower n'' value than SN-NIM with same $n' < 0$ and $\mu' > 0$. In addition DN-NIM can provide better impedance matching

Comments regarding negative refractive index at optical wave-length from nano-rod pairs

In the paired nano-rods system, both the $\varepsilon' < 0$ and $\mu' < 0$ can be realized based on electric and magne- -tic resonances respectively, it is not a good practice to combine these two resonances at overlapping frequ -ency range. However, it is very difficult to obtain a system where both the resonances occur at same frequ -ency. Moreover any plasmonic resonance brings loss to system. Since in this case the electric resonance (due to symmetric currents induced by E field) is not really necessary in order to have negative ε', we must therefore try and avoid this electric resonance in the system.

A possible solution to this generic problem is to use a resonant magnetic structure along with a non-resonant metallic structure that provides a "background" of negative permittivity in broad spectral range, including the frequency band where magnetic resonance (MPR) occurs. We know that noble metals gold silver have negative permittivity at optical frequencies below their plasma frequency. Hence by adding a metal film above and below the magnetic resonator (i.e. paired nano-rod/strips), should provide negative ε'

Chettiar UK, Kildishev AV, Shalaev VM (et al), 2006, Negative indexed meta-material combining magnetic resonator with metal film; Opt. Express 14: 7872-7877.

An alternative method to achieve the background of negative permittivity is to use pairs of continuous metal wires that do not have electric resonance at frequency of interest, then a magnetic resonance with negative permeability is obtained by including appropriately designed pairs of metallic wires or plates.

Zhou JF, et al (2006), Negative indexed materials using simple short wire pairs, Phys. Rev B 73: 041101

Overlapping electric and magnetic resonances can be achieved by placing a continuous metal sheet in between the upper and lower metal strips of nano-rod s (and adjusting its width) , but the structure is lossy.

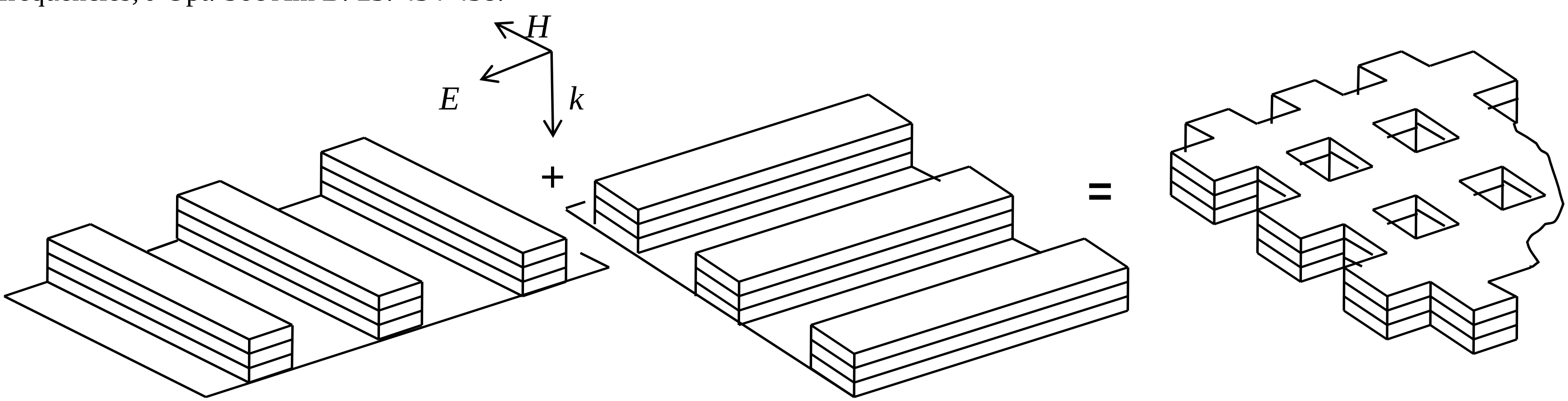
Inserting the negative permittivity background on magnetic resonator at optical frequency for optical DN-NIM meta-material

From the recipe of the previous discussion, results a ‘fishnet’ structure also called double grating. In this fishnet the pairs of broader metal strips provide negative permeability via asymmetric currents, whereas the pairs of narrower metal strips provide negative permittivity acts as diluted metal.

The fishnet can be viewed as a resonant magnetic structure combined with a non-resonant electric structure.

It is important to note that in a fishnet structure the pairs of narrower strips acts as-such off-resonant wires and at the wavelength where magnetic resonance occurs in the broader strips, they simply provide a back- -ground of negative permittivity.

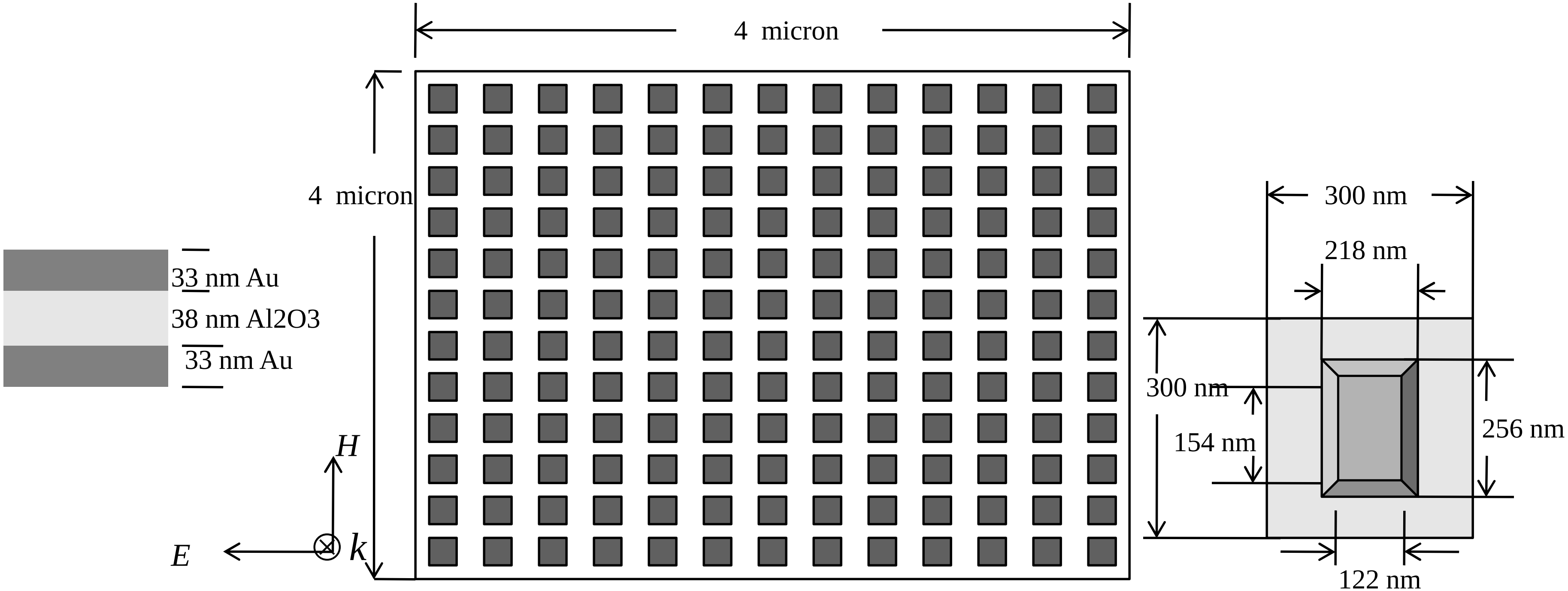
Zang S, Fan WJ, Osgood RO et al, (2006) Demonstration of metal dielectric negative indexed metamaterials with improved performance at Optical frequencies; J Opt. Soc Am B: 23: 434-438.



$\mu < 0$, (resonant) $\varepsilon < 0$, (non-resonant) $n < 0$, (resonant)

Magnetic strip Electric strip DN-NIM structure

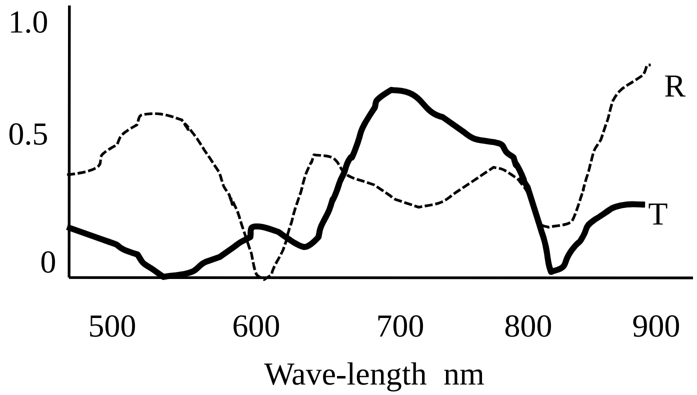
Double Negative-Negative Index (DN-NIM) structure with fishnet at visible frequency

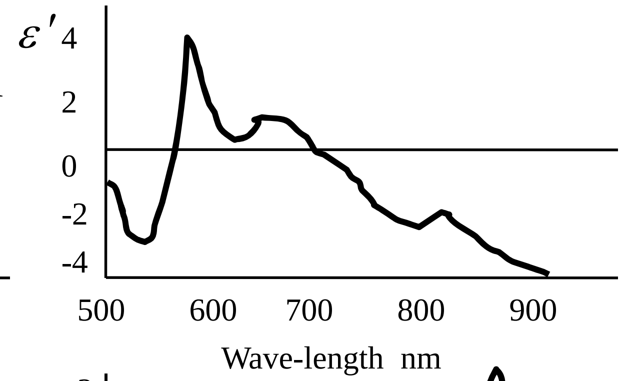


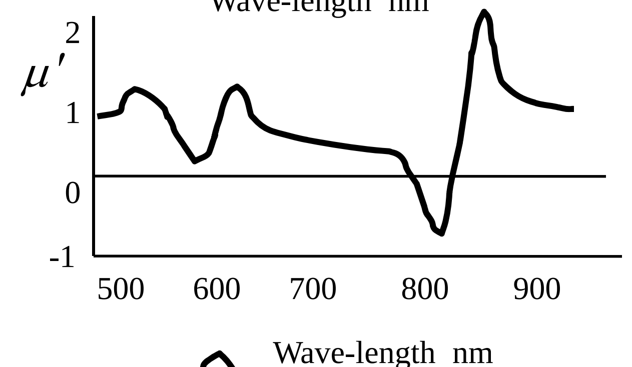
The fishnet structure is made of two 33 nm layers of perforated silver sheet, separated by a 38 nm layer of alumina. The lattice constant of the structure is 300 nm in both lateral directions . A 10 nm thick layer of alumina deposited on top and bottom of structure to protect silver from deterioration and to improve adhesion This is primary polarization of incident light which creates DN-NIM. The incident H field is polarized along the set of wider parallel strips, (these 'magnetic strips' of upper and lower layer are coupled at magnetic reso- -nance). In the 'optimized design' the magnetic resonance should be strong resulting in negative value for the effective permittivity. The other set of the parallel strip is aligned with incident E field (and show no diffrac- -tion, (since period is sub wavelength). These are 'electric strips', behave as diluted metal; as a result this structure behaves as negative ε' and μ' demonstrating DN-NIM behavior.

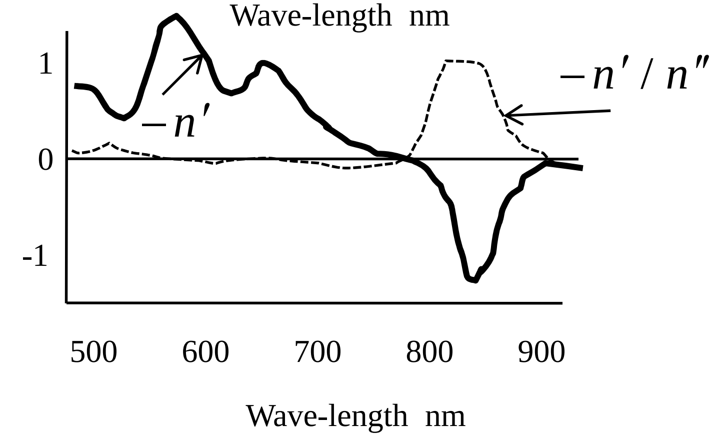
Double Negative-Negative Index (DN-NIM) structure with fishnet at visible frequency

Spectral analysis



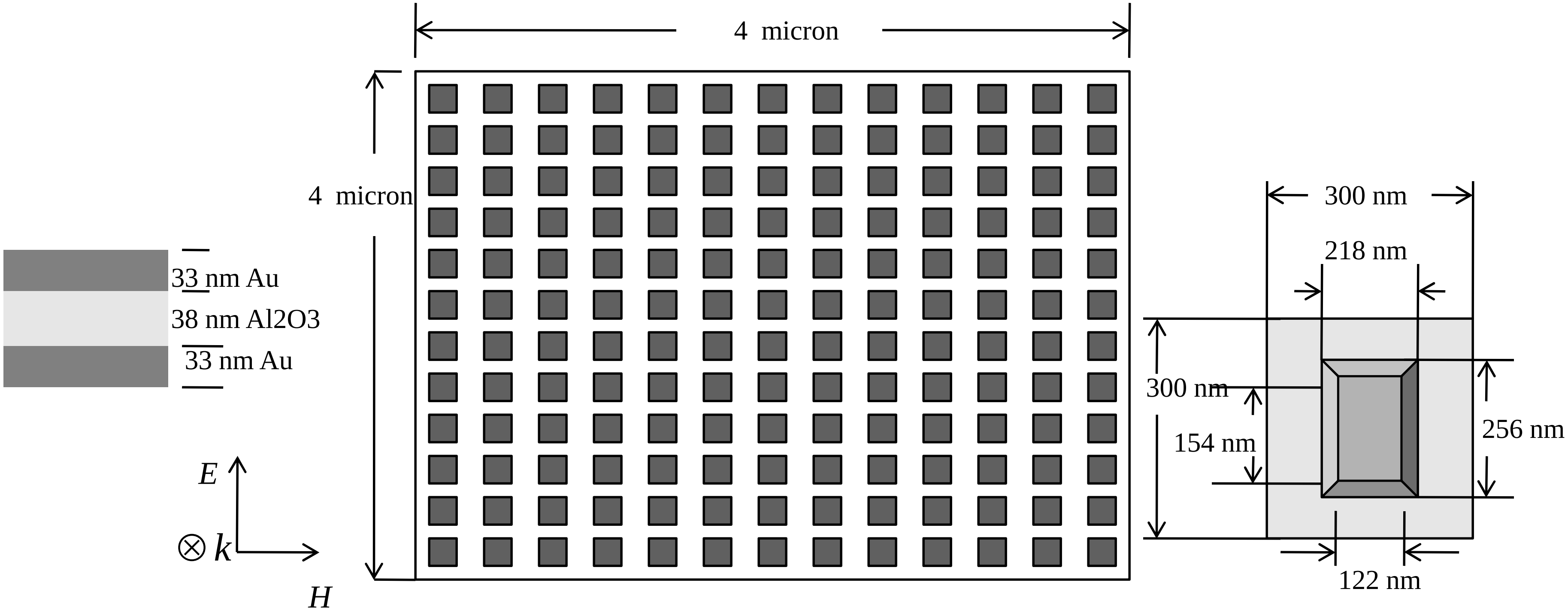






The spectral measurements are from 500 nm to 950 nm; shows an electric resonance at around 600 nm and a magnetic resonance close to 800 nm. The magnetic resonance is due to anti-symmetric currents in silver and loop closed by displacement currents at the edges. At 560 and 625 nm the electric displacements in both the strips are in phase, dominant is electric resonance. The electric and magnetic resonance is caused by the 'magnetic strips' –aligned with the H field the electric. The electric strips have no resonant behavior., just acting as diluted metal. The double negative-negative index is around 800 nm where we are having resonantly excited negative permeability and a background of negative permittivity.

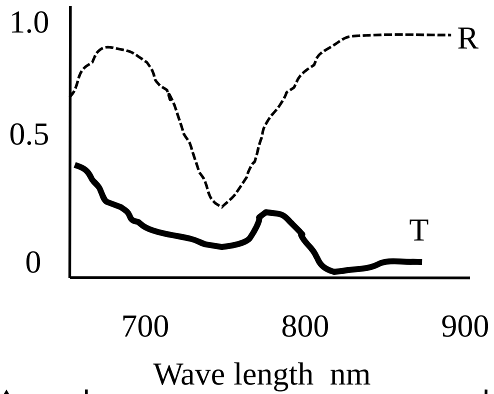
Single Negative-Negative Index (SN-NIM) structure with fishnet at visible frequency

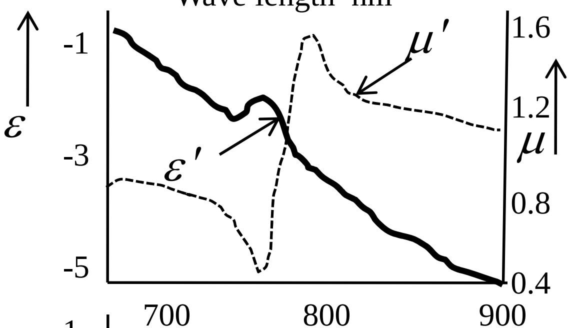


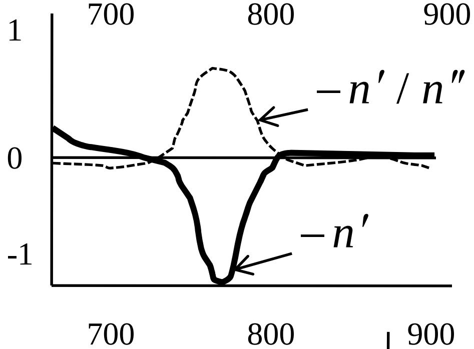
This is secondary linear polarization which is orthogonal to the previous case (primary polarization) of obtaining DN-NIM. In this case the fishnet structure changes its behavior from DN-NIM to SN-NIM. This polarization has incident electric field aligned along the wider parallel strip set, while the *H* field is aligned with the narrower strips. Here in this case a different set of parallel strip is coupled at magnetic resonance.

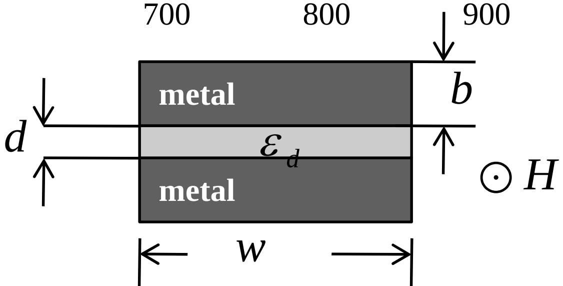
Single Negative-Negative Index (SN-NIM) structure with fishnet at visible frequency

Spectral analysis









In comparison with the DN-NIM regime, the magnetic strips in the Single-negative case are narrower. The resonance wave-length λ_m Is roughly proportional to the average width of the nano-strip, as a Result the magnetic resonance is blue shifted to 770 nm.

Recall that $\omega_m = N\pi\omega_p\sqrt{\dfrac{d\,b}{(w)^2\,\varepsilon_d}}$; thus $\lambda_m \propto w$. However The resonance is not sufficiently strong ; only positive values for effect--ve permeability are obtained. The magnetic strips in this secondary polarization are not optimized for a strong negative magnetic response. The electric strips in the secondary polarization are too wide resulting in non-optimal negative permittivity, which suppresses the magnetism further. Hence structure exhibits only SN-NIM with positive permea--bility

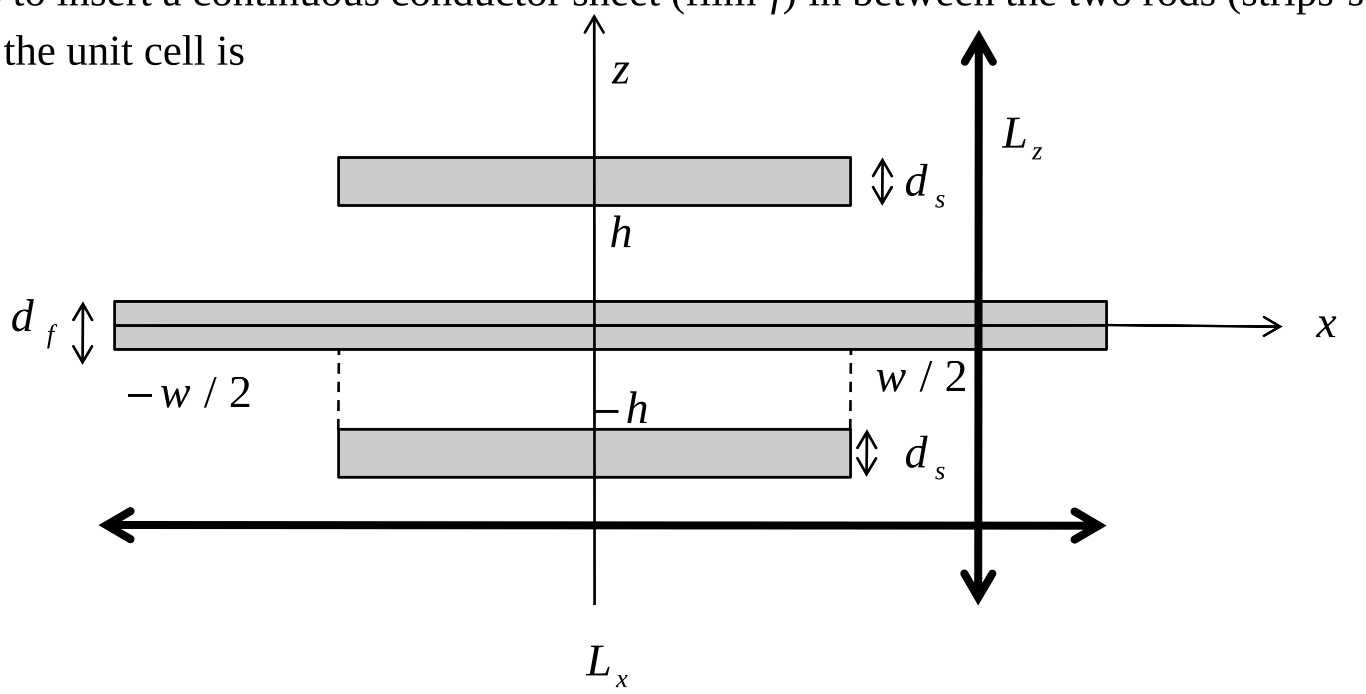
With these two polarization as DN-NIM, and SN-NIM this structure can show a dual-band magnetic response at linear polarization of 45 degrees !!

Obtaining DN-NIM via electric & magnetic resonance

In earlier discussed scheme we had magnetic resonance superimposed on the background of negative per--mittivity to get doubly negative structure thereby giving negative refractive index. The basic scheme of getting negative permeability is via paired nano-rods/strips etc, where the H field of the incident light gives anti-symmetric currents in both rods giving extra magnetic moment, plus the E field parallel to the wire rods generates symmetric currents in both wires giving electric resonance ; though at different frequency than magnetic resonance

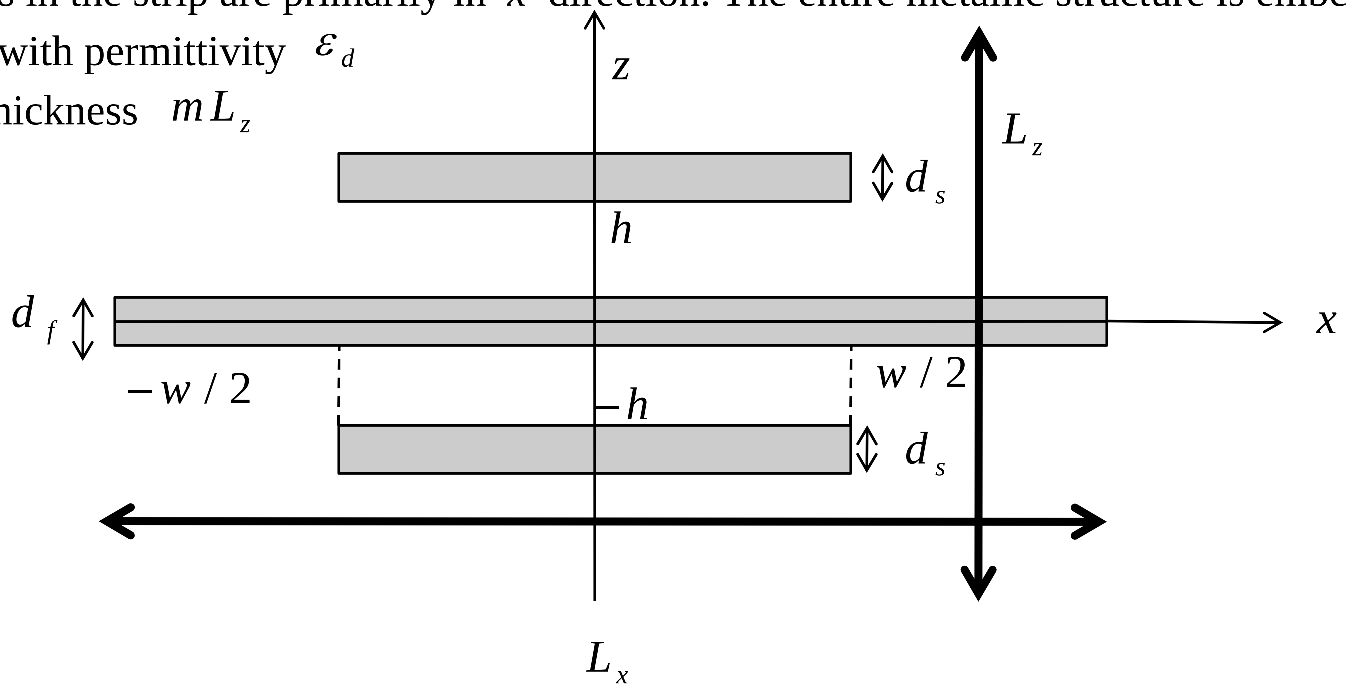
If we can tune that electric resonance frequency to overlap the magnetic resonance frequency, then we are likely to get DN-NIM.

The basic is to insert a continuous conductor sheet (film-*f*) in between the two rods (strips-*s*) separated by di-electric, the unit cell is

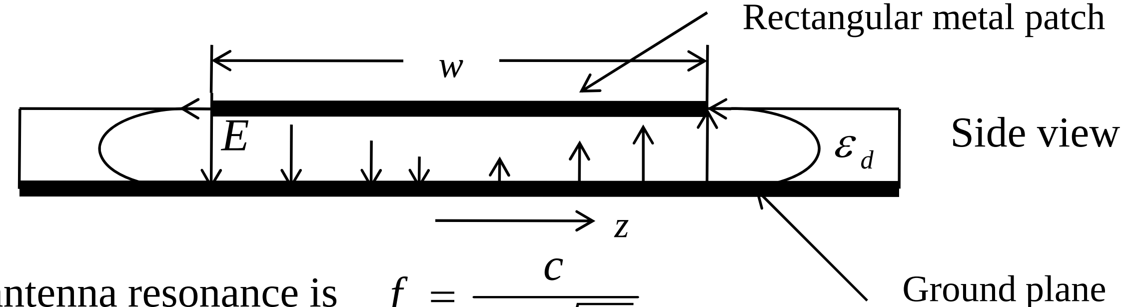


DN-NIM via electric & magnetic resonance via continuous metal film inserted between pair of nano-rods

The inhomogeneous nano composite consists of periodic array of unit cell, arranged periodically in *x*-*z* plane, with period L_x and L_z respectively. The structure contains m number of layers z direction and infinite number of unit cells in x direction. Every layer comprises an infinite metal film of thick--ness d_f, and an infinite array of metal strips of width w and thickness d_s. In z direction the strips are arranged in pairs symmetrically with w.r.t. unit cell's symmetry plane ($z = 0$). The distance between the bottom face of the top strip and the top face of the bottom strip is $2h$. The structure is uniform in *y*-direction. The strips and films are made with identical metal characterized by relative permittivity ε_m with $\mathrm{Re}\,\varepsilon_m < 0$ in optical frequency regime. All the sizes are smaller than free space wave-length λ The thickness of strip and film are smaller than w such that possible charge and current distribution variations in the strip are primarily in x direction. The entire metallic structure is embedded in dielectric material with permittivity ε_d of total thickness mL_z



Resonant cavity formation like patch antenna!



Side view of patch antenna

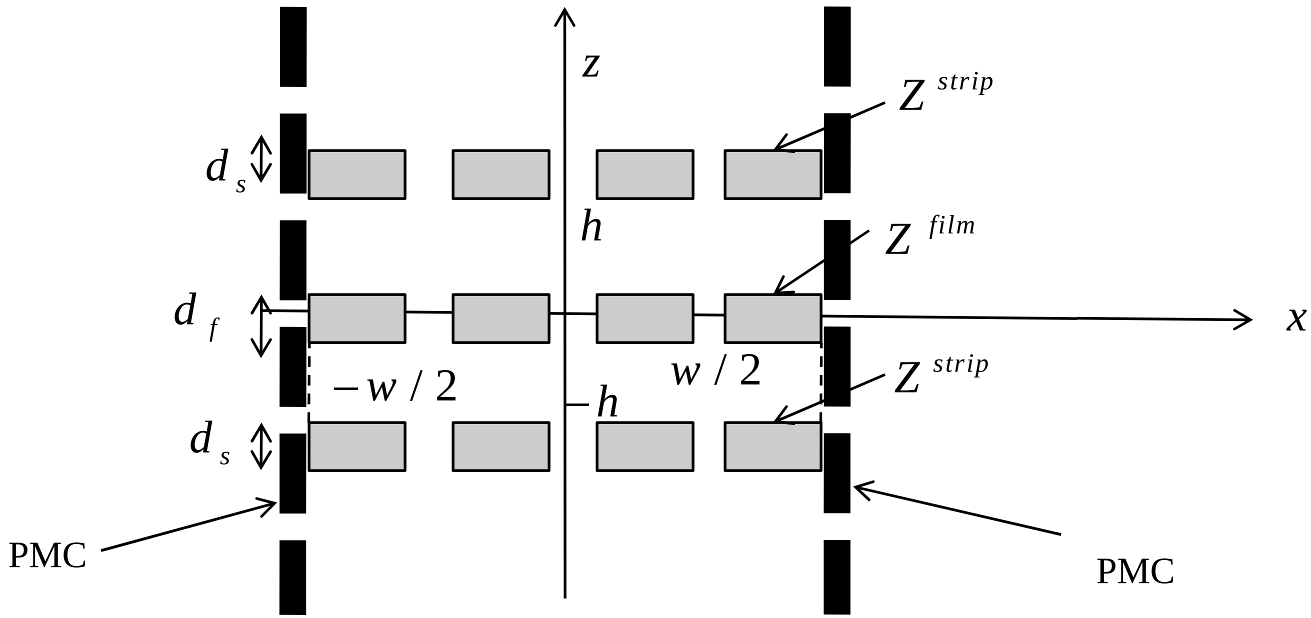
Patch antenna resonance is $f = \dfrac{c}{2w\sqrt{\varepsilon_d}}$

For the meta-material structure we have admittances as

$$Y_s^{strip} = ik_0(\varepsilon_m - 1)d_s \quad \text{at} \quad z = \pm h \qquad k_0 = 2\pi/\lambda$$

$$Y_s^{film} = ik_0(\varepsilon_m - 1)d_f \quad \text{at} \quad z = 0 \qquad \varepsilon_m \approx -f_p^2/f^2$$

$$Y_z = 2\pi f_{mag}\varepsilon_d / ck_z \qquad k_z = \sqrt{\varepsilon_d\left(2\pi f_{mag}/c\right)^2 - \left(N\pi/w\right)^2}$$



Resonant cavity with even magnetic field symmetry

No current flows in the film admittance sheet ($z = 0$), and anti-symmetric currents flow in the top sheet and bottom sheet, gives an effective magnetic moment. This resonance is magnetic response, the magnetic field behaves as

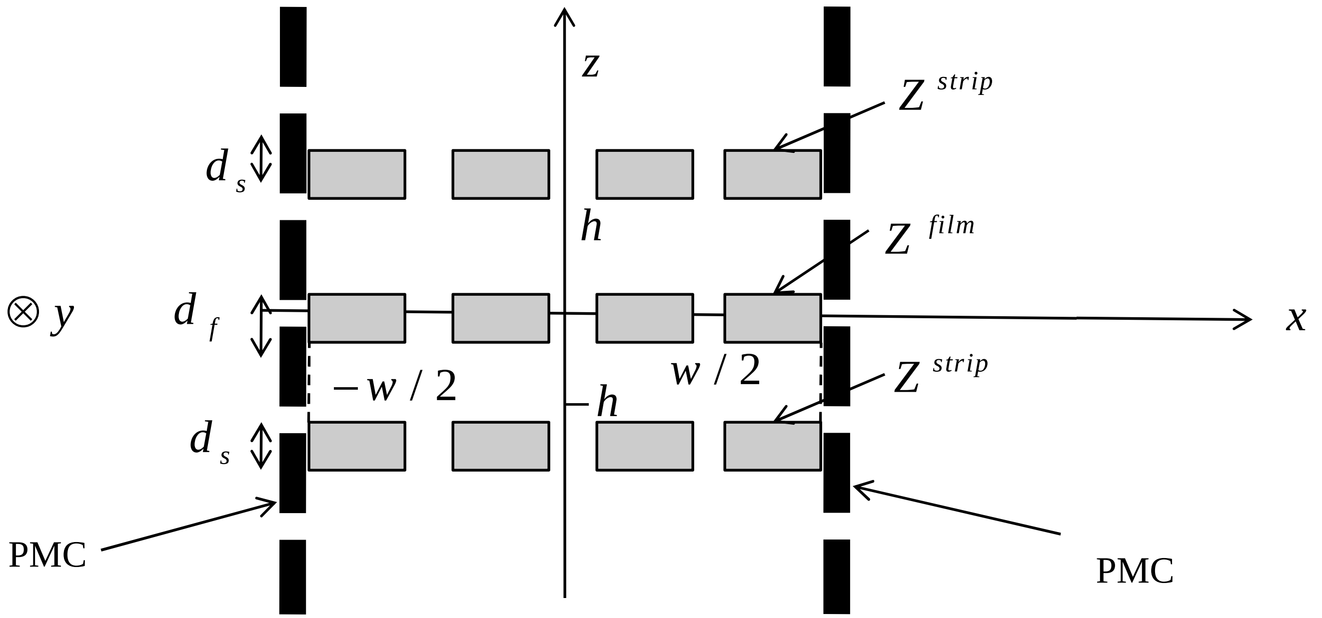
$$\vec{H} = \hat{y} A(z) \sin\left(N\pi \frac{(x - w/2)}{w} \right)$$

$A(z)$ is even function of z .

Dispersion relation is by field matching $Y_s^{strip} + Y_z (1 - i \cot k_z h) = 0$

Gives resonant frequency as

$$f_m \cong N\pi f_p \sqrt{\frac{d_s h}{\varepsilon_d w^2}}$$



Resonant cavity with odd magnetic field symmetry

This resonance is electric resonance, symmetric currents flow in the strips and also in film the magnetic field behaves as

$$\vec{H} = \hat{y} B(z) \sin\left(N\pi \frac{(x - w/2)}{w} \right)$$

$B(z)$ is odd function of z.

Dispersion relation is by field matching

$$Y_s^{strip} + Y_z + Y_z \left(\frac{Y_s^{film} + 2iY_z \tan k_z h}{2Y_z + iY_s^{film} \tan k_z h} \right) = 0$$

This above equation gives two electric resonance frequencies for each N. We have to choose such that

$$\mathrm{Re}\, f_e^{(1)} < \mathrm{Re}\, f_m < \mathrm{Re}\, f_e^{(2)}$$

Also with $\left|Y_s^{film}\right| \ll 1$, the inequality $\mathrm{Re}\, f_e^{(2)} \gg \mathrm{Re}\, f_m$ holds. As $\left|Y_s^{film}\right|$ increases, $\mathrm{Re}\, f_e^{(2)}$ decreases towards $\mathrm{Re}\, f_m$ and in limit $f_e^{(2)} \to f_m$ as $\left|Y_s^{film}\right| \to \infty$

Such behavior of $f_e^{(2)}$ allows the electric and magnetic resonances to overlap in the same frequency range by modifying d_f, the width of the film.

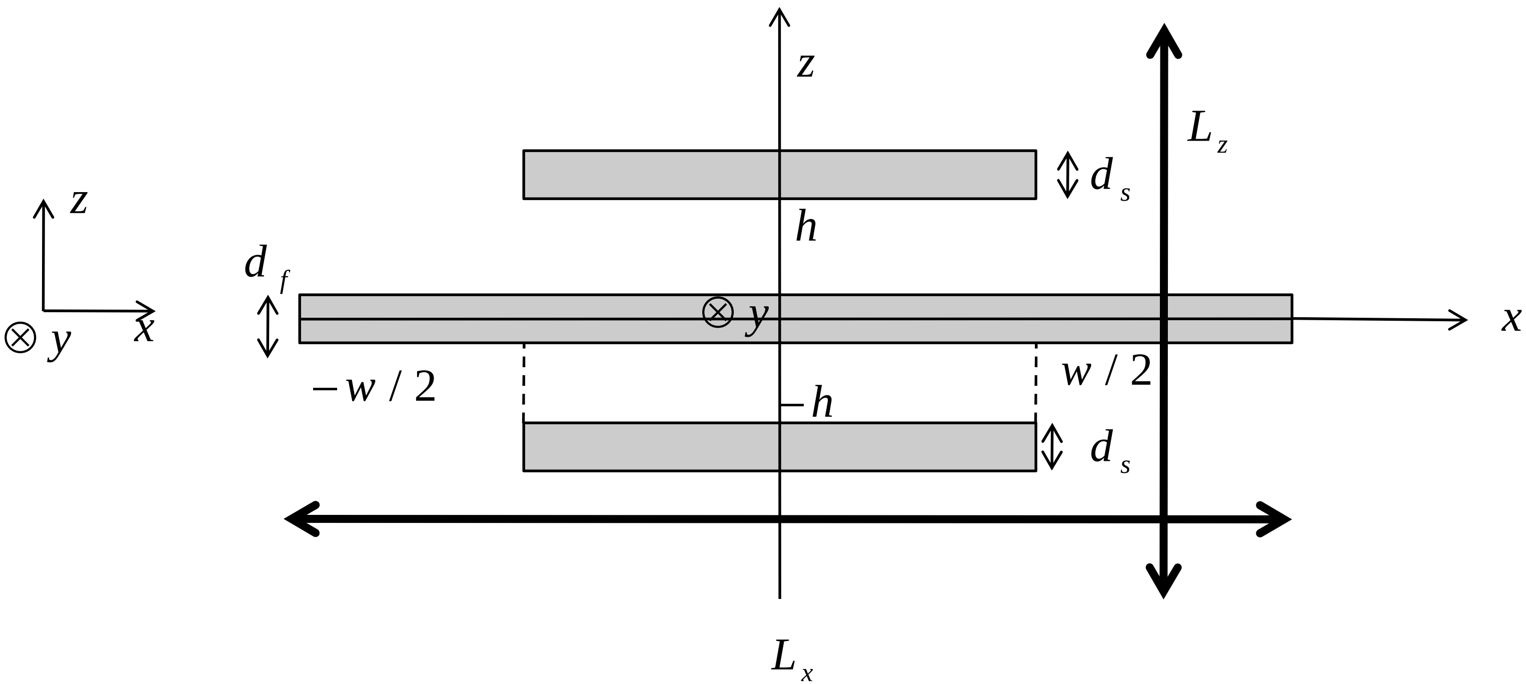
The magnetic and electric dipole moments

For the TM excitation the relevant 'tensor' components affecting the structure's e.m. properties are $\mu_{eff,yy}$, $\varepsilon_{eff,xx}$ & $\varepsilon_{eff,zz}$. By judicious choice of parameters simultaneous negative permittivity and negative permeability can be achieved for a finite frequency band-width. Interactions of magnetic and electric resonances with an external field can be described by magnetic and electric dipole moments

$$p_m(f) = \frac{p_{m0}}{f - f_m}\hat{y} \qquad p_e^{(i)}(f) = \frac{p_{e0}^{(i)}}{f - f_e^{(i)}}\hat{x} \qquad i = 1, 2$$

Where p_{m0}, $p_{e0}^{(i)}$ are constants determining the strength of the excitation

$$\otimes\, y \leftrightarrow p_m(f) \qquad x \rightarrow p_e(f)$$



The magnetic and electric dipole and effective mu and epsilon

Based on this understanding the effective parameters are

$$\mu_{eff,yy} = \mu_{0,eff,yy} - \frac{f_{p,m,yy}}{f - f_m}$$

$$\varepsilon_{eff,jj} = \varepsilon_{0,eff,jj} - \frac{f^{(1)}_{p,e,jj}}{f - f_e^{(1)}} - \frac{f^{(2)}_{p,e,jj}}{f - f_e^{(2)}} \qquad jj = xx \quad \text{or} \quad zz$$

The effective parameters comprise of non-resonant components $\mu_{0,eff,yy}$, $\varepsilon_{0,eff,jj}$ and the resonant components described by resonance frequencies f_m , $f_e^{(i)}$ together with constants $f_{p,m,yy}$, $f^{(i)}_{p,e.jj}$ determined by resonance excitation strength. The resonance excitation may depend on the frequency and the angle and may be different for $jj = xx$ and zz

These are analytical formulations and are difficult to determine these effective parameters.

There is experimental ways to extract these effective parameters from Transmittance and Reflectance experiments.

D R Smith, S. Schultz, et al, ‘ Determination of effective permittivity and permeability of meta-material from reflection and transmission coefficients’ Phys. Rev. B 65 195104 (2002)

Effective parameter extraction from scattering data

The effective permittivity and permeability and index of refraction can be obtained by calculating or measuring the structure of zero order scattering (reflection & transmission), coefficients and matching the appropriate effective parameter.

D R Smith, S. Schultz, et al, ‘ Determination of effective permittivity and permeability of meta-material from reflection and transmission coefficients’ Phys. Rev. B 65 195104 (2002)

For an oblique incidence the formula is , by taking zeroth order TM reflection and transmission coefficient and for an incident field at angle θ on a slab thickness L made of isotropic or uni-axially anisotropic material T and R are

$$\frac{1}{T} = \cos(k_0 n_{z,eff} L) + \frac{i}{2}\left(\frac{Z_{z,eff}}{\cos\theta} + \frac{\cos\theta}{Z_{z,eff}}\right)\sin(k_0 n_{z,eff} L) \qquad \frac{R}{T} = \frac{i}{2}\left(\frac{Z_{z,eff}}{\cos\theta} + \frac{\cos\theta}{Z_{z,eff}}\right)\sin(k_0 n_{z,eff} L)$$

Where $n_{z,eff}$ is the effective refractive index for the field propagating in z direction, the $Z_{z,eff}$ is the corresponding normalized impedance defined as the ratio between tangential components of electric and magnetic fields in the $x-y$ plane. From above

$$n_{z,eff} = \left(\frac{1}{k_0 L}\right)\left(\cos^{-1}\left(\frac{1-(R^2-T^2)}{2T}\right) + 2\pi l\right) \qquad Z_{z,eff} = \cos\theta\left(\sqrt{\frac{(1+R^2)-T^2}{(1-R^2)-T^2}}\right)$$

Where l is appropriately chosen integer. In contrast assuming the effective medium is anisotropic, and recalling that the field is TM polarized, we also have

$$n_{z,eff} = \sqrt{\mu_{eff,yy}\varepsilon_{eff,xx} - \frac{\sin^2\theta}{\varepsilon_{eff,zz}}} \qquad Z_{z,eff} = \frac{n_{z,eff}}{\varepsilon_{eff,xx}}$$

About the extraction expressions usage

Here assume that $\varepsilon_{eff,zz}$ is positive constant that is chosen as ε_d . This is because the metal films and strips are all arranged along $(x-y)$ plane so that the field components along z axis are weakly affected by the presented resonance so that excitation constant $f^{(i)}_{p,e,zz}$ are weak.

$$\varepsilon_{eff,zz} = \varepsilon_d$$

In addition this assumption is based on the fact that $\varepsilon_{eff,zz}$ extracted by assuming static fields is nearly constant in range of interest. Based on these we write

$$\varepsilon_{eff,xx} = \frac{n_{z,eff}}{Z_{z,eff}} \qquad \mu_{eff,yy} = n_{z,eff} Z_{z,eff} + \frac{\sin^2\theta}{\varepsilon_{eff,zz}}$$

Where

$$n_{z,eff} = \left(\frac{1}{k_0 L}\right)\left(\cos^{-1}\left(\frac{1-(R^2-T^2)}{2T}\right) + 2\pi l\right) \qquad Z_{z,eff} = \cos\theta\left(\sqrt{\frac{(1+R^2)-T^2}{(1-R^2)-T^2}}\right)$$

and $\varepsilon_{eff,zz} = \varepsilon_d$

$$\mu_{eff} = \mu_{eff,yy}; \quad \varepsilon_{eff} = \varepsilon_{eff,xx}; \quad n_{eff} = \sqrt{\mu_{eff}\varepsilon_{eff}}$$

For embedding material SiO_2 $\varepsilon_d = 2.25$, for gold as metal

$$\varepsilon_m = 1 - \frac{f_p^2}{f(f - i\Gamma)} \qquad f_p = 1.32\times 10^4/(2\pi)\mathrm{THz} \qquad \Gamma = 1.2\times 10^2/(2\pi)\mathrm{THz}$$

Structural parameters

$$L_x = 100\text{nm}, \quad w = 50\text{nm}, \quad d_s = 15\text{nm}$$

For these values of the strip-symmetry plane separation and metal-film thickness are

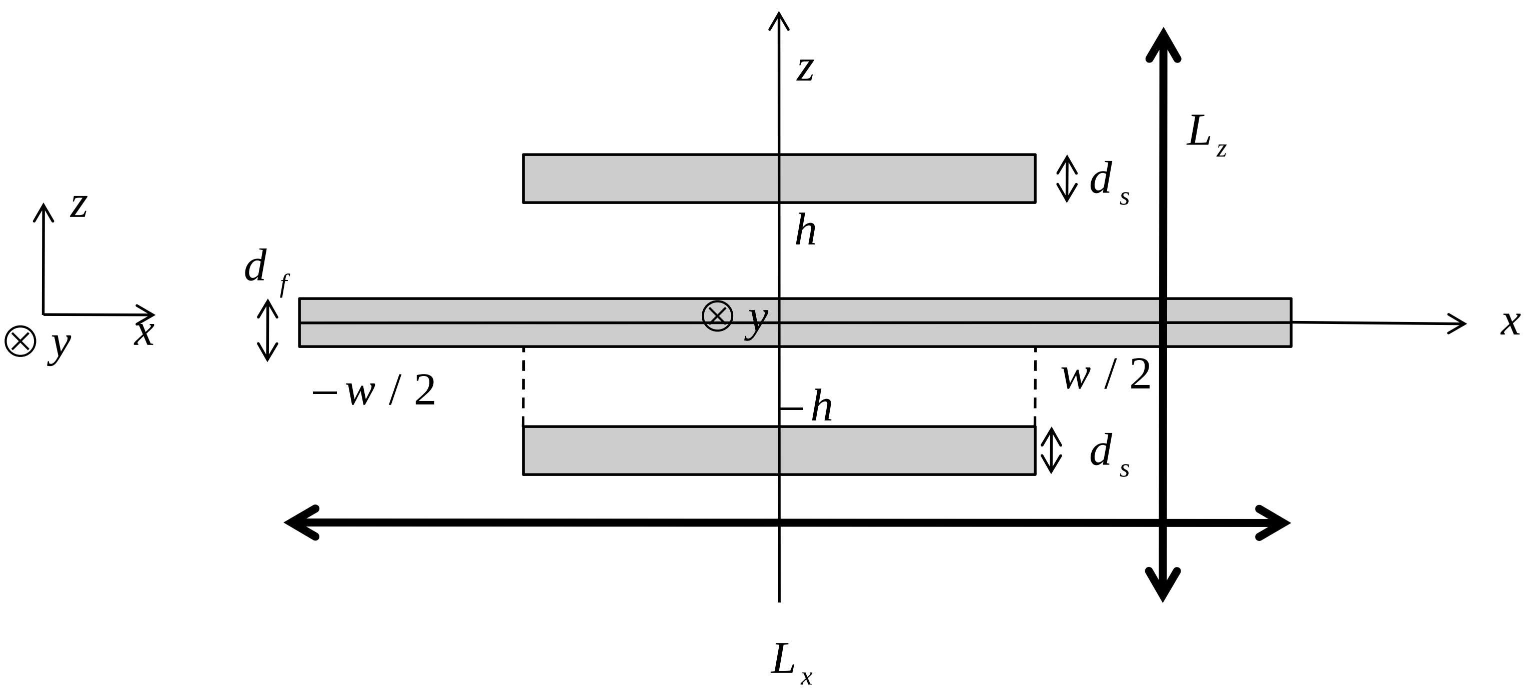
$$h = 7\text{nm}, \quad d_f = 0, \quad L_z = 44.5\text{nm}$$

$$h = 10.25\text{nm}, \quad d_f = 6.5\text{nm}, \quad L_z = 50.5\text{nm}$$

$$h = 11.25\text{nm}, \quad d_f = 8.5\text{nm}, \quad L_z = 52.5\text{nm}$$

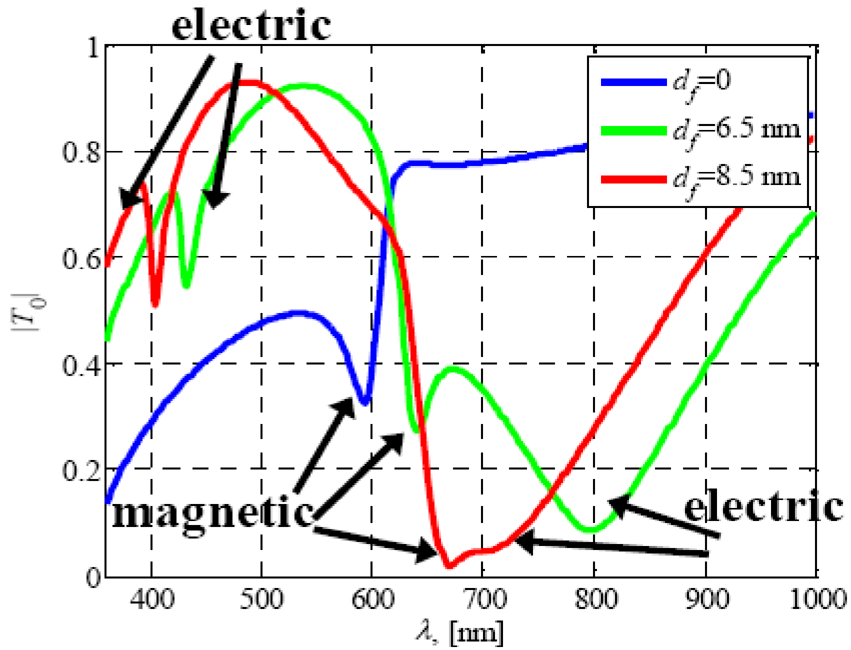
Vitaliy Lomakin, Y Fainman, Y Urzhumov, G. Shvet, 'Doubly negative meta-material in the near IR and visible regimes based on thin film nano composites', Vol. 14, No. 23 Optics Express 11164 (2006)

G Shvets & Y Urzhumov, ' Negative index meta-material based on two dimensional metallic structures', J. Opt. A 8 S122 (2006)



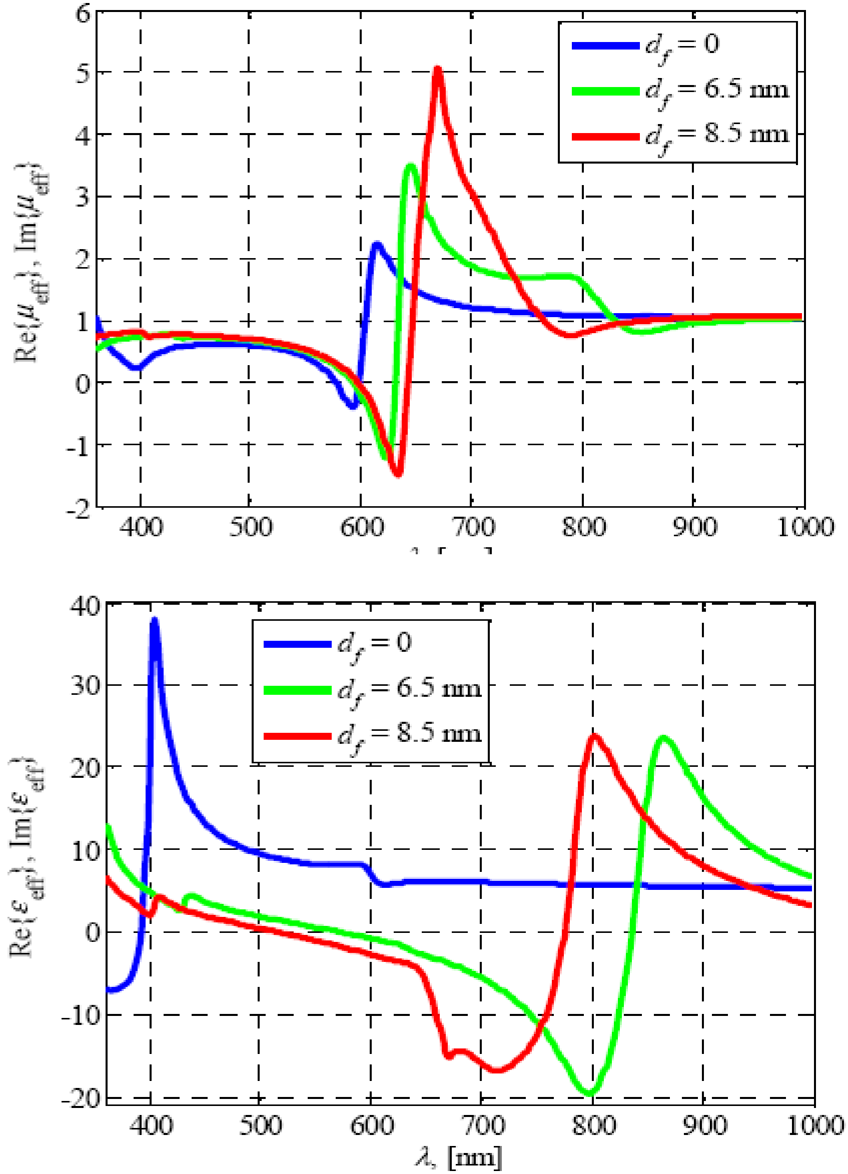
Magnitude of zeroth order Transmission coefficient

Vitaliy Lomakin, Y Fainman, Y Urzhumov, G. Shvet, ‘Doubly negative meta-material in the near IR and visible regimes based on thin film nano composites’, Vol. 14, No. 23 Optics Express 11164 (2006)



This shows magnitudes of the structure’s normal transmission coefficients for a single layer $m = 1$
In the absence of the middle metal film and in presence of the film. In the absence of the middle film the two non overlapping electric and magnetic resonance are obtained for wave length 350 nm, and 600 nm respectively. In the presence of the middle film for smaller thickness three separate dips are observed at 435 nm, 640 nm and 800 nm corresponding to electric magnetic and electric resonance respectively. As the film thickness increases the two longer wavelength (magnetic and electric) resonances approach and they almost merge at 680 nm at film thickness 8.5 nm. The longer wavelength resonance for $d_f = 0$ and middle resonance for $d_f \neq 0$ corresponds to bands of negative $\mathrm{Re}\,\mu_{eff}$. The longest wavelength resonances for $d_f \neq 0$ corresponds to band of negative $\mathrm{Re}\,\varepsilon_{eff}$

Effective permeability and permittivity



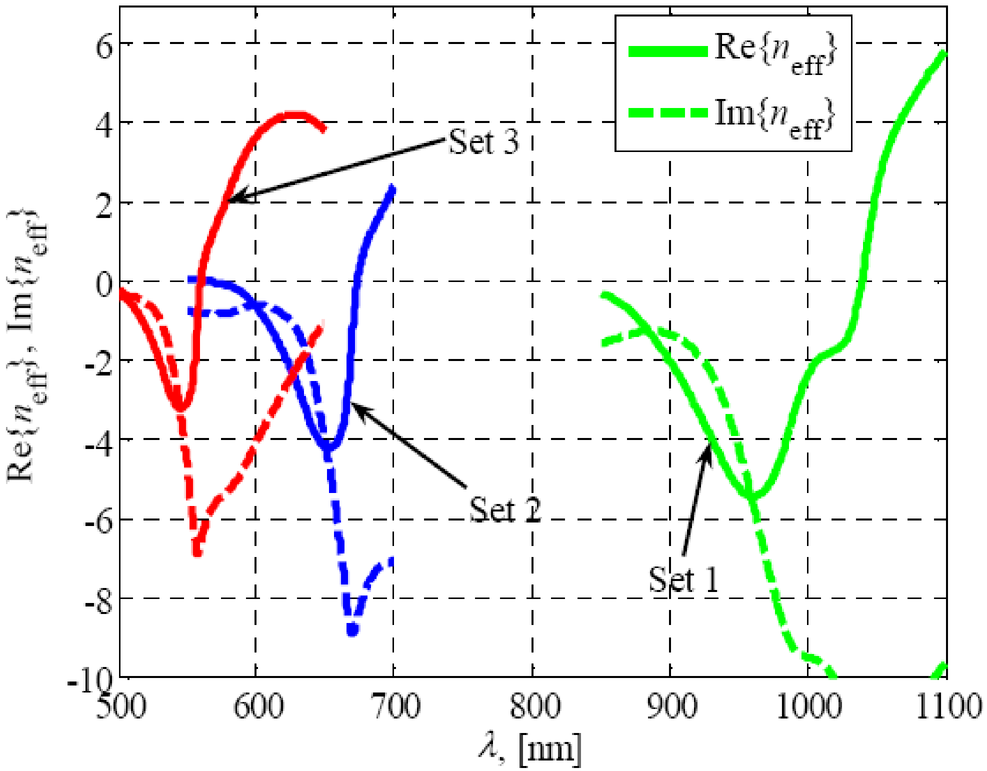
Vitaliy Lomakin, Y Fainman, Y Urzhumov, G. Shvet, 'Doubly negative meta-material in the near IR and visible regimes based on thin film nano composites', Vol. 14, No. 23 Optics Express 11164 (2006)

Observations

The observation is it is evident that no simultaneous bands of negative $\mathrm{Re}\{\mu_{eff}\}$ and $\mathrm{Re}\{\varepsilon_{eff}\}$ are Obtained when the middle slab is absent. Introducing the middle layer of thin metal film at $z = 0$ causes Simultaneous overlap bands of negative $\mathrm{Re}\,\mu_{eff}$ and $\mathrm{Re}\,\varepsilon_{eff}$ as required to make DNM. This is because $\mathrm{Re}\,\varepsilon_{eff}$ tends to be negative between two electric resonances.

Vitaliy Lomakin, Y Fainman, Y Urzhumov, G. Shvet, ‘Doubly negative meta-material in the near IR and visible regimes based on thin film nano composites’, Vol. 14, No. 23 Optics Express 11164 (2006)

Effective refractive index



Set 1: $L_x = 150\,\mathrm{nm}$, $L_z = 52\,\mathrm{nm}$, $w = 90\,\mathrm{nm}$, $d_s = 15\,\mathrm{nm}$, $d_f = 8\,\mathrm{nm}$, $h = 11\,\mathrm{nm}$;

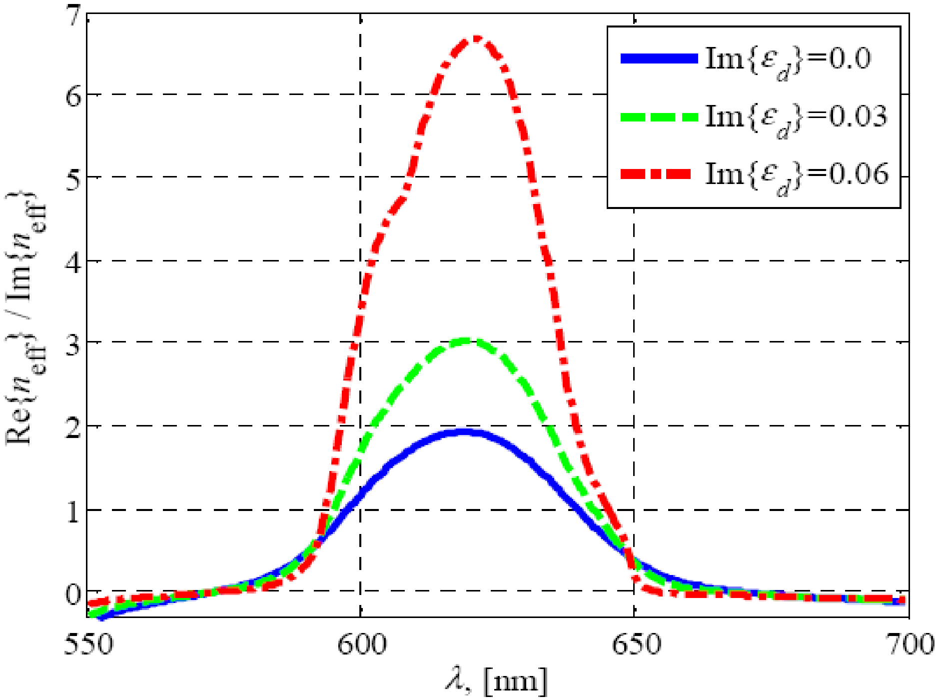
Set 2: $L_x = 100\,\mathrm{nm}$, $L_z = 52.5\,\mathrm{nm}$, $w = 50\,\mathrm{nm}$, $d_s = 15\,\mathrm{nm}$, $d_f = 8.5\,\mathrm{nm}$, $h = 11.25\,\mathrm{nm}$;

Set 3: $L_x = 100\,\mathrm{nm}$, $L_z = 57\,\mathrm{nm}$, $w = 40\,\mathrm{nm}$, $d_s = 15\,\mathrm{nm}$, $d_f = 10\,\mathrm{nm}$, $h = 13.5\,\mathrm{nm}$

It is evident that the structure can be tuned to operate in range from near-IR to entire visible spectra, here the wavelength / periodicity ratio being around 7 i.e. in deep sub wavelength regime.

Vitaliy Lomakin, Y Fainman, Y Urzhumov, G. Shvet, 'Doubly negative meta-material in the near IR and visible regimes based on thin film nano composites', Vol. 14, No. 23 Optics Express 11164 (2006)

Losses in meta-material DNM and 'active-meta-material'



The ratio $\text{Re}\, n_{eff} / \text{Im}\, n_{eff}$ indicates losses in the system as function of gain $\text{Im}\, \varepsilon_d$ in the di-electric layer for a single DNM layer. The structural parameters are $L_x = 100\text{nm}$, $L_z = 51.5\text{nm}$ $w = 50\text{nm}$ $d_s = 15\text{nm}$ $d_f = 7.5\text{nm}$ $h = 10.75\text{nm}$

It is evident that the loss is not high without any gain and it further improves by increasing the gain by use of active substrate of semiconductor polymer or laser dyes .

Vitaliy Lomakin, Y Fainman, Y Urzhumov, G. Shvet, 'Doubly negative meta-material in the near IR and visible regimes based on thin film nano composites', Vol. 14, No. 23 Optics Express 11164 (2006)

N M Lawandy, 'Localized surface plasmon singularities in amplifying media', Appl. Phys. Lett. 85, 5040 (2004)

F Hide, B J Schwartz et al, 'Conjugated polymers as solid state laser materials', Synth. Met. 91, 35 (1997)

Conclusions

We have seen that there is possibility of having plasmonic meta-material to have negative refractive index material (NIM). The two methods we discussed to get optical NIM are to have magnetic meta atoms with magnetic plasmonic resonances embedded in epsilon negative background, and second way is to have both electric and magnetic resonances overlapped at a particular frequency band.

These two techniques discussed here are well with in fabrication methods of nano-fabrication facility available.

The Double Negative and Single Negative indexed Material (DN-NIM and SN-NIM) composites have applications in optical signal manipulation & control, which are not perhaps possible with natural materials

End of part-7