Left Handed Maxwell Systems In Optical Regime

PART-6

Theory of Magnetic Plasmon Resonance at optical frequencies

Shantanu Das
RR&PS
Reactor Control Division, B.A.R.C. Mumbai-400085
shantanu@barc.gov.in

Few salient points

For micro-wave (uW) artificial magnetic meta-atoms/particles providing $\mu < 0$ are the split ring resonators (SRR). In the uW part of e.m. spectrum metals can be considered as perfect conductors as the skin depth at those frequencies (order of micron) is very -very small than the metallic feature (size of order of mm). There the strong magnetic response is achieved by operating in the vicinity of the $L\,C$ resonance. The same technique of obtaining $\mu < 0$ using SRR extended to mid IR, by scaling down the dimensions.

In the uW region and to a lesser degree in the mid-IR part, metals can be approximated as perfect conductors as because skin depth is smaller than size. Therefore the frequency of $L\,C$ resonance is determined entirely by geometry of SRR and size, but not by the e.m. property of metal. In accordance the SRR response is (resonantly) enhanced at some particular ratio of the radiation wavelength λ, and structural size a. This we had referred to the $L\,C$ resonance, of perfectly conducting metal as Geometric LC (GLC) resonance.

The situation drastically changes in the optical part of the spectrum, where the thin (sub-wave length) metal components behave very differently, when the sizes become less than skin depth, in the optical and near-IR region. Here for instance, 'electrical' surface plasmon resonance (SPR) occurs at optical and near-IR, due to collective oscillations of free electrons of metal structure (the surface here extends to bulk dimmension)

Plasmonic nature of the e.m. response in metals at these frequencies is the main reason why the original GLC resonance of uW to mid-IR is not extendable at higher frequencies; along with the saturation effect due to 'kinetic-inductance' of the free electrons as we studied in earlier parts

Importance of plasmonic effect and Magnetic Plasmonic Resonance (MPR)

Plasmonic effects must be correctly accounted for to design meta-material at optical regime.

Magnetic Plamonic Resonance (MPR) frequency ω_m can be made independent of the absolute characteristic size a and $\lambda / a \equiv 2\pi c / \omega a$

The only defining factor are the plasmonic permittivity of metal $\varepsilon_m(\omega)$, not the conductivity σ

Such nano-structures acts as ‘nano-antenna’ by concentrating the large electric and magnetic energies on the nano-meter scale at optical frequency.

Metal nano-wires pair support electrical SPR and magnetic plasmonic resonance (MPR) together; thus we find in the transmission spectra experiments stop bands at ω_m ; or (λ_m) and ω_e ; or (λ_e) , due to asymmetric current and symmetric current distributions respectively.

The magnetic response is characterized by magnetic polarizability α_M , the real part of α_M shall change the sign near the resonance and may becomes negative for $\omega > \omega_m$ as required for negative refractive indexed meta-material.

The mechanism is that the electrostatic resonances must replace (or strongly modify) the GLC resonances in the optical/mid IR region if a strong magnetic response is desired.

Approaches for making optical meta-materials of for DNG applications

These optical Doubly Negative Material (DNG or DNM) can be classified as two types.

The first one is incorporates arrays of plasmonic rods or spheres of sub wavelength size to construct two-three dimensional DNM. The operation of these structures is based on the existence of resonances supported by sub wavelength particles when the frequency of the operation approaches the plasma frequency of the particle in the ambient environment. Unfortunately due to this property the DNM will not operate in spectral ranges extended to near IR. Moreover these designs may lead to excessive high losses and cannot be fabricated by standard nano-fabrication technique.

G Shvets, Y A Urzhumov, Engineering the electromagnetic properties of the periodic nano structures using electrostatic resonances; Phys. Rev. Lett. 93 243902- 243901 (2004)

A Alu, A Salandrino, N Enghta, Negative effective permeability and Left Handed Material at Optical frequencies, Optic Express 14 1557 (2006)

The second type of optical DNM represents several variations of pairs of patterned thin metal films including arrays of paired wires, paired strips etc. The operation of these structure is based on the existence of plasmonic resonances of magnetic and electric type supported by the cavities formed between the pair of particles. These structures can be fabricated by nano-fabrication methods.

G Shevet, Y Urzhumov, Negative indexed meta-material based on two dimmensional metallic structures, J Opt. A8, S122 (2006).

Z. Shuang, F. Wenjun, B K Minhas, et al, Mid-infrared resonant magnetic nano-structures exhibiting a negative permeability, Phys. Rev. Lett. 94 037402-037401, (2005).

V M . Shalaev, et al, Negative index of refraction in optical meta-material, Opt. Lett. 30 3356-3358 (2005)

The electrostatic resonance inducing magnetic properties!!

Strong electrostatic resonances of regularly shaped nano-particles, (nano-rods, nano-spheres) occur for $-2 < \varepsilon'_m < -1$.

From effective medium approximation (MGT) we have for sphere of ε'_m embedded in host of ε'_h , the effective permittivity with inclusion filling f is ε'_{eff}

$$\varepsilon'_{eff} = \varepsilon'_h + 3 f \varepsilon'_h \frac{\varepsilon'_m - \varepsilon'_h}{\varepsilon'_m + 2\varepsilon'_h}$$

The resonance is occurring at $\varepsilon'_m = -2\varepsilon'_h$ which represents surface plasmon resonance of an isolated metal spherical inclusion embedded in the host dielectric. With $\varepsilon'_h = 1$, we have $\varepsilon'_m = -2$ for spherical inclusion . By replacing 2 above by $(d-1)$; the d is dimension of inclusion we had generalized MGT and EMT formula, and for rods $d = 2$ we have resonance as $\varepsilon'_m = -\varepsilon'_h$

Drude metal is $\varepsilon_m{}' = 1 - \omega_p^2 / \omega^2 = -2\varepsilon_h$ gives electrostatic resonance frequency $\omega_{sp} = \omega_p / \sqrt{1+2\varepsilon_h}$

$\omega_{sp} = 0.57\omega_p$; for $\varepsilon_h = 1$ is surface plasmon resonance frequency

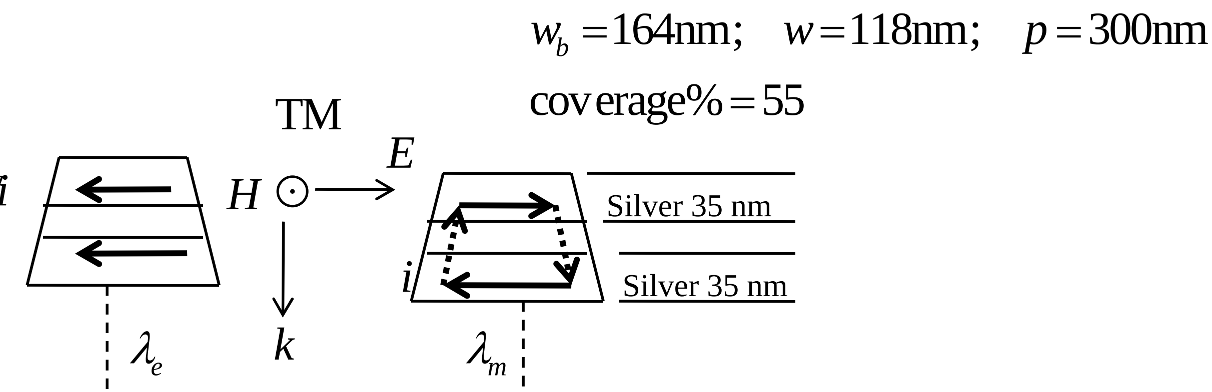
The resistive damping is defined by $\varepsilon''_m / \varepsilon'_m$ of metal is fairly strong for those frequency corresponding to $|\varepsilon'_m| \sim 1$. However, $\varepsilon''_m / \varepsilon'_m$ is known to decrease for $|\varepsilon_m| \gg 1$. Thus there is considerable incentive to design nano-structures exhibiting resonances for $\varepsilon'_m \ll -1$

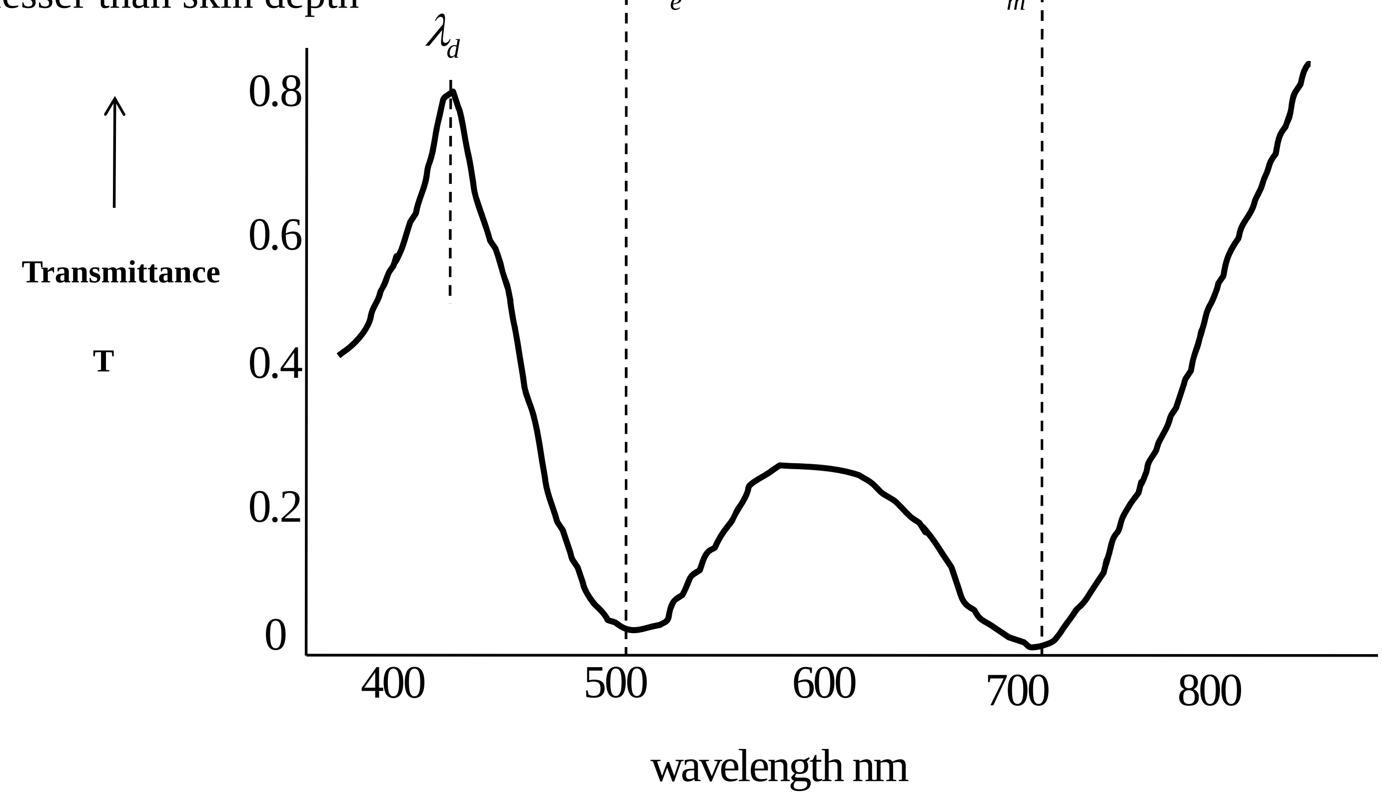
The idea is of using electrostatic resonance for inducing optical magnetism and thus to get magnetic plasmon resonances is new, these resonances in periodic plasmonic nano-structures employed to induce magnetic properties due to close proximity of adjacent nano-wires.

Sarychev A K, Shalaev, SPIE Proceedings 2004, 128, 5508.

The T spectra of coupled nano-strips showing two resonances

Skin depth at optical frequency is of order 50 nm, here the conductor thickness is 35 nm. lesser than skin depth





Yuan HK, Chettiar UK et al; A negative permeability material at red light; Opt Express 15: 1076-1083

Electrical conductivity from Drude-a revision of concepts

Typical charged particle motion in electric field is $m(\ddot{x} + \Gamma\dot{x} + \omega_0^2 x) = qE$

Electric current density is $J = Nqv = Nq\dot{x}$, Fourier transforming we get $\tilde{J} = Nq(-i\omega)\tilde{x}$

Recall $\tilde{x}(\omega) = \dfrac{q}{m}\dfrac{\tilde{E}(\omega)}{(\omega_0^2 - \omega^2 - i\omega\Gamma)}$ got from the motion equation's Fourier

Ohm's law is $\tilde{J}(\omega) = \sigma(\omega)\tilde{E}(\omega)$

Therefore conductivity expression is:

$$\sigma(\omega) = \frac{Nq^2}{m}\frac{-i\omega}{\omega_0^2 - \omega^2 - i\omega\Gamma}$$

A material with $\sigma(0) = 0$ is an electrical insulator, it cannot transport dc currents

A material with $\sigma(0) > 0$ is an electrical conductor

In metal, free charged particles (electrons) we set $\omega_0 = 0$, giving $\sigma(\omega) = \dfrac{Nq^2}{m}\dfrac{1}{\Gamma - i\omega}$

$$\frac{\sigma(\omega)}{\sigma(0)} = \frac{1}{1 - i\omega/\Gamma} \qquad \sigma(0) = \frac{Nq^2}{m\Gamma} \qquad \sigma(0) > 0$$

Note the dc conductivity is always positive.

Conduction currents in metals-a revision of concepts

For a general case we find that $\tilde{J} = \left[\dfrac{\sigma(0)}{1 - i(\omega / \Gamma)} \right] \tilde{E} = \sigma(\omega) \tilde{E}$

For $\omega = 0$, we have static conductivity or dc conductivity as $\sigma_0 = \sigma(0) = Nq^2 / m\Gamma$

For a very low frequency $(\omega / \Gamma) \ll 1$, the dynamic conductivity $\sigma(\omega)$ is purely real and electrons follow the electric field *E*.

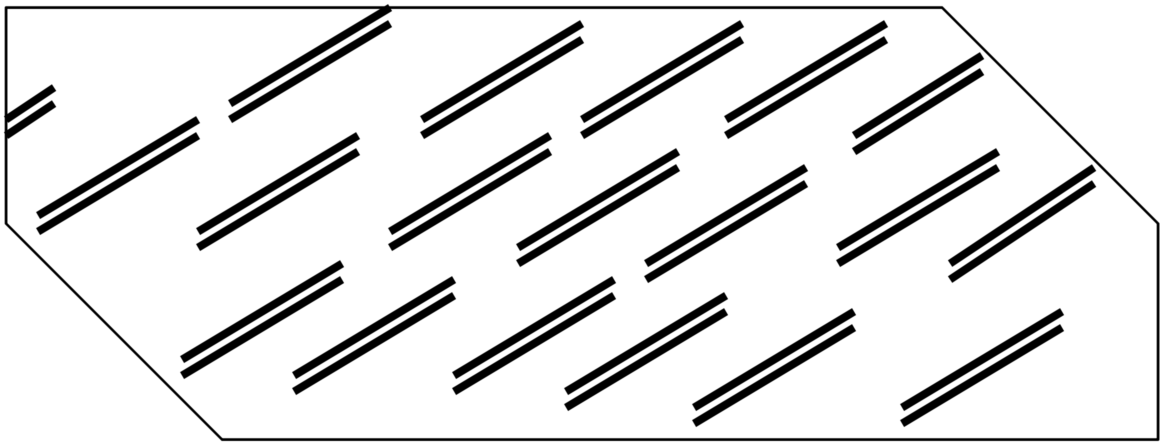
As the applied frequency is increased, the inertia of the electrons introduce a phase-lag in the electrons response to the field, and dynamic conductivity is complex.

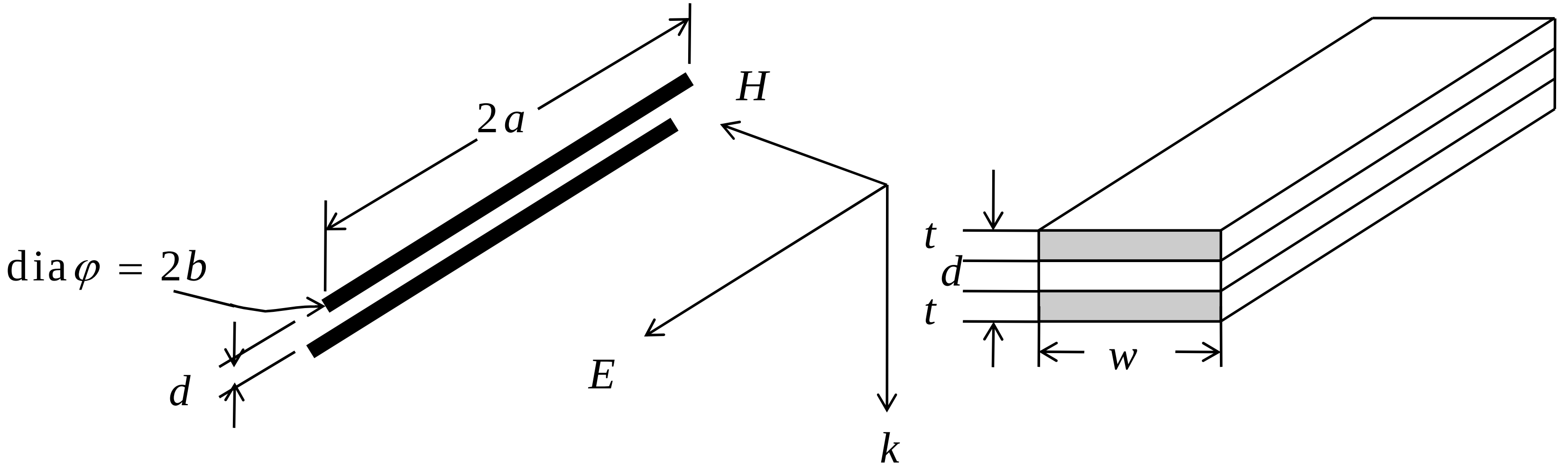
At very high frequency $(\omega / \Gamma) \gg 1$ the dynamic conductivity is purely imaginary and electron oscillations are 90 degree out of phase with the applied field *E*-an inductive behavior.

So at optical frequency we shall consider metal as inductive admittance $Y = ik_0(\varepsilon_m - 1)b, \quad k_0 = 2\pi / \lambda$ where b is the thickness of the conductor sheet and $\varepsilon_m \sim \omega_p^2 / \omega^2$; ω_p is plasma frequency

The static conductivity or dc conductivity of copper is $\sigma_0 = 5.76 \times 10^7 \, \Omega^{-1} \mathrm{m}^{-1}$

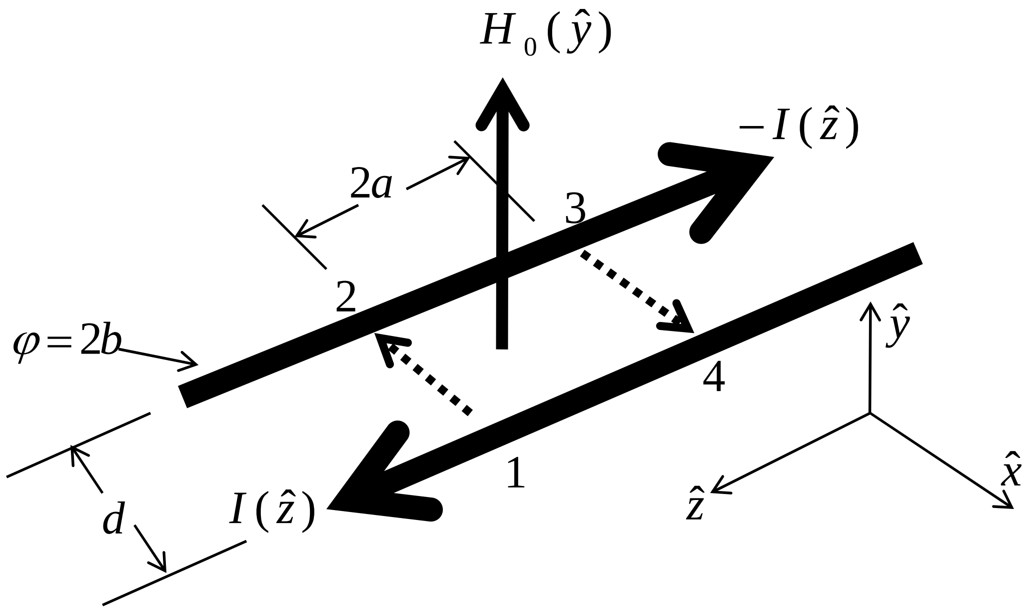
Composite consists of pair of metallic nanorods for magnetic response at optical frequencies





For ease of calculations we are assuming circular wires of diameter *b*, actually it is strip of width *w* and thickness *t* separated by distance *d*.

Analytical Theory of Magnetic Plasmon Resonance (MPR) in pair of nano wires



Current in two parallel metal nano-wires excited by external magnetic field the displacement currents closing the circuit –generates a magnetic dipole; due to circular current flowing in 1-2-3-4; which is opposed to external excitation *H*.

Suppose the external magnetic field is

$$\vec{H} = \left\{0, H_0 e^{-i\omega t}, 0\right\}$$

applied perpendicular to the plane of wires. the circular current $I(\hat{z})$, as shown, excited by the time-varying magnetic field, flows opposite so as to oppose the cause; this current in turn generates a induced magnetic field in *y*-direction H_{ind}. The displacement currents flowing between the nano-wires close the circuit. We thus have a “potential drop”, which is varying with *z*, as

$$U(z) = \int_1^2 \vec{E}.\mathrm{d}\vec{l}$$

The integration is along the line $\{1(z), 2(z)\}$ We will have to find current $I(z)$, and to do so we use Faraday's law $\mathrm{curl}\vec{E} = -\dot{\vec{B}} = i\omega\vec{B}$ & integrate over contour 1-2-3-4

Assumptions $2a \gg d$; $2a \gg b$; $kd \ll 1$ where $k = \omega / c$; $\mathrm{d}/\mathrm{d}t \equiv -i\omega$

$kd \ll 1$; $(2\pi/\lambda)d \ll 1$; $\lambda/2 \gg \pi d$ is the criteria for meta-material

A K Sarychev, G Shevet, V M Shalaev, Magnetic Plasmon Resonance, Phys Rev. E 73, 036609 (2006)

Important equations of electromagnetic theory and its variants used in MPR theory

$$\nabla \times E = \mathrm{curl}E = -\frac{\partial}{\partial t}B = i\omega B ; \qquad B = \mu_0 H$$

$$\mathrm{curl}E = i\omega\mu_0 H ; \qquad \omega = kc ; \quad k = 2\pi / \lambda$$

$$\mathrm{curl}E = ikc\mu_0 H ; \qquad c = \left(\varepsilon_0\mu_0\right)^{-1}$$

$$\mathrm{curl}E = ikH \qquad \text{putting} \quad \varepsilon_0 = \mu_0 = 1$$

The magnetic field H_0 is external plus induced field H_{ind} so we write Faraday's law as

$$\mathrm{curl}E = ik(H_0 + H_{ind})$$

If the A is magnetic vector potential, with the assumption that length of nano-wire ($2a$) is much larger than the distance d between the wires and its radius b and with assumption of $kd << 1$; the magnetic vector potential A, is primarily directed along the nano-wires (i.e. z, direction). We have thus the induced magnetic field as $H_{ind} = \mathrm{curl}A$

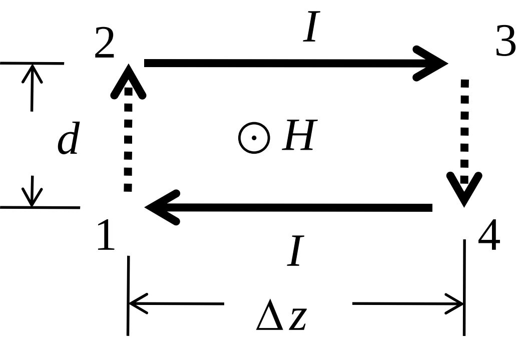
The electric field E in terms of scalar potential ϕ and magnetic vector potential A is

$$E = -\nabla\phi - \frac{\partial}{\partial t}A = -\nabla\phi + i\omega A \qquad \text{put} \qquad \omega = kc$$

$$= -\nabla\phi + ikcA \qquad c = \left(\varepsilon_0\mu_0\right)^{-1} \qquad \varepsilon_0 = \mu_0 = 1$$

$$E = -\nabla\phi + ikA$$

Application of Faraday's law for nano-wire pair



$\text{curl}E = ikH$ working in area $(\Delta z)\, d$

Integral-form is $\oint_{1-2-3-4} E.\mathrm{d}l = ikH\,(d\,\Delta z)$

$E = -\nabla\phi + ikA$

Since the vector potential A is perpendicular to the line ;

the 'potential drop' between the lines is $\{1(z), 2(z)\}$

$$U_{1-2} = \int_{1(z)}^{2(z)} E.\mathrm{d}l = \phi_1 - \phi_2 \qquad U_{3-4} = \int_{3(z)}^{4(z)} E.\mathrm{d}l = \phi_3 - \phi_4$$

loop-area $= (d)(\Delta z)$

Let $R/2$ denote resistance per length Ω / cm for segment 2-3

For segment 2-3, we write integration along the line 2-3

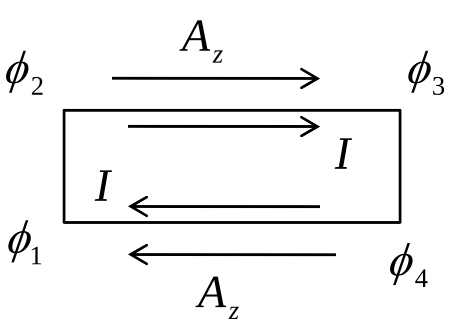
$$\phi_2 - \phi_3 = -I(R/2)\Delta z$$

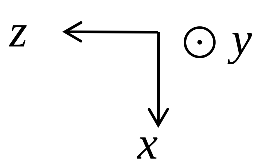
$$\int_2^3 E.\mathrm{d}l = [\phi_2 - \phi_3] + [ik(-A_z)](-\Delta z) = -I(R/2)\Delta z + ikA_z\Delta z = (-IR/2 + ikA_z)\Delta z$$

For segment 4-1, we write the integration along the line 4-1

$$\phi_4 - \phi_1 = -I(R/2)\Delta z$$

$$\int_4^1 E.\mathrm{d}l = [\phi_4 - \phi_1] + (ikA_z)(\Delta z) = -I(R/2)(\Delta z) + ikA_z\Delta z = (-IR/2 + ikA_z)\Delta z$$



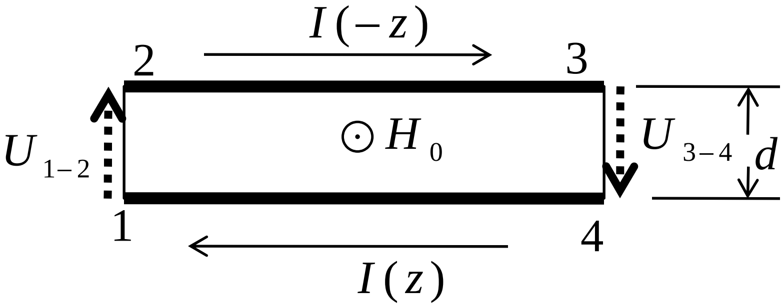


For in-between the lines segments 1-2 and 3-4 we have

$$\int_1^2 E\,\mathrm{d}l + \int_3^4 E\,\mathrm{d}l = U_{1-2} + U_{3-4} = -U_{2-1} + U_{3-4} = -(U_{2-1} - U_{4-3})$$

$$= -\Delta U(z) = -\frac{\mathrm{d}U(z)}{\mathrm{d}z}\Delta z$$

Derivation of constitutive expression for nano-wire pair



$$\oint_{1-2-3-4} E.\mathrm{d}l = ikH\,(d\,\Delta z)$$

$$\int_1^2 E\,\mathrm{d}l + \int_3^4 E\,\mathrm{d}l = -\frac{\mathrm{d}U(z)}{\mathrm{d}z}\Delta z$$

$$\int_2^3 E.\mathrm{d}l = (-IR/2 + ikA_z)\Delta z \qquad \int_4^1 E.\mathrm{d}l = (-IR/2 + ikA_z)\Delta z$$

$$\oint_{1-2-3-4} E.\mathrm{d}l = -IR\,\Delta z + 2ikA_z\Delta z - \frac{\mathrm{d}U(z)}{\mathrm{d}z}\Delta z = ikH_0(d\,\Delta z)$$

$$\left(IR - 2ikA_z + \frac{\mathrm{d}U(z)}{\mathrm{d}z}\right)\Delta z = -ikH_0 d\,\Delta z$$

In this equation i.e. Faraday's law applied to the two wire pairs, having thickness and separation of the order or rather less than skin depth (at optical wavelengths) , the expression for R, U and A, to get the current $I(z)$ equation; for resonance conditions.

A K Sarychev, G Shevet, V M Shalaev, Magnetic Plasmon Resonance, Phys Rev. E 73, 036609 (2006)

Per unit length of resistance capacitance and inductance of nano-wire pairs

At the high frequency of optical regime, we write the resistance per unit length of the wire of dia, 2 b as

$R \simeq \dfrac{2}{\sigma \pi b^2} \approx \dfrac{8i}{\omega \varepsilon_m b^2}$ where the complex permittivity of metal is $\varepsilon_m = 1 + i\dfrac{4\pi\sigma}{\omega}$, the imaginary part of the complex permittivity are losses appears as resistance. $\sigma \cong (\varepsilon_m \omega)/(i4\pi)$

Write the capacitance between the wires as $C = Q(z)/U(z); \quad U(z) = Q(z)/C$

$$C = [4\ln(d/b)]^{-1}$$

The magnetic vector potential A is proportional to electric current with L as wire pair inductance $A_z(z) = \left(\dfrac{L}{2c}\right) I(z)$

Write inductance as $L = 4\ln(d/b)$

Note $LC = 1$

This $L\,C$ we will discuss later

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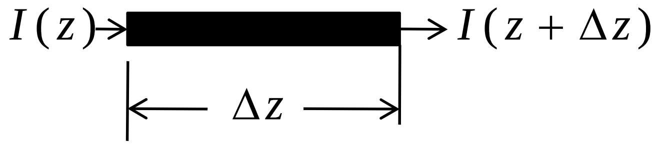
The conservation of charge

$Q(z)$ is the linear charge density



$$I(z) - I(z+\Delta z) = \frac{\mathrm{d}}{\mathrm{d}t} Q(z)\Delta z = \Delta z \frac{\mathrm{d}}{\mathrm{d}t} Q(z)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} Q(z) = \frac{I(z) - I(z+\Delta z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \left(- \frac{\mathrm{d}}{\mathrm{d}z} I(z) \right)$$

The charge conservation law we write as $\dfrac{\mathrm{d}I(z)}{\mathrm{d}z} = -\dfrac{\mathrm{d}}{\mathrm{d}t} Q(z) = i\omega Q(z)$

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The equation for currents in wire-pair

Following are steps to get equation for current after application of Faraday's law to the nano-wire pair

$$\left(IR - 2ikA_z + \frac{\mathrm{d}U(z)}{\mathrm{d}z} \right) \Delta z = -ikH_0 d\,\Delta z$$

$$\left(IR - 2ikA_z + \frac{\mathrm{d}U(z)}{\mathrm{d}z} \right) = -ikH_0 d$$

Put in above $\quad R = \dfrac{8i}{\varepsilon_m b^2 \omega}; \quad A_z = \left(\dfrac{L}{2c}\right) I(z); \quad U(z) = \dfrac{Q(z)}{C}; \quad Q(z) = \dfrac{\mathrm{d}I(z)/\mathrm{d}z}{i\omega}$

$$i\frac{8}{\varepsilon_m b^2 \omega} I - i\frac{kL}{c} I + \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{Q(z)}{C}\right) = -ikH_0 d\,; \qquad i\frac{8}{\varepsilon_m b^2 \omega} I - i\frac{kL}{c} I + \frac{1}{C}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{i\omega}\frac{\mathrm{d}I}{\mathrm{d}z}\right) = -ikH_0 d$$

$$i\frac{8}{\varepsilon_m b^2 \omega} I - i\frac{kL}{c} I - i\frac{1}{\omega C}\frac{\mathrm{d}^2 I}{\mathrm{d}z^2} = -ikH_0 d\,; \qquad \frac{1}{\omega C}\frac{\mathrm{d}^2 I}{\mathrm{d}z^2} + \left(\frac{kL}{c} - \frac{8}{\varepsilon_m b^2 \omega}\right) I = kdH_0$$

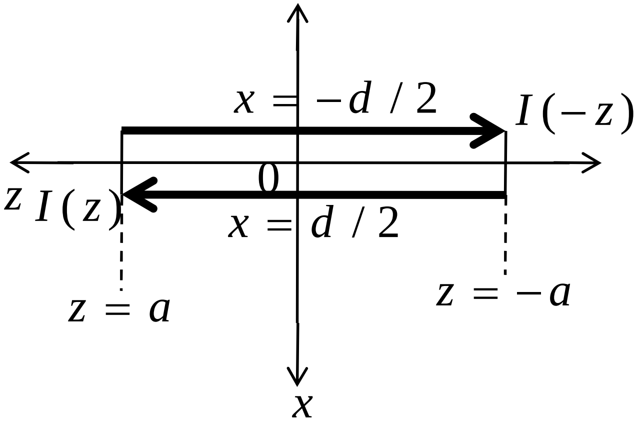
$$\frac{1}{\omega C}\frac{\mathrm{d}^2 I}{\mathrm{d}z^2} + k^2\left(\frac{L}{ck} - \frac{8}{\varepsilon_m (kb)^2 \omega}\right) I = kdH_0 \qquad \text{put} \qquad ck = \omega$$

$$\frac{1}{\omega C}\frac{d^2 I}{dz^2} + \frac{k^2}{\omega}\left(L - \frac{8}{\varepsilon_m (kb)^2}\right) I = kdH_0$$

Rearrange above to get final equation in currents in the wire-pair as

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2} I + k^2\left[LC - \frac{8C}{\varepsilon_m (kb)^2}\right] I = Ck\omega dH_0 \qquad \text{put}\, k = \frac{\omega}{c}; \qquad \frac{\mathrm{d}^2}{\mathrm{d}z^2} I + k^2\left[LC - \frac{8C}{\varepsilon_m (kb)^2}\right] I = \frac{Cd\omega^2}{c} H_0$$

Current equation for nano-wire pair



$I(a) = I(-a) = 0$

$$\frac{d^2}{dz^2} I(z) + g^2 I(z) - \frac{C d \omega^2}{c} H_0 = 0$$

$$-a < z < a \qquad I(-a) = I(a) = 0$$

The parameter g is given as

$$g^2 = k^2 \left[LC - \frac{8C}{\varepsilon_m (kb)^2} \right] \qquad LC \sim 1 \qquad g^2 = k^2 \left[1 - \frac{8C}{\varepsilon_m (kb)^2} \right]$$

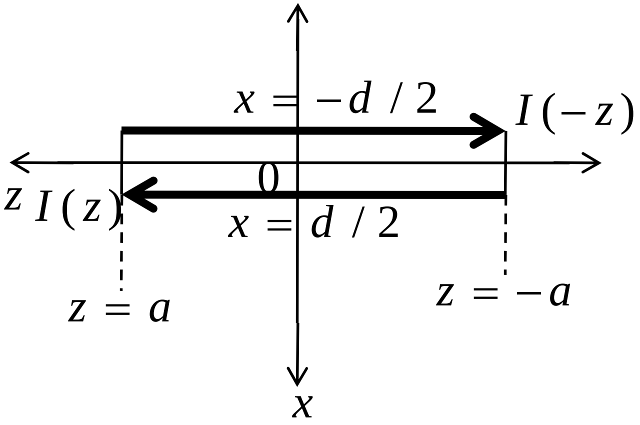
This two nano-wire 'antenna' system is resonantly excited when $G = ga = N\pi/2; \quad N \in \mathbb{Z}^+; \quad N = 1, 2, 3...$

Here in this nano-wire-pair system the material property of metal at optical frequency, enters the resonant parameter above through ε_m . While at the (G Hz) uW range to mid IR the wire thickness (in this case radius b) is >> skin depth $\delta \sim 1/\sqrt{k^2 |\varepsilon_m|}$; the material property factor i.e.

$$b \gg \delta \qquad \frac{8C}{k^2 \varepsilon_m b^2} \equiv \frac{8C\delta^2}{b^2} \ll 1 \qquad \text{gives} \quad g/\sqrt{LC} = k \sim g$$

And the resonant condition in this case is also known as 'antenna resonance' is $ka = \pi/2$; the material parameter does not enter the resonance: or this is GLC Geometric *LC* resonance, of SRR type at G Hz to few T Hz.

Current equation for nano-wire pair and resonance with material parameters



$$I(a) = I(-a) = 0$$

The parameter g is given as

$$g^2 = k^2\left[LC - \frac{8C}{\varepsilon_m (kb)^2}\right] \qquad LC \sim 1 \qquad g^2 = k^2\left[1 - \frac{8C}{\varepsilon_m (kb)^2}\right]$$

This two nano-wire ‘antenna’ system is resonantly excited when $G = ga = N\pi/2;\quad N \in \mathbb{Z}^+;\quad N = 1,2,3...$

The opposite case when the wire thickness or radius b << skin depth δ ; that is in our case of these nano-antennas; then

$$b < \delta \qquad\qquad \frac{8C}{k^2\varepsilon_m b^2} \equiv \frac{8C\delta^2}{b^2} \gg 1$$

Here the resonance parameter is $G^2 = g^2a^2 \cong k^2a^2\left(-\dfrac{8C}{\varepsilon_m k^2 b^2}\right) = -\left(\dfrac{a}{b}\right)^2\left(\dfrac{8C}{\varepsilon_m}\right)$

Sharp resonance requires that G^2 be positive, possibly with very less imaginary part. Indeed for the IR and visible frequencies, the ε_m is ‘negative’ (with smaller imaginary part) for noble metals Ag / Au etc.

The resonance condition for magnetic plasmon resonance requires metal property, and geometry features-we will see that C in above depends on geometry only $C \approx \left(4\ln(d/b)\right)^{-1}$

The resonance parameter G^2 at optical frequency

When the skin depth $\delta \sim (k^2 |\varepsilon_m|)^{-1/2}$ is larger than the metal feature, wire thickness $\sim b$

We have
$$g^2 \cong k^2 \left(-\frac{8C}{(kb)^2 \varepsilon_m} \right)$$

Therefore, the resonant parameter G becomes

$$G^2 = g^2 a^2 = -k^2 \frac{8C}{(kb)^2 \varepsilon_m} a^2 = -\left(\frac{a}{b}\right)^2 \frac{8C}{\varepsilon_m} \quad \text{put} \quad C = \frac{1}{4\ln(d/b)}$$

$$G^2 = -2\left(\frac{a}{b}\right)^2 \frac{1}{\varepsilon_m \ln(d/b)} \qquad G = \sqrt{2}\left(\frac{a}{b}\right) \frac{1}{\sqrt{|\varepsilon_m| \ln(d/b)}}$$

Sharp resonance requires that G^2 be positive, possibly with very less imaginary part. Indeed for the IR and visible frequencies, the ε_m is ‘negative’ (with smaller imaginary part) for noble metals Ag / Au etc. makes the resonant parameter G^2 positive.

$$\varepsilon_m \cong \varepsilon_\infty - \left(\frac{\omega_p^2}{\omega^2}\right) \frac{1}{(1 - i\omega_\tau/\omega)} \approx -\frac{\omega_p^2}{\omega^2}$$

Where ε_∞ is polarization constant , ω_p is the plasma frequency, and relaxation rate ω_τ is $1/\tau$

For silver $\varepsilon_b \approx 5, \quad \omega_p = 9.1\,\mathrm{eV}\,(\mathrm{or}\ 14\times10^{15}\,\mathrm{Hz}), \quad \omega_\tau = 0.02\,\mathrm{eV}$

At wavelength $\lambda = 1.5\,\mu\mathrm{m}, \quad \varepsilon_m' = -120, \quad \varepsilon_m'' / |\varepsilon_m'| = 0.025$

Magnetic plasmon resonance frequency

$$g = k\sqrt{1-\frac{8C}{(kb)^2\varepsilon_m}}, \quad G = ga = ka\sqrt{1-\frac{8C}{(kb)^2\varepsilon_m}}$$

For $\delta = (k^2|\varepsilon_m|)^{-1/2} > b$, we have resonance condition as $G = \pi/2$, gives following

$$G = \frac{\pi}{2} = ka\sqrt{-\frac{8C}{(kb)^2\varepsilon_m}} \quad \text{put} \quad C = [4\ln(d/b)]^{-1}, \quad k = \omega/c, \quad \varepsilon_m = -\omega_p^2/\omega^2$$

$$\frac{\pi}{2} = \frac{\omega}{c}a\sqrt{\frac{2}{\frac{\omega^2}{c^2}b^2\left[\ln\left(\frac{d}{b}\right)\right]\left(\frac{\omega_p^2}{\omega^2}\right)}} = \frac{\omega}{\omega_p}\frac{a}{b}\sqrt{\frac{2}{\ln(d/b)}}$$

$$\omega = \omega_m = \left(\frac{\pi b\sqrt{\ln(d/b)}}{a2\sqrt{2}}\right)\omega_p$$

This is magnetic plasmon resonance frequency

Here we get magnetic plasmon resonance frequency as proportional to plasma frequency of metal scaled down by geometric factors. The property of metal determines the magnetic plasmon resonance at the optical regime, (in contrast to the SRR structures at uW where only geometric factors govern the resonance)

$$\omega_m < \omega_p$$

Comment about the magnetic plasmon resonance frequency

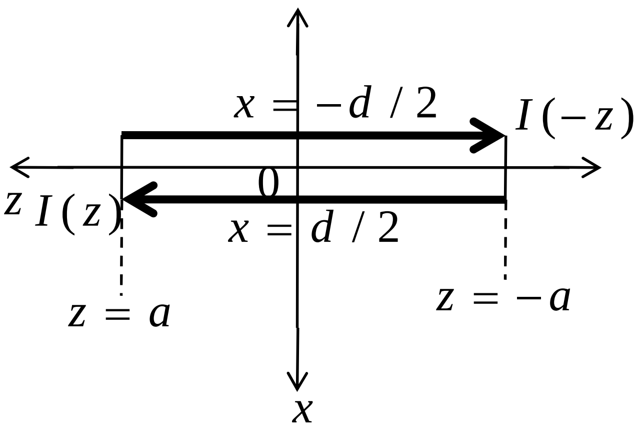
This is what was stated that the MPR frequency depends on material property of metal at optical wavelengths, instead of purely geometrical dependence as for resonances in G Hz for SRR (antenna resonance).

The electric resonance (plasma resonance) frequency of metal is responsible for Magnetic Plasmon Resonance (MPR) when at optical wave-lengths the size of metal is of skin depth size.

Shevet G , Urzhumov V A, 2006 Negative indexed metamaterial based on two-dimensional metallic structures, J Opt, A Pure Appl Opt S 122, S130

A K Sarychev, G Shevet, V M Shalaev, Magnetic Plasmon Resonance, Phys Rev. E 73, 036609 (2006)

Current distribution for nano-wire pair



$$I(a) = I(-a) = 0$$

$$\frac{d^2}{dz^2} I(z) + g^2 I(z) - \frac{Cd\omega^2}{c} H_0 = 0$$

$$-a < z < a \qquad I(-a) = I(a) = 0 \qquad G = ga$$

We need to solve this differential equation for $I(z)$

The homogeneous equation is

$$\frac{d^2}{dz^2} I(z) + g^2 I(z) = 0$$

Has a standard solution as $I_h(z) = c_1 \cos gz + c_2 \sin gz$

The particular solution is solution of $\frac{d^2}{dz^2} I(z) + g^2 I(z) = \frac{Cd\omega^2}{c} H_0 = Ke^{\in z}$ $\qquad \in \approx 0 \qquad K = \frac{Cd\omega^2}{c}$

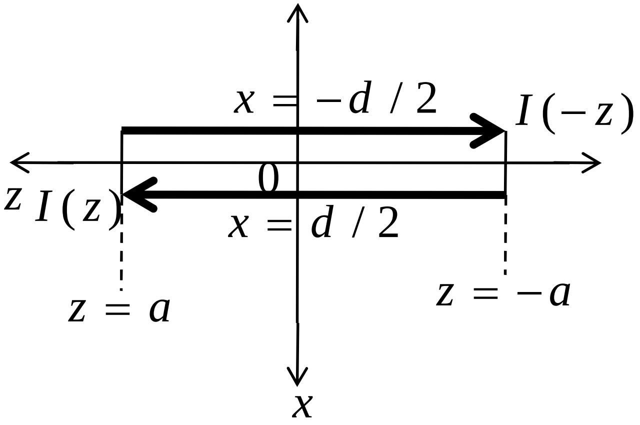
Let $I_p(z) = AKe^{\in z}$ be the particular solution, and we substitute in above, we get following

$$AK(\in^2 + g^2)e^{\in z} = Ke^{\in z} \qquad A(\in^2 + g^2) = 1; \quad \in = 0 \quad \text{give} \quad A = 1/g^2$$

Therefore particular solution is

$$I_p(z) = \frac{1}{g^2} \frac{Cd\omega^2}{c} H_0$$

Solution for current distribution for nano-wire pair



$$\frac{d^2}{dz^2} I(z) + g^2 I(z) - \frac{C d \omega^2}{c} H_0 = 0$$

$$-a < z < a \qquad I(-a) = I(a) = 0 \qquad G = ga$$

The total solution is homogeneous solution plus particular solution

$$I(z) = I_h(z) + I_p(z)$$

$$I(z) = c_1 \cos gz + c_2 \sin gz + \frac{1}{g^2} \frac{C d \omega^2}{c} H_0$$

Now we apply the boundary conditions $I(a) = I(-a) = 0$

$$I(a) = c_1 \cos ga + c_2 \sin ga + \frac{C d \omega^2 H_0}{g^2 c} = 0 \qquad I(-a) = c_1 \cos ga - c_2 \sin ga + \frac{C d \omega^2 H_0}{g^2 c} = 0$$

Gives $c_1 = -\dfrac{C d \omega^2 H_0}{g^2 c \cos ga} \qquad c_2 = 0$

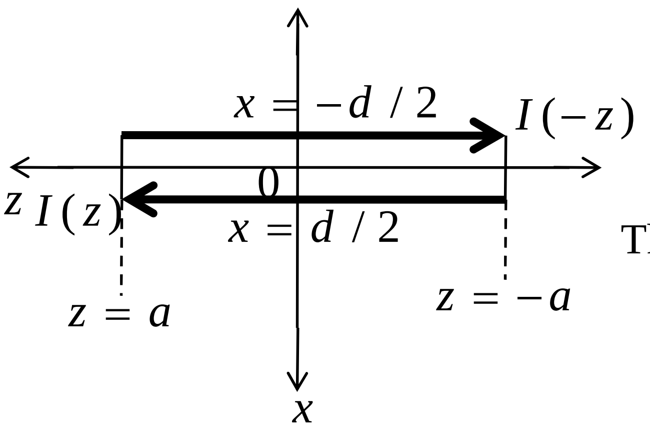
Current expression is

$$I(z) = -\frac{C d \omega^2 H_0}{g^2 c \cos ga} \cos gz + \frac{C d \omega^2 H_0}{g^2 c}$$

$$= \frac{C d \omega^2 H_0}{g^2 c} \left[1 - \frac{\cos gz}{\cos ga} \right] \qquad \text{put} \qquad g = G / a$$

$$I(z) = \frac{C d \omega^2 H_0}{g^2 c} \left(1 - \frac{\cos(Gz/a)}{\cos G} \right)$$

The current distribution *I (z)* for nano-wire pair



$$\frac{d^2}{dz^2} I(z) + g^2 I(z) - \frac{C d \omega^2}{c} H_0 = 0$$

$$-a < z < a \qquad I(-a) = I(a) = 0 \qquad G = ga$$

The total solution is homogeneous solution plus particular solution

$$I(z) = \frac{C d \omega^2 H_0}{g^2 c}\left(1 - \frac{\cos(Gz/a)}{\cos G}\right)$$

Manipulating the term outside the bracket we obtain

By using $g = G/a; \quad c = \omega / k$ we obtain $\dfrac{C d \omega^2 H_0}{g^2 c} = \dfrac{C d \omega^2 k a^2 H_0}{G^2 \omega} = \dfrac{C d \omega k a^2 k H_0}{(G)(G)}$

Now use $G = \sqrt{2}\left(\dfrac{a}{b}\right) / \sqrt{|\varepsilon_m| \ln(d/b)}, \quad C = [4 \ln(d/b)]^{-1}$

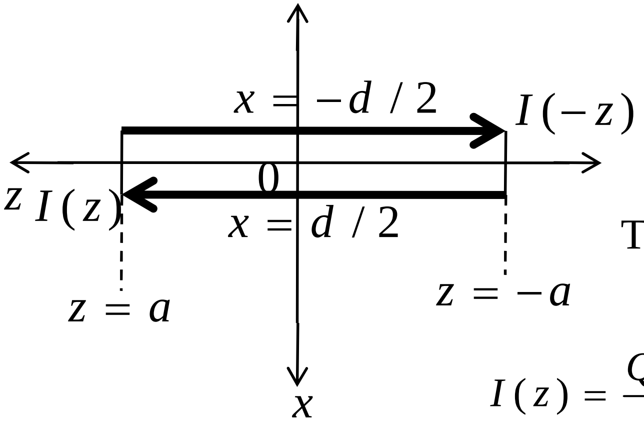
to get

$$\frac{C d \omega a^2 k H_0}{(G)(G)} = \frac{k \omega a b d H_0 \sqrt{|\varepsilon_m|}}{G\, 4\sqrt{2 \ln(d/b)}} = \frac{Q_0 a \omega}{G}; \qquad Q_0 = k b d H_0 \sqrt{|\varepsilon_m|} / 4\sqrt{2 \ln(d/b)}$$

Final current distribution is:

$$I(z) = \frac{Q_0 a \omega}{G}\left(1 - \frac{\cos(Gz/a)}{\cos G}\right); \qquad Q_0 = k b d H_0 \sqrt{|\varepsilon_m|} / 4\sqrt{2 \ln(d/b)}$$

Solution for charge distribution $Q(z)$ for nano-wire pair



$$\frac{d^2}{dz^2} I(z) + g^2 I(z) - \frac{C d\omega^2}{c} H_0 = 0$$

$$-a < z < a \qquad I(-a) = I(a) = 0 \qquad G = ga$$

The total solution for current giving current distribution is

$$I(z) = \frac{Q_0 a\omega}{G}\left(1 - \frac{\cos(Gz/a)}{\cos G}\right); \quad Q_0 = kbdH_0\sqrt{|\varepsilon_m|}/4\sqrt{2\ln(d/b)}$$

The charge conservation law gives gradient of current as $\frac{d}{dz} I(z) = i\omega Q(z)$, from this we have

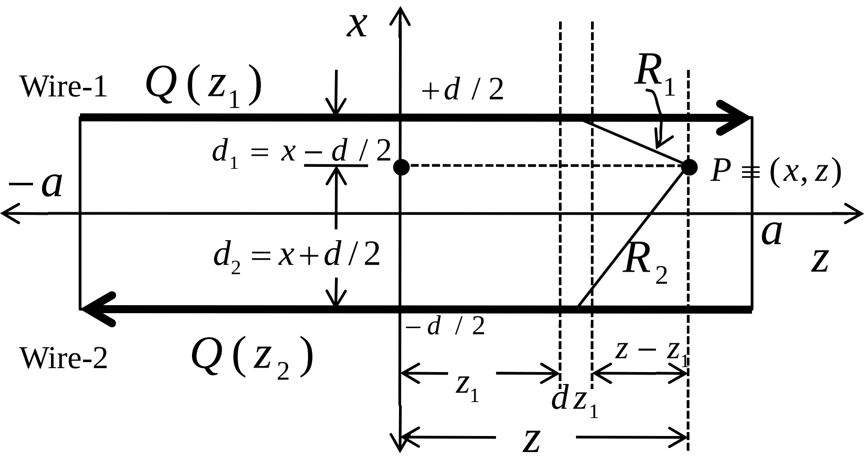
$$Q(z) = \frac{1}{i\omega}\frac{d}{dz} I(z) = \frac{1}{i\omega}\frac{d}{dz}\left(\frac{Q_0 a\omega}{G}\left(1 - \frac{\cos(Gz/a)}{\cos G}\right)\right)$$

$$= -\frac{1}{i} Q_0 \frac{\sin(Gz/a)}{\cos G}$$

$$Q(z) = iQ_0 \frac{\sin(Gz/a)}{\cos G}; \quad Q_0 = kbdH_0\sqrt{|\varepsilon_m|}/4\sqrt{2\ln(d/b)}$$

$Q(z)$ is the distribution of charge per unit length

The scalar potential function in between the nano-wires



The electric potential in Lorentz gauge is:

$$\phi(r) = \int \frac{q(r_1)e^{ikR_1}}{R_1} \mathrm{d}r_1 - \int \frac{q(r_2)e^{ikR_2}}{R_2} \mathrm{d}r_2$$

$q(r_1)$, $q(r_2)$ electric charges distributed over wire-1 and wire-2 surface $R_1 = |r - r_1|$, $R_2 = |r - r_2|$

The integration is over wire-1 and wire-2

We will consider the electric field between the wires, i.e. in {z, x} plane; and assume $|x| \ll a$, $|z| < a$ and the distances to the wire-1 and 2 $d_1 = |x - d/2| \gg b$, and $d_2 = |x + d/2| \gg b$

$$Q(z)\big|_{\text{wire}-1} = -Q(z)\big|_{\text{wire}-2} = iQ_0 \frac{\sin(Gz/a)}{\cos G}$$

The 1D form of $\phi(r)$ is:

$$\phi(P) = \phi(x,z) = \int_{-a}^{+a} Q(z)\left[\frac{e^{ikR_1}}{R_1} - \frac{e^{ikR_2}}{R_2}\right]\mathrm{d}z_1 \qquad R_1 = \sqrt{d_1^2 + (z - z_1)^2}\,; \quad R_2 = \sqrt{d_2^2 + (z - z_1)^2}$$

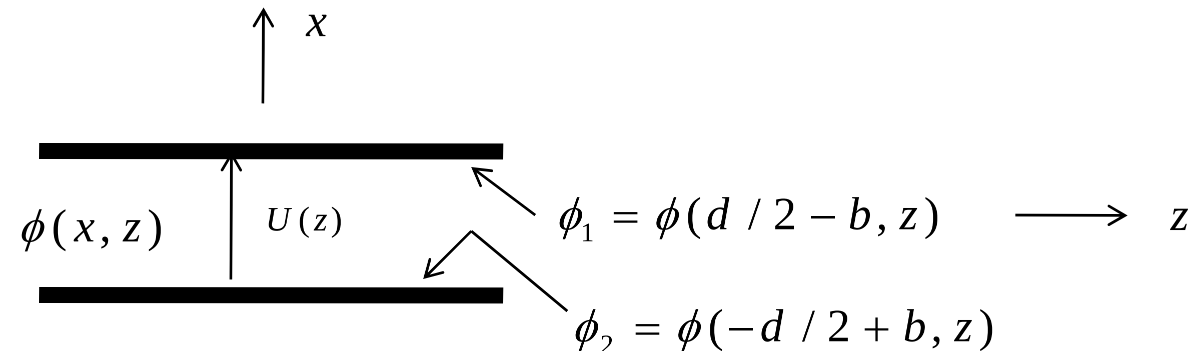
Since we have assumed $d << a$ and $k\,d << 1$, the exponentials in above equation can be equated to one $e^{ikR_1} \simeq e^{ikR_2} = 1$, and also extending the limits to $\pm\infty$ we write

$$\phi(x,z) = Q(z)\int_{-\infty}^{+\infty}\left[\frac{1}{\sqrt{(z - z_1)^2 + (x - d/2)^2}} - \frac{1}{\sqrt{(z - z_1)^2 + (x + d/2)^2}}\right]\mathrm{d}z_1$$

$$\phi(x,z) = 2\ln\left|\frac{x + d/2}{x - d/2}\right| Q(z)$$

A K Sarychev, G Shevet, V M Shalaev, Magnetic Plasmon Resonance, Phys Rev. E 73, 036609 (2006)

The potential drop between the two nano-wires and capacitance calculations



$$\phi(x,z) = 2\ln\left|\frac{x+d/2}{x-d/2}\right| Q(z)$$

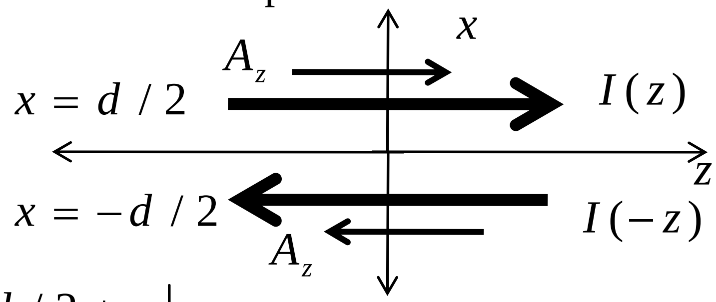
$$\begin{aligned}
U(z) &= \phi_1 - \phi_2 = \phi(d/2 - b, z) - \phi(-d/2 + b, z) \\
&= 2\ln\left|\frac{d/2 + (d/2 - b)}{d/2 - (d/2 - b)}\right| Q(z) - 2\ln\left|\frac{d/2 + (-d/2 + b)}{d/2 - (-d/2 + b)}\right| Q(z) \\
&= 2\ln\left|\frac{d-b}{b}\right| Q(z) - 2\ln\left|\frac{b}{d-b}\right| Q(z) = 4\ln\left|\frac{d-b}{b}\right| \cong 4Q(z)\ln\left(\frac{d}{b}\right)
\end{aligned}$$

$$C = \frac{Q(z)}{U(z)} = \frac{1}{4\ln(d/b)}$$ Per unit length capacitance

A K Sarychev, G Shevet, V M Shalaev, Magnetic Plasmon Resonance, Phys Rev. E 73, 036609 (2006)

The magnetic vector potential of nano-wire system and its inductance

The magnetic vector potential is calculated in similar way as done for scalar potential. The vector potential A is z directed i.e. $\{0,0,A\}$



$$A(x,z) \simeq \frac{1}{c}\int_{-a}^{+a} I(z_1)\left[\frac{e^{ikR_1}}{R_1} - \frac{e^{ikR_2}}{R_2}\right]\mathrm{d}z_1$$

$$\simeq \frac{1}{c}\int_{-\infty}^{+\infty} I(z)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]\mathrm{d}z_1 \simeq \frac{2}{c}\ln\left|\frac{d/2+x}{d/2-x}\right| I(z)$$

Where the electric current is given by:

$$I(z) = \frac{Q_0 a\omega}{G}\left(1 - \frac{\cos(Gz/a)}{\cos G}\right); \quad Q_0 = kbdH_0\sqrt{\varepsilon_m}\,/\,4\sqrt{2\ln(d/b)}$$

Extrapolating the vector potential to the surface of wire-1 ($x = d/2 - b$) we get

$$A(d/2-b,z) = \frac{2}{c}\ln\left|\frac{d/2+(d/2-b)}{d/2-(d/2-b)}\right| I(z) = \frac{2}{c}\ln\left|\frac{d-b}{b}\right| I(z) \approx \frac{2}{c}\ln\left(\frac{d}{b}\right) I(z) = \frac{1}{c}\left(\frac{L}{2}\right) I(z)$$

$$L = 4\ln\left(\frac{d}{b}\right)$$ Per unit length inductance

A K Sarychev, G Shevet, V M Shalaev, Magnetic Plasmon Resonance, Phys Rev. E 73, 036609 (2006)

Expression for Electric Field between two-nano-wire in transverse direction (x)

Electric field in terms of scalar and vector potential is

$$\vec{E} = -\nabla \phi + ik\vec{A}$$

We are interested in the electric field in the { x , z } plane. The magnetic vector potential is z directed and we do not get contribution of that in E field in x direction, therefore:

$$E_x = -\nabla \phi(x,z) = -\frac{\partial}{\partial x}\left(2Q(z)\ln\left|\frac{d/2+x}{d/2-x}\right|\right)$$

$$= -\frac{\partial}{\partial x}\left(2iQ_0 \frac{\sin(Gz/a)}{\cos G}\ln\left|\frac{d/2+x}{d/2-x}\right|\right)$$

$$= -2iQ_0 \frac{\sin(Gz/a)}{\cos G}\left[\frac{1}{d/2+x} + \frac{1}{d/2-x} \right]$$

$$= -2i\frac{Q_0 d}{(d/2)^2 - x^2}\sin(Gz/a)\sec(G)$$

Expression for Electric Field between two-nano-wire in longitudinal direction (*z*)

$$\vec{E} = -\nabla\phi + ik\vec{A}$$

$$E_z = -\nabla\phi(x,z) + ikA_z = -\frac{\partial}{\partial z}\left(2Q(z)\ln\left|\frac{d/2+x}{d/2-x}\right|\right) + ik\left(2I(z)\ln\left|\frac{d/2+x}{d/2-x}\right|\right)$$

$$= -\frac{\partial}{\partial z}\left(2iQ_0\sin(Gz/a)\sec(G)\ln\left|\frac{d/2+x}{d/2-x}\right|\right) + 2ik\frac{Q_0 a\omega}{G}\left[1-\cos(Gz/a)\sec(G)\right]\ln\left|\frac{d/2+x}{d/2-x}\right|$$

$$= \frac{2iQ_0G}{a}\cos(Gz/a)\sec(G)\ln\left|\frac{d/2+x}{d/2-x}\right| + \left[\frac{2ikQ_0a\omega}{G} - \frac{2ikQ_0a\omega}{G}\cos(Gz/a)\sec(G)\right]\ln\left|\frac{d/2+x}{d/2-x}\right|$$

$$= \frac{2iQ_0}{aG}\ln\left|\frac{d/2+x}{d/2-x}\right|\left[G^2\cos(Gz/a)\sec(G) + a^2\omega k\left(1-\cos(Gz/a)\sec(G)\right)\right]; \quad \text{put} \quad \omega = ck$$

$$= \frac{2iQ_0}{aG}\ln\left|\frac{d/2+x}{d/2-x}\right|\left[G^2\cos(Gz/a)\sec(G) + ca^2k^2\left(1-\cos(Gz/a)\sec(G)\right)\right]$$

$$E_x = \frac{-2iQ_0 d}{(d/2)^2 - x^2}\sin(Gz/a)\sec(G)$$

$$E_z = \frac{2iQ_0}{aG}\ln\left|\frac{d/2+x}{d/2-x}\right|\left(G^2\cos(Gz/a)\sec(G) + ca^2k^2\left[1-\cos(Gz/a)\sec(G)\right]\right)$$

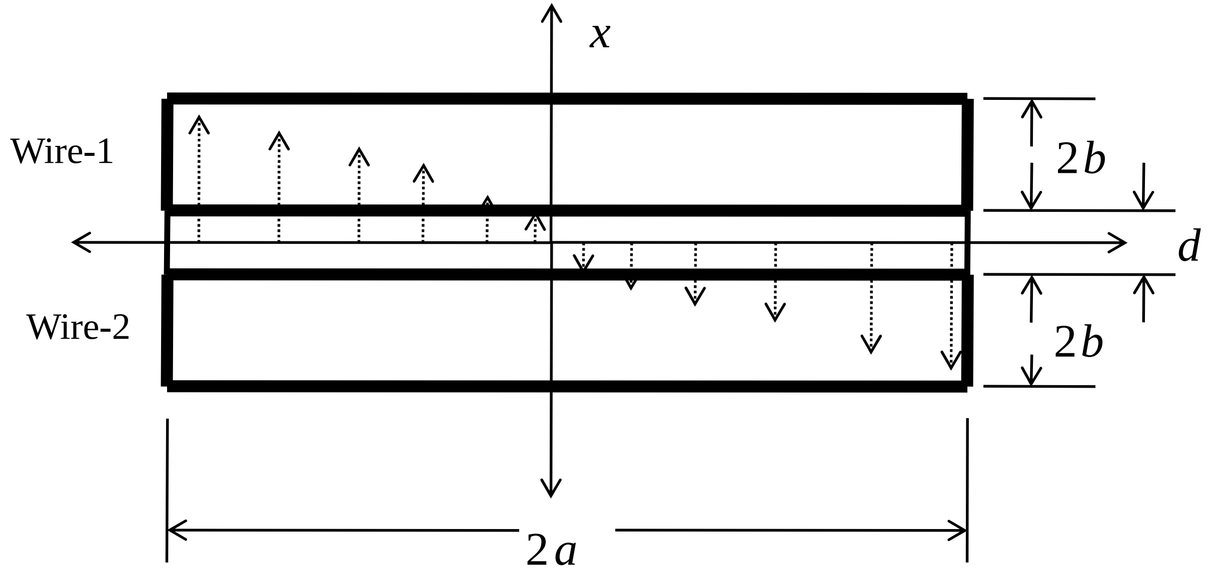
About the transverse (x) and longitudinal (z) electric fields

$$E_x = \frac{-2iQ_0 d}{(d/2)^2 - x^2}\sin(Gz/a)\sec(G)$$

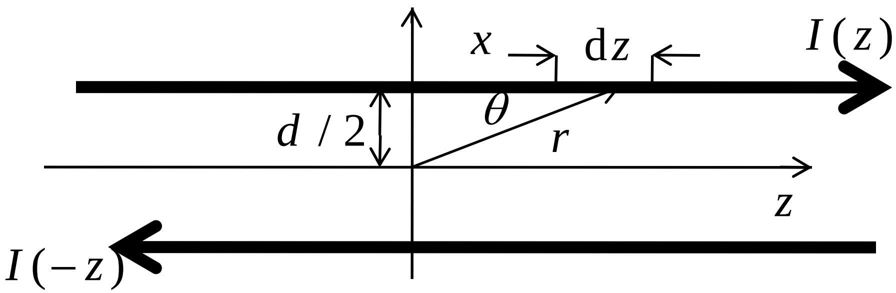
$$E_z = \frac{2iQ_0}{aG}\ln\left|\frac{d/2+x}{d/2-x}\right|\left(G^2\cos(Gz/a)\sec(G) + ca^2k^2\left[1-\cos(Gz/a)\sec(G)\right]\right)$$

We still have $|x| \ll a$, $|z| < a$, $|x - d/2| \gg b$, $|x + d/2| \gg b$

The transverse E field E_x changes sign with the coordinate z, and is zero at $z = 0$. yet on the average (near resonance $G \approx \pi/2$) $|E_x|/|E_z| \sim a/d \gg 1$. That is the transverse E field is much larger than the longitudinal field on average at Magnetic Plasmon resonance (MPR). Near the wires at $x = d/2 - b$ the E_x increases even more and $|E_x| \sim 2Q_0 d/[(d/2)^2 - (d/2-b)^2] \approx Q_0/b$



The magnetic moment generated by the nano-wire pair



$$p_m \triangleq \frac{1}{2c}\int \vec{r}\times\vec{j}(r)\mathrm{d}r$$

$$j(r) = I(z)$$

$$I(z) = \frac{Q_0 a\omega}{G}\left(1-\frac{\cos(Gz/a)}{\cos G}\right); \quad Q_0 = kbdH_0\sqrt{|\varepsilon_m|}/4\sqrt{2\ln(d/b)}$$

$$p_m = \frac{1}{2c}\int_{-a}^{a} rI(\mathrm{z})\sin\theta \mathrm{dz}; \quad \text{assumption} \quad \mathrm{dz}=\mathrm{d}r \quad \sin\theta = (d/2)/r$$

$$p_m = \frac{1}{2c}\int_{-a}^{a} rI(\mathrm{z})\frac{(d/2)}{r}\mathrm{dz} = \frac{d}{2}\int_{-a}^{a} I(\mathrm{z})\mathrm{dz}$$

$$= \frac{d}{4c}\int_{-a}^{a}\frac{Q_0 a\omega}{G}\left(1-\frac{\cos(G\mathrm{z}/a)}{\cos G}\right)\mathrm{dz} = \frac{Q_0 a\omega d}{2G}\int_{-a}^{a}\left(1-\frac{\cos(G\mathrm{z}/a)}{\cos G}\right)\mathrm{dz}$$

$$= \frac{Q_0 a\omega d}{4cG}\left(\mathrm{z}\Big|_{-a}^{a} + \frac{a}{G\cos G}\sin(G\mathrm{z}/a)\Big|_{-a}^{a}\right) = \frac{Q_0 a\omega d}{4cG}\left(2a+\frac{2a\sin G}{G\cos G}\right)$$

$$= \frac{Q_0 a^2\omega d}{2cG}\left(\frac{G+\tan G}{G}\right) = \frac{1}{2c}Q_0 a^2\omega d\left(\frac{G+\tan G}{G^2}\right)$$

$$= \frac{kbdH_0\sqrt{\varepsilon_m}}{8c\sqrt{2}\sqrt{\ln(d/b)}}a^2\omega d\left(\frac{G+\tan G}{G^2}\right) = \frac{kbd^2H_0\sqrt{\varepsilon_m}}{8c\sqrt{2}\sqrt{\ln(d/b)}}Ga^2\omega\left(\frac{G+\tan G}{G^3}\right) \quad \text{put} \quad G=\sqrt{2}\left(\frac{a}{b}\right)\frac{1}{\sqrt{\varepsilon_m}\sqrt{\ln(d/b)}}$$

$$p_m = \frac{1}{8c\ln(d/b)}a^3d^2\omega kH_0\left(\frac{G+\tan G}{G^3}\right)$$

The magnetic effect generated by nano-wire pair-at optical frequency

We derived the magnetic moment by the asymmetric currents in the two nano-conductors as

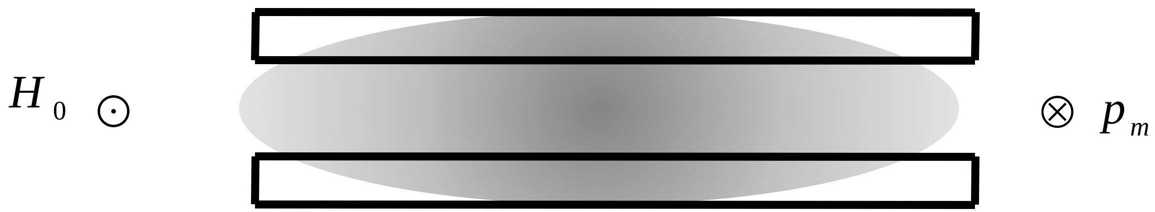
$$p_m = \frac{1}{8c\ln(d/b)} a^3 d^2 \omega k H_0 \left(\frac{G + \tan G}{G^3} \right) \qquad \text{put} \qquad \omega = ck$$

$$p_m = \frac{1}{8\ln(d/b)} a^3 d^2 k^2 H_0 \left(\frac{G + \tan G}{G^3} \right) \qquad G^2 = k^2 a^2 \left(-\frac{8C}{(kb)^2 \varepsilon_m} \right)$$

The metal permittivity ε_m has got large negative value at the optical/near-IR, while its imaginary part is small, therefore the magnetic moment p_m has resonance at $G \approx \pi / 2$, when the p_m attains a large value.

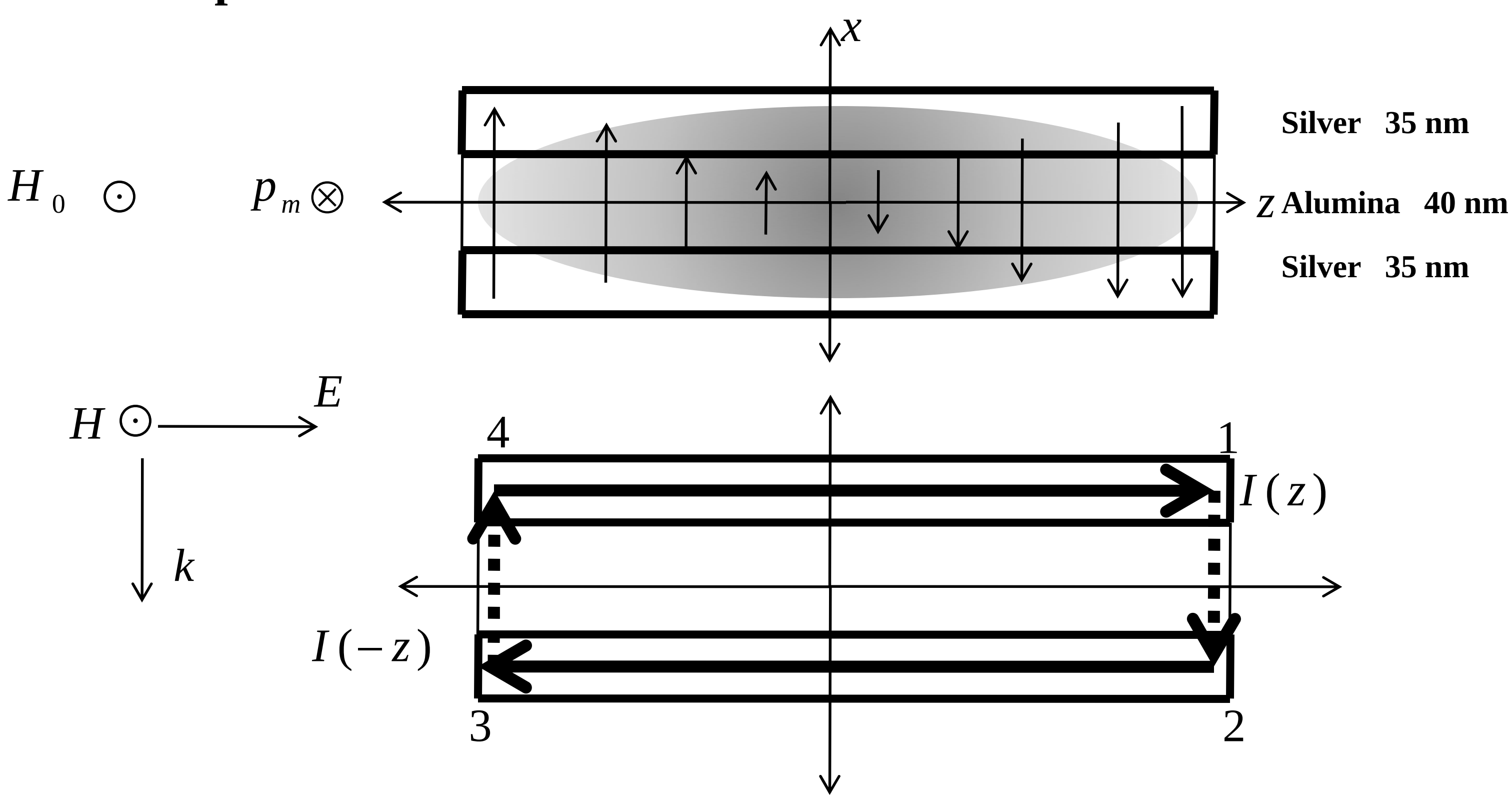
The magnetic polarizabilty can be obtained from $\alpha_m = \dfrac{4\pi p_m}{H_0 V}$

The volume is approximated as $V = 4abd$



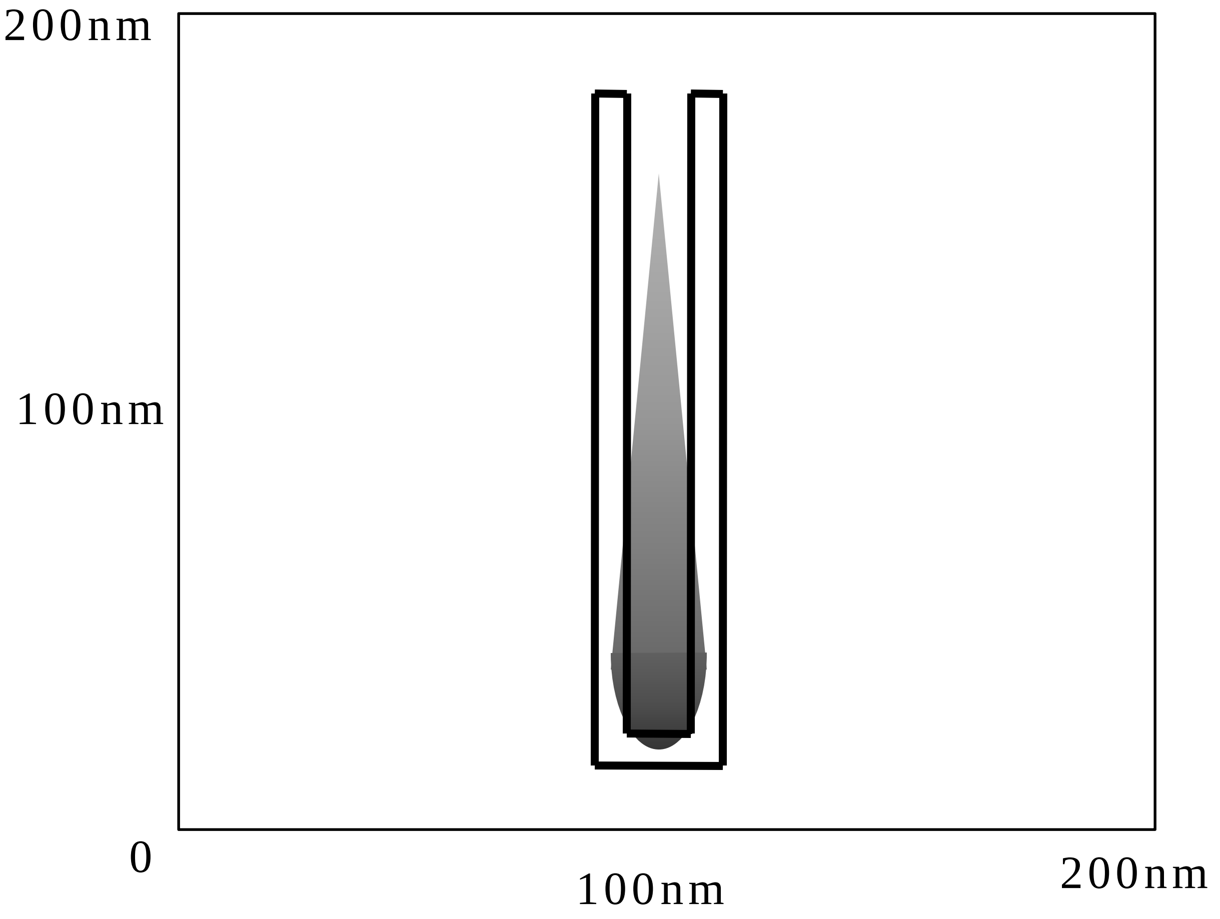
The magnetic field do get penetrated inside the metal wires. A large magnetic moment p_m appearing as opposed to the external H field

The (dominant) electric field E_x and magnetic moment in asymmetrical current excitation by external *H* field; generating magnetic response p_m in a meta-magnetic atom/particle



The dominant electric field is transverse in *x*-direction, the magnetic moment is opposite to external *H*. This dominant *E* field does not aid Poynting vector thus waves decay as in SNG material, without showing propagation. The incident e.m wave in –ve *x* direction will decay inside this meta--magnetic atom/particle structure and will be evanescent wave.

Horse shoe type nano-antenna



Exhibits magnetic resonance with large magnetic dipole moment at about 1.5 um

A K Sarychev, G Shevet, V M Shalaev, Magnetic Plasmon Resonance, Phys Rev. E 73, 036609 (2006)

Cavity model

The structure of two nano-strips (wires) of silver can be viewed as 'cavity' formed between the volume of the strips that support resonance viz. source free fields. To describe the cavity resonances the cavity model successfully used to analyze patch antennas , which can be of use here.

J.R. James and P.S Hall , ' Handbook of micro-strip antenna' (1988).

The modifications are to take into account the penetration of the fields through the metal films.

In the first step anticipating that the resonance (magnetic) field are concentrated primarily between The strips the region $|z| < a$ is closed by (virtual) perfect magnetic conductor walls (i.e. no tangential H exists there and no normal E exists there at walls $z = \pm a$

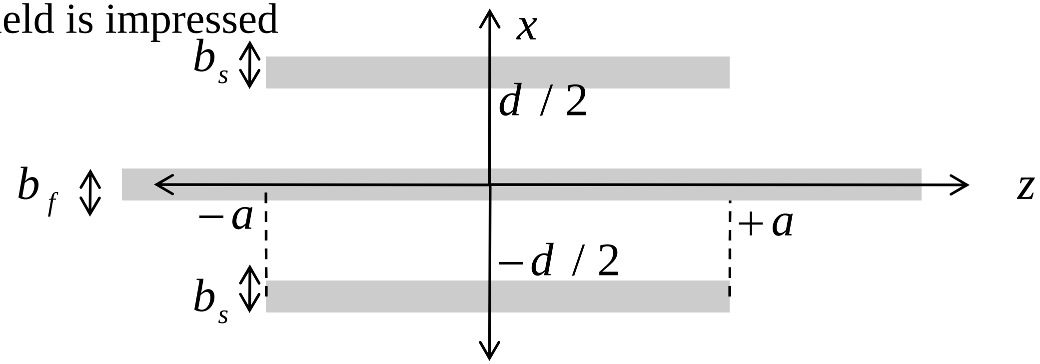
In the second step, the optically thin top and bottom silver strips (wires), are replaced by thin (inductive) admittance sheets characterized by normalized surface admittance at $x = \pm d / 2$ $Y_s^{strip} = ik_0(\varepsilon_m - 1)b_s$, where b_s is metal strip width.

For analysis we consider a metal film (of silver) at $x = 0$ having width b_f will have nor- -malized sheet admittance as $Y_s^{film} = ik_0(\varepsilon_m - 1)b_f$; with $k_0 = 2\pi / \lambda$ as free space wave number.

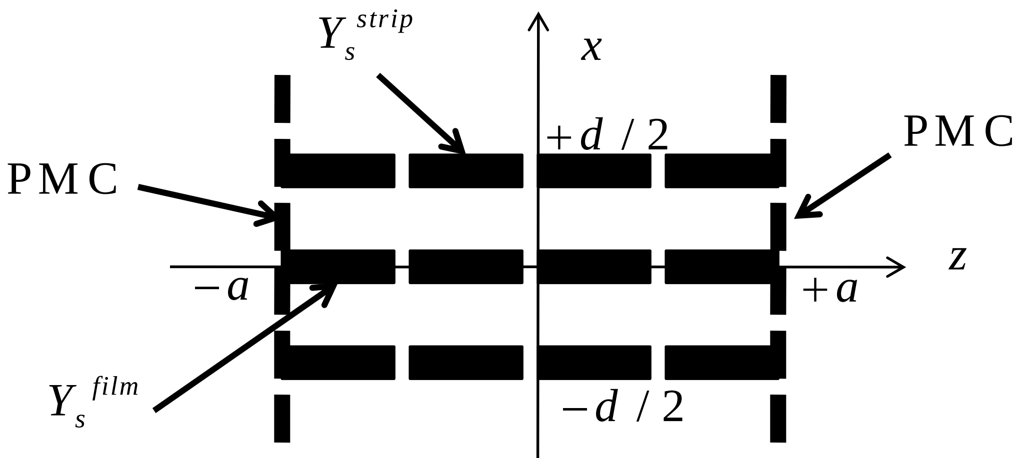
C. A. Balanis, 'Antenna Theory : Analysis and Design III-edition John Willey 2005

The geometry of cavity model-single unit cell

The width of strip b_s , b_f are assumed to be smaller than the width $2a$ so that the charge and current distribution variation in the strip are primarily in the horizontal (z) dimension. The entire structure is embedded into a dielectric host with permittivity ε_d . A harmonic time dependence $e^{-i\omega t}$ em (optical) field is impressed



$$2\pi\omega = c / \lambda$$



Due to symmetry around the $z = 0$, the cavity supports the resonances for which magnetic field has either odd parity (when currents in the strips are symmetrical), or even parity (with currents in the strips anti-parallel or asymmetrical)

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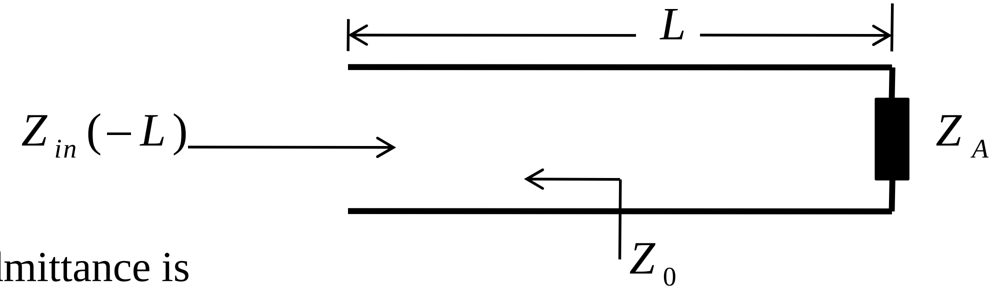
About impedance of a finite length of transmission line terminated at end with impedance

A transmission line (TL), that could be a free space too, is having length L, and characteristic impedance Z_0 is terminated by impedance Z_A

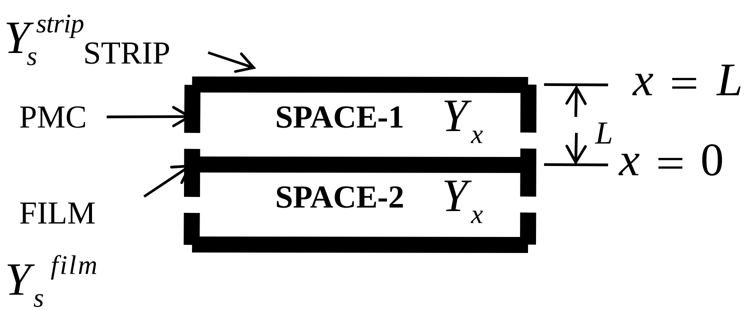


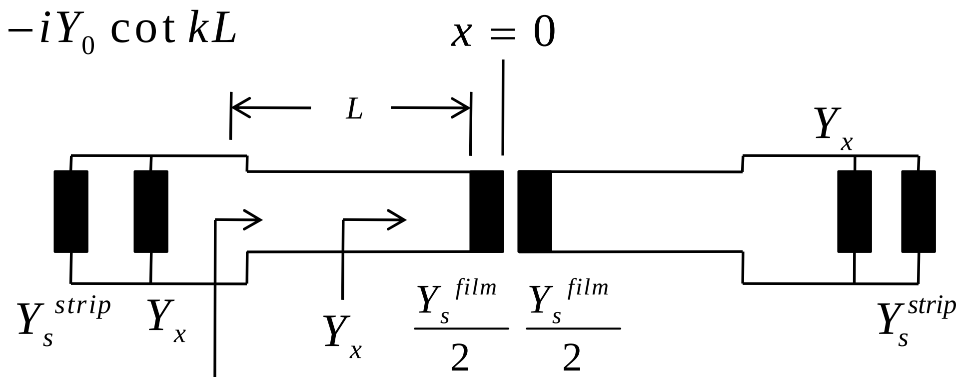
$$Z_{in}(-L) = Z_0\left[\frac{Z_A + iZ_0 \tan kL}{Z_0 + iZ_A \tan kL}\right]$$

Admittance is

$$\frac{1}{Y_{in}} = \frac{1}{Y_0}\left[\frac{\frac{1}{Y_A} + \frac{i}{Y_0}\tan kL}{\frac{1}{Y_0} + \frac{i}{Y_A}\tan kL}\right] = \frac{1}{Y_0}\left[\frac{Y_0 + iY_A \tan kL}{Y_A + iY_0 \tan kL}\right] \qquad Y_{in} = Y_0\left[\frac{Y_A + iY_0 \tan kL}{Y_0 + iY_A \tan kL}\right]$$

For shorted termination $Y_A \to \infty$, $Y_{in}(-L) = -iY_0 \cot kL$





PMC $Y^{PMC} = 0$

$$Y_{in} = Y_x \frac{Y_s^{film} + 2iY_x \tan kL}{2Y_x + iY_s^{film} \tan kL}$$

Total admittance at $x = L$ of cavity is

$$Y_{total} = Y_s^{strip} + Y_x + Y_{in}$$

Admittance at boundary for symmetric and asymmetric cases of current flow

For asymmetric case the film has no role thus no currents flow in the film we take the admittance of the film at $x = 0$ as infinity

$$Y_{in} = Y_x \frac{Y_s^{film} + 2iY_x \tan kL}{2Y_x + iY_s^{film} \tan kL} \qquad \text{put} \quad Y_s^{film} = \infty ; \quad L = d/2, \quad k = k_x$$

$$Y_{in} = Y_x \frac{1}{i \tan(k_x d/2)} = -iY_x \cot(k_x d/2)$$

Total admittance at $x = d/2$ is $Y_{total} = Y_s^{strip} + Y_x + [-iY_x \cot(k_x d/2)]$ and this should be zero

$$Y_s^{strip} + Y_x - iY_x \cot(k_x d/2) = 0$$

For asymmetric case the film carries the 'return current' of the two strips and thus film admittance is finite and we have the total admittance summed up at the boundary x = d / 2 to zero and get

$$Y_s^{strip} + Y_x + Y_x \frac{Y_s^{film} + 2iY_x \tan(k_x d/2)}{2Y_x + iY_s^{film} \tan(k_x d/2)} = 0$$

Magnetic resonance with even magnetic field symmetry cavity mode

For this symmetry, no current flows in the film admittance sheet ($z = 0$) and hence this sheet has no effect on the modal field structure. In contrast the currents in the top and bottom sheets at x =+/-($d/2$) are strong and they flow in the opposite directions, thus resulting in the effective magnetic dipole response: and this is what we called Magnetic Plasmonic Resonance, or refer as magnetic response. This is y –directed an the magnetic field of this magnetic resonance behaves as

$$\vec{H} = (\hat{y}) A(\omega_m, x) \sin\left(\frac{N\pi(z-a)}{a}\right) \qquad N = 1, 2, 3\ldots$$

N : integer counting number of field oscillations in z direction with in $2a$; while $A(\omega_m, x)$ is an even function separately be defined for inside and outside the cavity. The ω_m is the magnetic resonance frequency (ies) satisfying the dispersion relation obtained from matching the fields inside and outside the cavity (impedance matching)

$$Y_s^{strip} + [Y_x - iY_x \cot(k_x d/2)] = 0 \qquad Y_x = (\omega_m \varepsilon_d)/(ck_x) \qquad k_x = \sqrt{\left(\frac{\omega_m}{c}\right)^2 \varepsilon_d - \left(\frac{N\pi}{2a}\right)^2}$$

When strip admittance $\left|Y_s^{strip}\right|$ is large, due to large $\left|(\varepsilon_m - 1)b_s\right|$ i.e. the case for normal GHz range structure, then $\left|\cot(k_x d/2)\right|$ should be large, and it happens at $k_x \approx 0$

$$k_x^2 = \frac{\omega_m^2}{c^2}\varepsilon_d - \frac{N^2\pi^2}{(2a)^2} = 0 \qquad \text{gives} \qquad \omega_m = \frac{\pi N c}{2a\sqrt{\varepsilon_d}}$$

This is simple patch antenna resonance-with no metal property: similar to GLC, when b > skin depth

Magnetic resonance with even magnetic field symmetry cavity mode for nano-rods

However, when the product $\left|(\varepsilon_m - 1)b_s\right|$ is small as for the nano-rods, the ω_m can be made much smaller than the patch antenna resonances, which is crucial in achieving sub wavelength Operation. To obtain an approximate expression in this regime, it is assumed that $k_x d/2 << 1$ $\omega_m\sqrt{\varepsilon_d}/c \ll (N\pi)/2a$, so the $k_x^2 \approx -(N\pi)^2/(2a)^2$ and $\cot k_x d/2 \approx 1/(k_x d/2)$
Put $\varepsilon_m \approx -\omega_p^2/\omega^2$ and apply all these approximations in admittance expression

$$Y_s^{strip} + Y_x[1 - i\cot(k_x d/2)] = 0$$

$$Y_s^{strip} + Y_x[1 - i(k_x d/2)^{-1}] = 0 \qquad Y_s^{strip} - iY_x(k_x d/2)^{-1}] = 0$$

$$-ik_0 b_s\left(\frac{\omega_p^2}{\omega^2}\right) - i\frac{\omega_m\varepsilon_d}{ck_x^2(d/2)} = 0 \qquad -ik_0 b_s\left(\frac{\omega_p^2}{\omega^2}\right) + i\frac{\omega_m\varepsilon_d}{c(N\pi)^2/(2a)^2} = 0$$

put $ck_0 = \omega$ make $\omega = \omega_m$

to get MPR resonance frequency as $$\omega_m = N\pi\omega_p\sqrt{\frac{(d/2)b_s}{(2a)^2\varepsilon_d}}$$

The expression shows that the nano-rods structure of unit cell supports Magnetic Resonances when the cavity has sub wavelength size. This also show that the MPR frequency no longer depends on the only size factor (as case with simple antenna), rather depends on size (geometry) plus the metal property (plasma frequency) at optical wavelengths-this we earlier showed via current wave equation

Electric resonance with odd magnetic field symmetry cavity mode

Now consider the resonance with odd magnetic field symmetry. The currents in the upper and lower metal strip admittance sheet at $x = +/- (d / 2)$ in the same direction and its return currents adds in the admittance sheet at $x = 0$; so the three sheets carry currents. The currents in top and bottom sheet are in the same direction-thus resulting in an effecting electric dipole response, thus these resonances we like to call as 'electric resonances'. The magnetic field will be described by

$$\vec{H} = (\hat{y}) B(\omega_e, x) \sin\left(\frac{N\pi(z-a)}{a} \right) \qquad N = 1, 2, 3 \ldots$$

Where $B(\omega_e, x)$ is an odd function around $x = 0$, and ω_e are the electric resonance frequency satisfying the impedance matching condition as:

$$Y_s^{strip} + Y_z \left(1 + \frac{Y_s^{film} + 2iY_x \tan(k_x d / 2)}{2Y_x + iY_x^{film} \tan(k_x d / 2)} \right) = 0$$

Suitable schemes to solve this numerically will give the electric resonance frequencies-here the size of the upper/lower strips and middle strips determine the electric resonance frequency .

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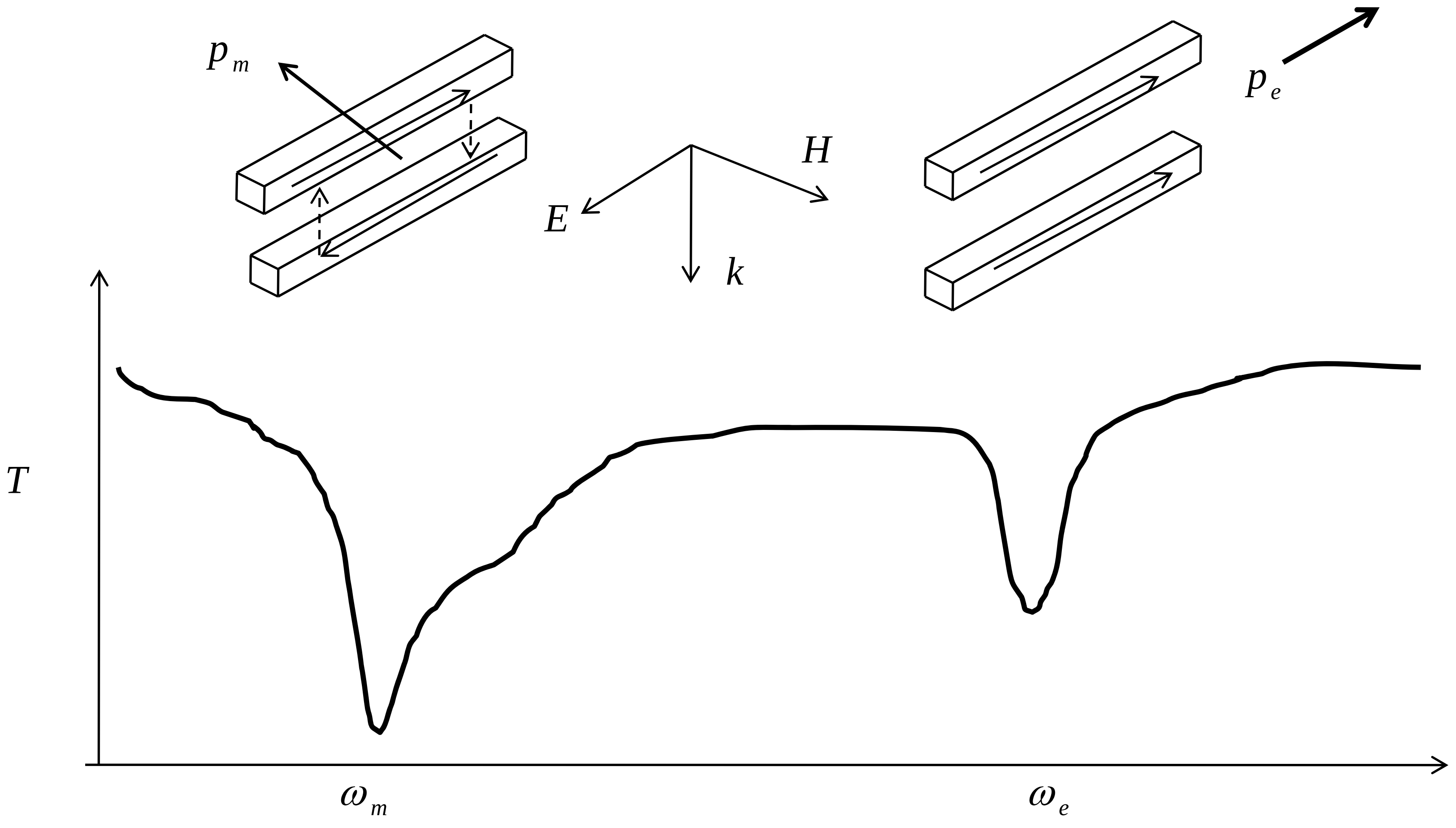
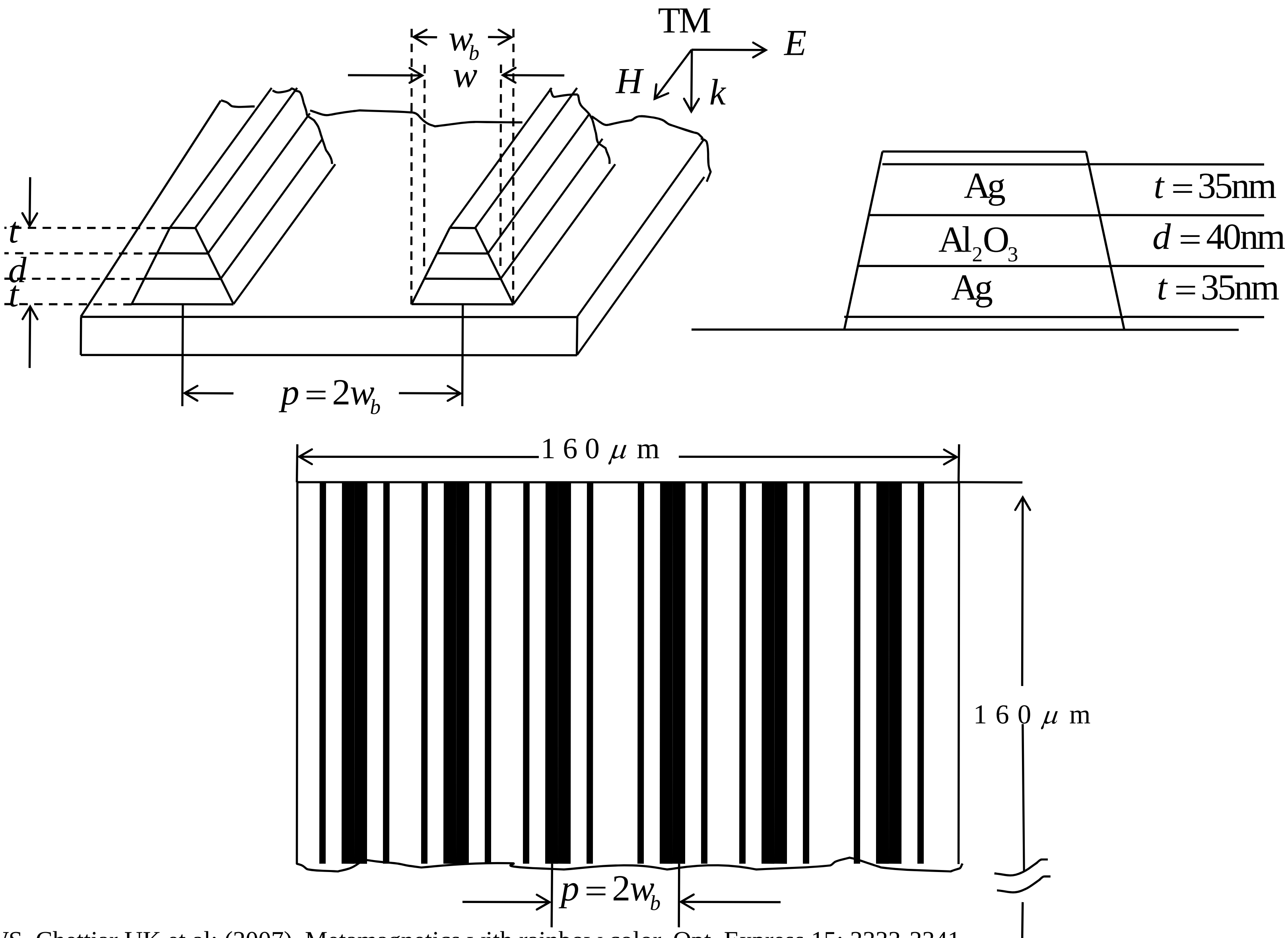
The magnetic and electric resonances



Magnetic meta-atom with coupled nano-strips



Cai WS, Chettiar UK et al; (2007), Metamagnetics with rainbow color, Opt. Express 15: 3333-3341

Size parameters for the magnetic-nano-strips

Sample	w_b (nm)	w (nm)	p (nm)	Coverage (%)
A	95	50	191	0.50
B	118	69	218	0.54
C	127	83	245	0.52
D	143	98	273	0.52
E	164	118	300	0.55
F	173	127	300	0.58

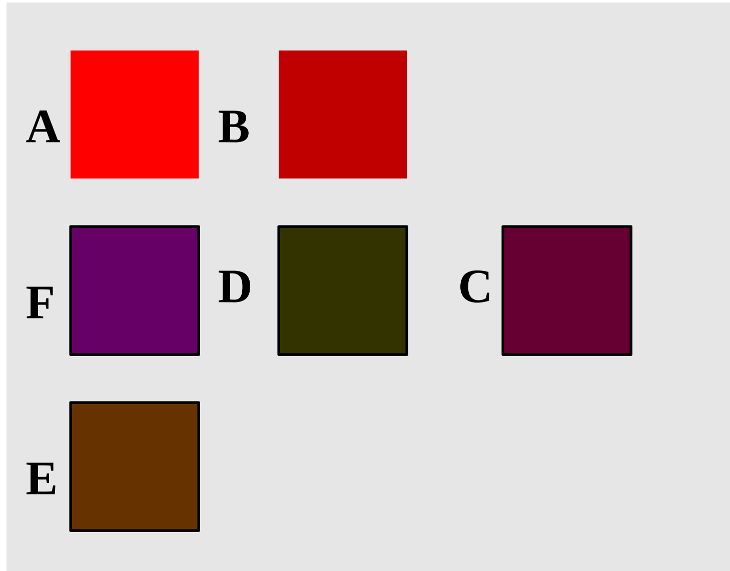
Cover ratio is calculated by the ratio of bottom width w_b to the periodicity p.

Magnetic plasmon resonances are observed for A to F samples from 491 to 754 nm. This frequency range covers most of the visible spectra. It is important to note that the positions of the resonant wavelength move towards ‘blue’ when decreasing the width of the nano-strip, from sample F to A.

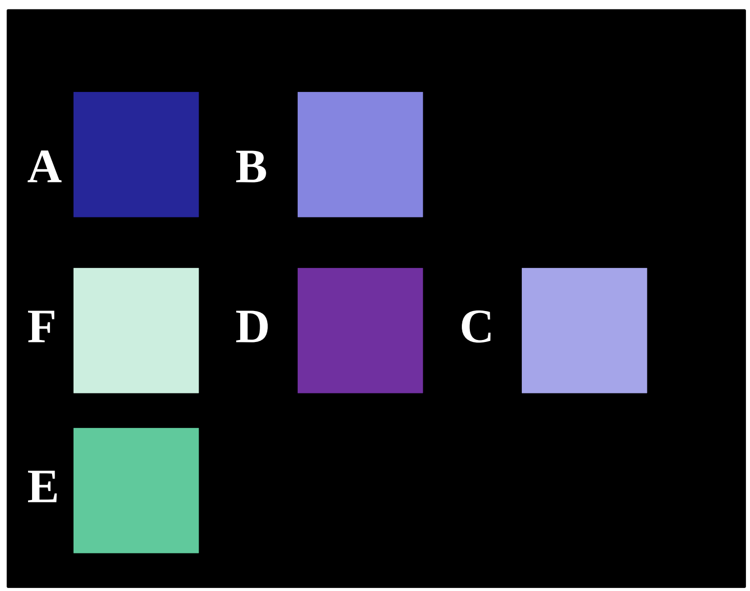
Cai WS, Chettiar UK et al; (2007), Metamagnetics with rainbow color, Opt. Express 15: 3333-3341

Optical microscopy images of magnetic-meta-atom

Transmission



Reflection



Cai WS, Chettiar UK et al; (2007), Metamagnetics with rainbow color, Opt. Express 15: 3333-3341

Conclusions

A new phenomena of magnetic plasmon resonance (MPR) is developed which is distinctly different from Geometric LC resonance (GLC), used in conventional split ring resonators ; because this MPR is determined by the plasmonic properties of metal. This paves way to design metallic meta-material that are magnetically active in optical visible and near IR.

End of part-6