

Left Handed Maxwell Systems In Optical Regime

PART-5

Magnetism-in optical regime

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Few salient points

At the optical frequencies, the dielectric permittivity of material ϵ is ‘generally’ different from that in vacuum.; also we have seen that ϵ takes negative values in metals at frequency less than their plasma frequency. In contrast the magnetic permeability μ for the naturally occurring materials is always close to unity (the free space value), at the optical frequencies.

Near absence of magnetic response is a general rule. Magnetic activity in most materials tends to ‘tail-off’ at high frequencies of even few Giga-Hertz! It is therefore challenge to get, magnetic activity let alone negative permeability, at micro-wave frequencies and higher. Landau and Lifschitz (1984) gave a general argument that magnetic activity arising from atomic orbital currents should be negligible at optical frequency if one neglects the polarization currents; there is certainly no meaning in using the magnetic susceptibility from optical frequencies onwards so we must put $\mu = 1$

Landau LD, Liftshitz EM, Pitaevskii LP (1984) , Electrodynamics of continuous media, Pergamon, New-York

The major reason of absence of optical magnetism is that magnetic field component H of the e.m. waves especially that of light couples to atom very weakly than the electric field components E .

Therefore the interaction of matter with light is ‘one-handed’. That is out of the two fields E and H only one hand i.e. E interacts with matter. The magnetic response (which is due to electronic spin states) in naturally occurring material diminishes at frequencies higher than few G Hz, moreover there are no ‘free-magnetic-monopoles’; and thus it is not possible to obtain the ‘magnetic-plasma’ frequency (as was the case with the free electrons’ in metals, we have thus electric plasma frequency).

Thus it is challenging to get magnetic response at μ W frequency and higher, let alone more extreme value like ‘negative permeability’.

Importance of “artificial-magnetism” the magnetic meta-material at optical frequencies

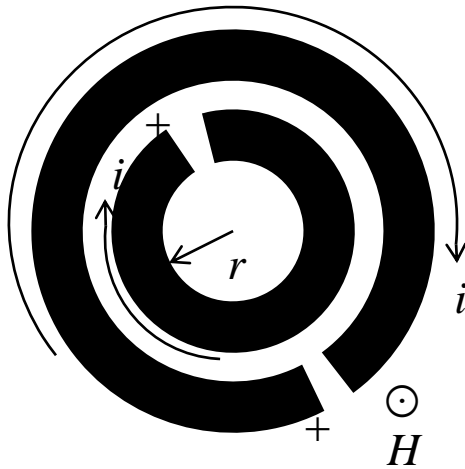
Meta-Magnetics-at optical frequency

In a landmark work by Veselago showed that (1968) a material with simultaneous negative permeability $\mu = \mu' + i\mu''$ and negative permittivity $\epsilon = \epsilon' + i\epsilon''$ has a negative refractive index. The necessary and strong condition of the negative index is given by $\epsilon'\mu'' + \epsilon''\mu' < 0$. This strong condition strictly imply $n' < 0$; $n = n' + in''$; cannot occur in passive meta-material with permeability $\mu = 1 + i0$. Consequently the magnetic response is essential in NIM. Unfortunately there is no magnetic response in nature at optical frequencies; hence to get an optical NIM an artificial magnetic response is prerequisite. At optical wavelengths (frequencies) obtaining $\epsilon' < 0$ is easy compared with getting $\mu' < 0$, as the noble metals have negative ϵ' value above the plasma wavelength. In a meta-material consisting of artificial sub wave-length structure with tailored properties, the magnetic response is not limited to the electronic spin states of individual atoms instead magnetism can be achieved (even in optical frequencies) by specially designed ‘meta-atoms’.

Therefore artificial magnetism is possible in meta-material as long as the H component incident light can interact effectively with meta-atoms. This was the idea of having split-ring-resonators (SRR) as micro-wave meta-atoms for artificial magnetism. Can this SRR be scaled to THz and optical zone ?

As E arm of the light interacts with the metals (better), could the E field excitation give an artificial magnetic moment in magnetic-meta-atom ?

Pendry's recipe for artificial magnetism at micro-wave (uW) region-the double SRR



d inter-ring gap
 c conductor width
 t conductor thickness

C_0 Capacitance between two ring shaped electrodes

$$C_0 \approx 2\pi r \epsilon_0 (c + t) / d; \quad L \approx 2\mu_0 r$$

R Ohmic loss in copper

$$R \approx \pi r / c \delta \sigma$$

with δ and σ as

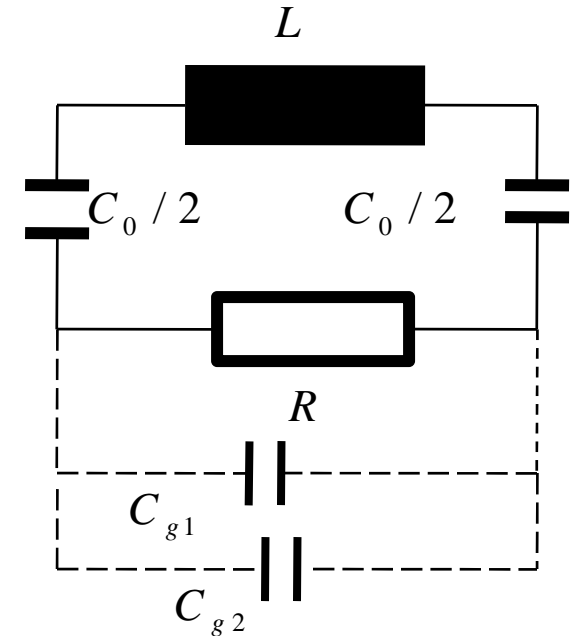
skin depth & conductivity

Magnetic resonance frequency of SRR is the lowest resonance which depends on circuital parameters

$$\omega_0 = \sqrt{\frac{1}{(L + R / i\omega_0)C}}$$

$$C = C_0 / 4$$

Gap capacities do not play significant role in the double SRR



The problem of low coupling to H field can be overcome by using meta-material that mimic the magnetism at high frequency. This is Pendry's SRR, that play significant role in artificial magnetism at G Hz range-is 'magnetic meta-atom'

Actually the term SRR was coined by Hardy in early 80's, to describe a hollow metallic cylinder with a linear cut that exhibited magnetic resonances at 1 G Hz.

Induced magnetism in metals

Consider a circular metal plate placed in an oscillating e.m. wave, with its H field polarized normal to the flat surface. This metal plate is magnetically active but weakly though; as the oscillating magnetic field induces a circular current in the round plate, producing a magnetic flux as to oppose the external magnetic field. Consequently the metal plate is weakly diamagnetic, and array of such metal plates would be exhibiting $\mu_{eff} \sim < 1$

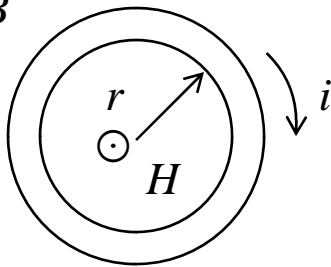
Because the currents are all in periphery of the metal plate, we can remove the central part of it, to make it ring like structure. This magnetic response via Lenz's law to the metal plate is purely 'inductive'; the induced emf $v(t)$ is just $L di/dt$; where the L is the self inductance of metallic ring, and i is the current. Thus response is purely inductive and non-resonant. To introduce the resonance behavior and to enhance the (weak) magnetic response, a capacitance is introduced, which prevents the formation of circular currents, and charges accumulates across the gaps. With both capacitance and inductance the SRR is a resonating element.

$$v(t) = L \frac{di}{dt} + Ri \quad i(\omega) = v(\omega) / [R + iL\omega] \quad v(t) = -\frac{d\phi_B}{dt} = -\pi r^2 \dot{B}$$

For ring with gap

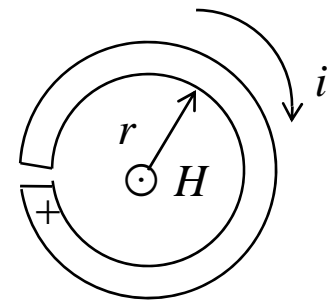
$$\phi_B = \pi r^2 B$$

$$B = \mu_0 H$$



$$v(t) = L \frac{di}{dt} + \frac{1}{C} \int_0^t i(\tau) d\tau + Ri$$

$$i(\omega) = \frac{v(\omega)}{R + i\omega L + \frac{1}{i\omega C}}$$



At resonance $\omega_0 L = (\omega_0 C)^{-1}$ the current is $v = e / R$ and as R is very small we get high i thus enhanced magnetic response

Generation of magnetic moment from SRR array

Capacitance is more effectively introduced by two SRR and the inter-ring capacitance dominates, where the electric dipole moments, appearing at the opposite gap gets almost cancelled thus, enhancing the magnetic activity. This was Pendry's recipe for obtaining artificial magnetism at GHz range of micro-wave region. The size of the SRR is very small compared to the wavelength at free space of the exciting radiation (e.m).

Magnetic moment per unit volume is (i.e. due to single SRR) $p_m = \pi r^2 i$

$$v(t) = -(\pi r^2) \dot{B} = -\pi r^2 \mu_0 \dot{H}$$

$$v(\omega) = -\pi r^2 \mu_0 (i\omega) H ; \quad i(\omega) = v(\omega) / Z(\omega) ; \quad Z(\omega) = R + i\omega L + (i\omega C)^{-1}$$

$$i(\omega) = -\pi r^2 \frac{\mu_0 (i\omega) H}{R + i\omega L + \frac{1}{i\omega C}} ; \quad \omega_0^2 = \frac{1}{LC} ; \quad R \cong 0$$

$$= -\pi r^2 \frac{\mu_0 (i\omega) H}{i\omega L - \frac{i}{\omega C}} = -\pi r^2 \frac{\mu_0 (i\omega) H}{i \left(\omega L - \frac{\omega_0^2 L}{\omega} \right)} = \pi r^2 \frac{\mu_0 H \omega^2}{L (\omega_0^2 - \omega^2)}$$

$$p_m = \frac{\pi^2 r^4 \mu_0 H}{\left(\frac{\omega_0^2}{\omega^2} - 1 \right) L}$$

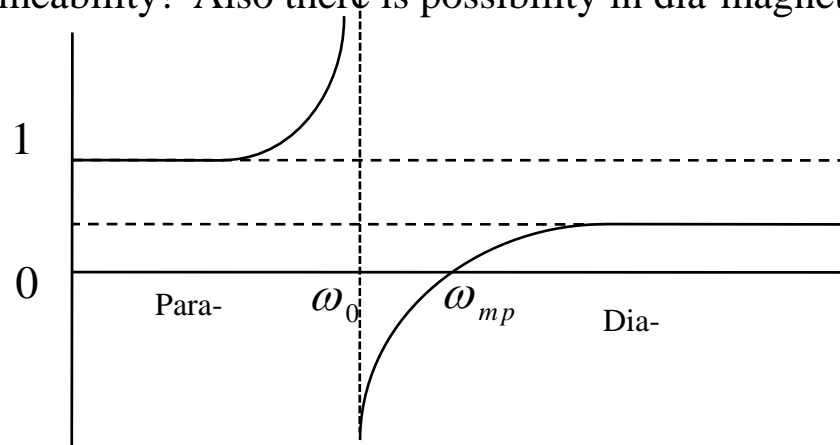
With N SRR in volume V we have

Effective permeability is
$$\mu_{eff} = 1 + \frac{N p_m}{V H}$$

The positive or negative magnetic response (permeability) depending on the frequency (wavelength) of radiation

$$\mu_{eff} = 1 + \frac{Np_m}{VH} = 1 + \frac{N}{V} \frac{\pi^2 r^4 \mu_0}{\left(\frac{\omega_0^2}{\omega^2} - 1\right)L} \quad \begin{array}{ll} \omega < \omega_0 & \text{Para -} \\ \omega > \omega_0 & \text{Dia -} \end{array}$$

Such an expression leads to a dia- or paramagnetic response to the external magnetic field depending on whether the wavelength of the incoming H is shorter or longer than the resonant wavelength of the SRR. Thus μ_{eff} is other than unity; even though the natural materials comprising the SRR (copper, RT-Duroid) have unity as effective permeability! Also there is possibility in dia-magnetic zone of getting permeability negative, for $\omega_0 < \omega < \omega_{mp}$



Note that this is very rough analysis; only to show that strong artificial magnetic activity can be obtained

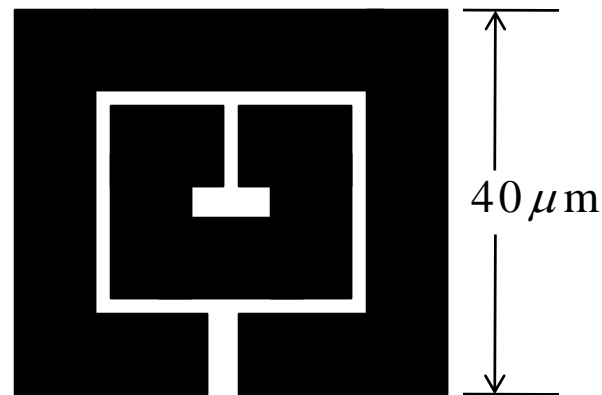
For details : <http://pdfcast.org/pdf/basic-theory-of-small-resonators-for-left-handed-maxwell-systems-lecture-4-by-shantanu-das>

Scaling the SRR structure to micron (μm) size for T Hz meta-magnetic structure

SRR can be moved towards higher frequency by scaling (shrinking). Since Maxwell's equations are scalable, shrinking the design in size could lead to similar artificial magnetic response at higher frequencies.

Here we note that the material properties of metals do not scale; metals which are important components of meta-material, have a very different property in the optical range, where the plasmon resonances are of critical importance, as opposed to micro-wave case. Also fabrication limitations comes in as for micro-wave range where SRR are of mm scale (at G Hz skin depth is of μm) to bring down to nm scale (comparable to skin-depth at optical region)!

Yen TJ, Smith DR, et al; (2004); Terahertz magnetic response from artificial material; Science 303: 1494-1496

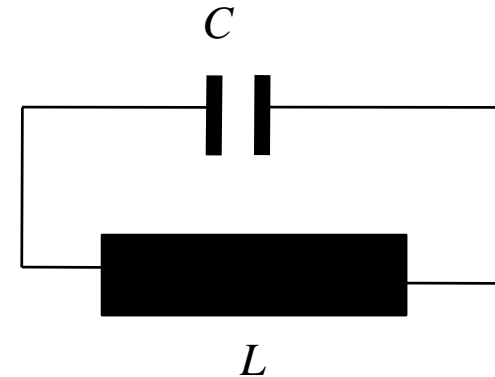
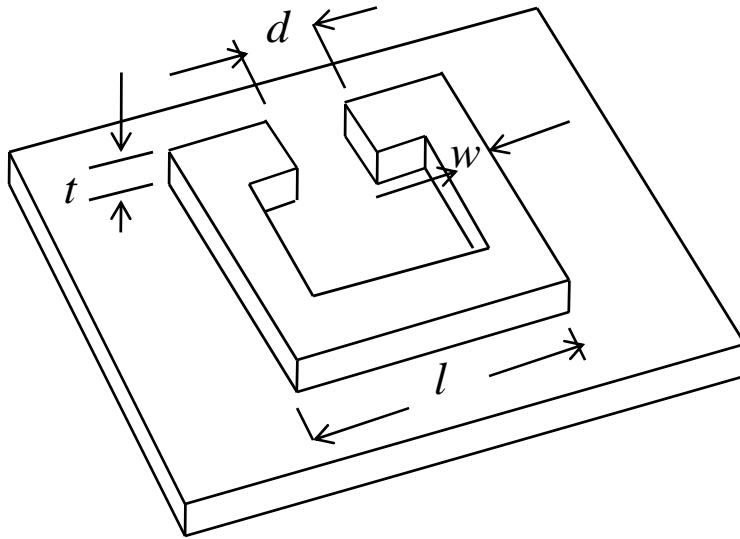


Using scaling technique; and Electron Beam Lithography method, the Pendry's double SRR has been scaled up to 1 T Hz . Each SRR here is about 40 μm in size; near resonance frequency the effective permeability exhibits +1 maximum and -4 minimum.

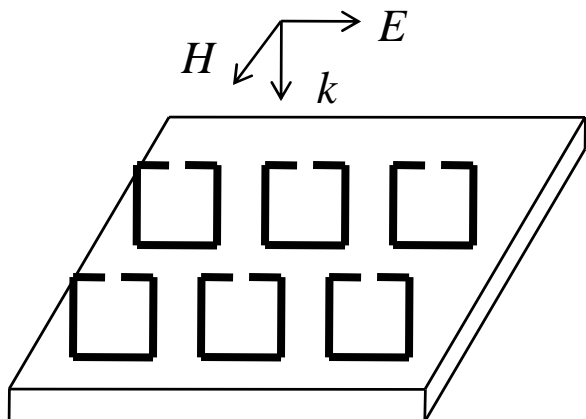
Scaling the SRR structure to nm size for T Hz and beyond to optical regime

The basic principle in design is that the unit structures can support a principal eigen-mode with a circular current distribution that gives rise to magnetic moment. Such magnetic moment give rise to resonance behavior and the magnetic moment parallel to the incident magnetic field H gives rise to $\mu_{eff} > 1$ below the resonance frequency ω_0 and the magnetic moment is anti-parallel to the incident magnetic field H produces $\mu_{eff} < 1$ for the structure, above the resonant frequency.

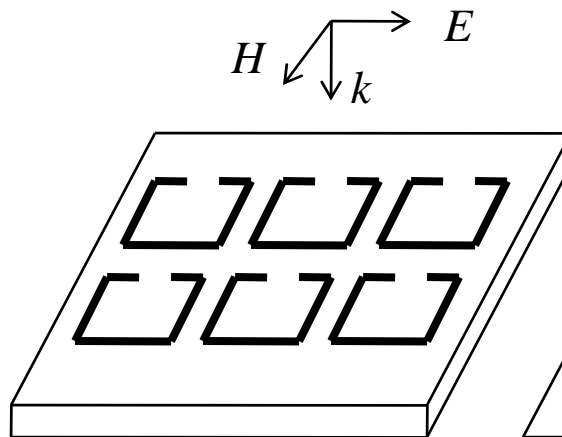
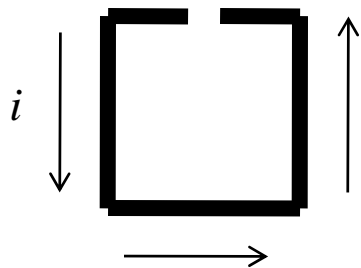
In order to obtain a meta magnetic response at higher frequencies different variations of SRR have been explored, one of the most important design for achieving magnetic resonance towards the optical regime is array of single SRR lying in a plane perpendicular to the direction of wave propagation.



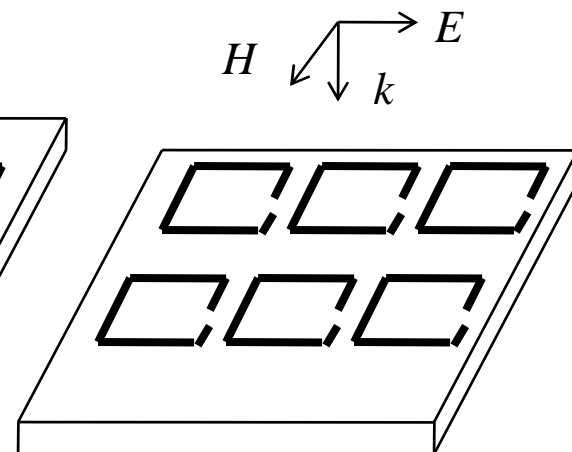
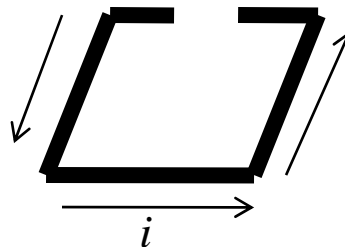
Considering different polarizations for single SRR



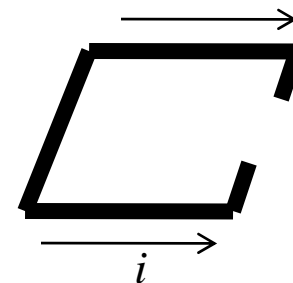
Standard SRR operation



Magnetic dipole possible
due to asymmetric current



No magnetic response when E
is parallel to arms of SRR

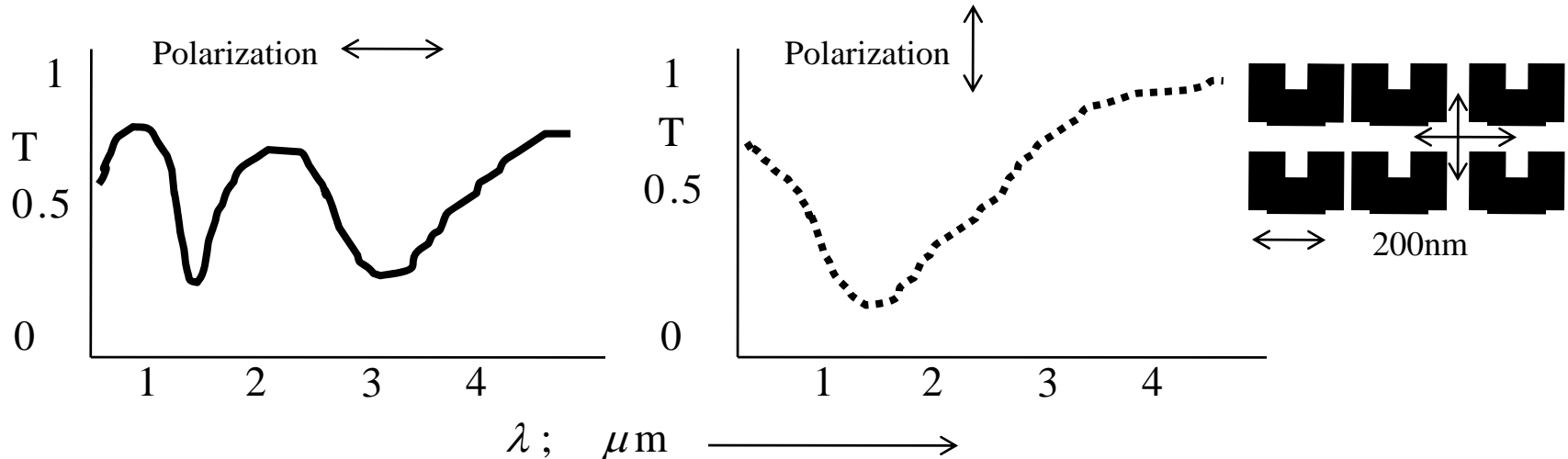


Working of single SRR

Besides the current symmetry considerations the underlying physics of the planer single SRR can be viewed in terms of equivalent circuit . The single SRR is viewed as an LC oscillator, with inductance corresponding to ring part, and capacitance corresponding to gap. For such single SRR there are two means to excite the resonance, either magnetic field vector H pass through the inductor coil (the ring), or the electric field E be aligned to the capacitor plates, (the cut surface).

Magnetic resonance with E field excitation has been achieved in mid IR of 3 μm wavelength. The size is about 320 nm. The transmission spectral (T) for two polarization is as follows

Liden S, Enkrich C, Wegener M, et al, (2004), Magnetic response of meta-material at 100 T Hz, Science 306; 1351-1353
Enkrich C, et al; (2005), Magnetic meta-materials at telecommunications and visible frequency. Phys. Rev. Lett 95: 203901



Difficulty in scaling and getting thus magnetic response at T Hz and beyond to visible range

It is important to note that the electrically coupled magnetic resonance is an indirect one, and is inevitably weak compared to the magnetic resonance excited by H field directly. In these experiments; the retrieved parameters from T and R spectra, says that effective permeability is about 0.8; at 3 μm wavelength, is marginally small than the magnetically inactive medium. This is major reason why meta-magnetics in planer single SRR cannot be used in interesting application of negative magnetism like negative refractive index experiments, em cloaking .

Another drawback of planer SRR is breaking of geometrical scaling and subsequent saturation of the resonance frequency. This saturation effect fundamentally prevents the SRR from providing the magnetic resonance at “true” optical frequencies-the visible range.

This saturation is an intrinsic property of SRR as a resonator and is independent of how the SRR is excited (either via H the standard practice or via E field).

The reason could be due to kinetic energy of electron and electron self inductance, which results from plasmonic behavior of noble metals in optical regime!

Zhou J, Pendry J B, et al; (2005) ; Saturation of magnetic response of split-ring-resonator at optical frequencies; Phys Rev Letts; 95 223902.

Tretyakov S (2007), On geometrical scaling of split-ring and double bar resonators at optical frequencies; Met-amaterials 1: 40-43.

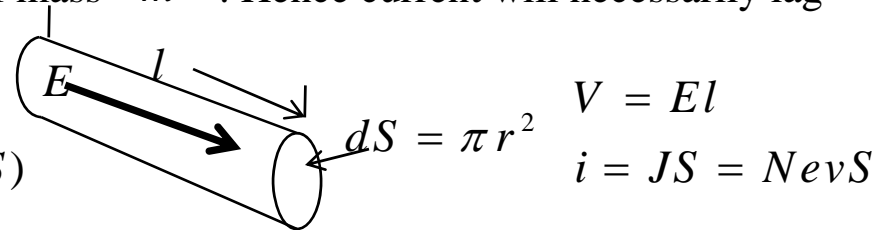
Kinetic/Inertial Inductance

We can see that limitation to scale up to IR comes due to inertia of electrons, the limit of magnetic resonance is about 350 THz. A normal L - R circuit excited by step voltage has following relation

$$V = L \frac{di(t)}{dt} + Ri(t) \quad i(t) = \frac{V_0}{R} [1 - \exp(-t/\tau)] \quad \tau = L/R$$

The current seem to be delayed due to the time constant. We may ask a question, is there anything else in electrical circuit that cause delay? Clearly in order to have current; the charge carriers must be accelerated and it takes time to accelerate particles of mass m . Hence current will necessarily lag behind voltage causing it to rise.

$$eE = m \left(\frac{dv}{dt} + \frac{v}{\tau_{coll}} \right); \text{ put } E = V/l; \quad v = i/(NeS)$$



To get $V = \frac{lm}{Ne^2 S} \left(\frac{di}{dt} + \frac{i}{\tau_{coll}} \right)$ Compare with L - R circuit equation, we have kinetic/inertial inductance and resistance, as: $L_i = \frac{lm}{Ne^2 S}$ $R_i = \frac{lm}{Ne^2 S \tau_{coll}}$

Looking at expression of kinetic/inertial resistance, it is nothing but ‘ordinary resistance’. On the other hand the expression for kinetic inductance is new. As the area of conductor becomes smaller and smaller the kinetic inductance becomes comparable to magnetic inductance. It can even become dominant inductance as in example in circuit containing nano-rods. Since $\omega_p^2 = Ne^2 / \epsilon_0 m$

$$\text{Kinetic inductance is } L_i = \frac{l}{\pi r^2 \epsilon_0 \omega_p^2}; \quad S = \pi r^2$$

Interpretation of additional inductance at very high frequency

The presence of this additional inductance can be explained by noting that at high frequencies the currents are hardly diffusive, and almost ballistic; because the distance through which electrons move within a period of wave becomes comparable to the mean free path in metals. This means that if the frequencies are too high, the electrons can hardly be accelerated and the response falls. The mass of electron contributes additionally to the inductance. Current density $J = N e v = N e (-j e E / \omega m)$, then the potential drop is $V \approx \{m l / N e^2\} (\partial J / \partial t)$, implying an inductance that is proportional to electron mass.

The effective damping factor Γ also increases, becomes much larger as the size of the ring is reduced. This is due to the fact that the proportional energy in ballistic motion of the electron increases as size gets reduced and resistive losses are then very large indeed.

Thus even if the size of the ring were negligible the inertial/kinetic inductances would still be present preventing scaling to higher frequencies. Well, this effect of inertial inductance is also there even if superconducting SRR are employed.

The large increase in damping as the sizes are scaled down broadens the resonance and permeability does not rapidly disperse; and the regions of MNG vanishes altogether. This increase in damping is matter of great concern for optical frequencies.

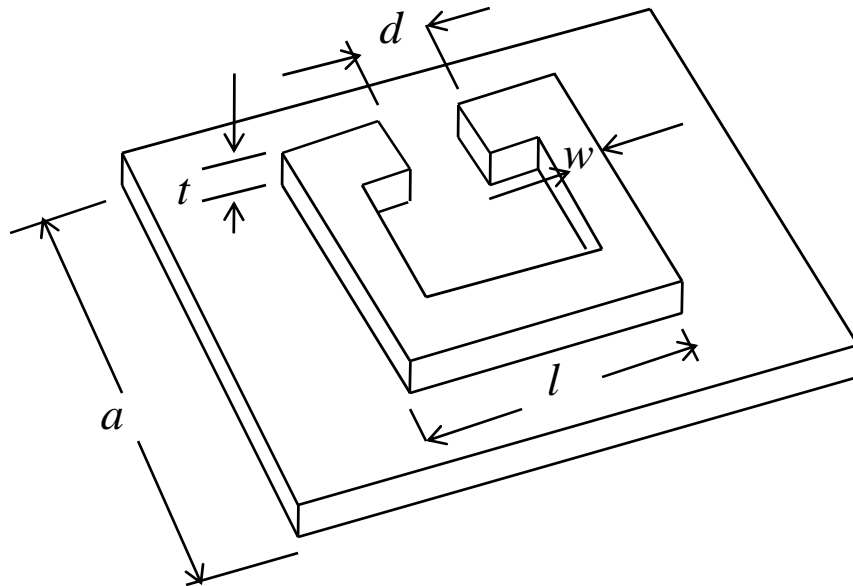
SRR with this two splits tends to tail off at wave length of 5 μ -m. (IR region). By adding more capacitive gaps to lower the net capacitance and adjusting the dielectric constant of substrate MNG with this scaled down is achievable at 1.5 μ -m.

Parallel metal sticks (pair of wire) of 100nm length periodically embedded in dielectric behaves as MNG at IR.

Scaling the SRR structure to nm size for T Hz and beyond to optical regime

As we discussed, the SRR is essentially a LC resonator, with magnetic resonance at $\omega_0 = 1 / \sqrt{LC}$. To first order approximation $L = \mu_0 l^2 / t$ and $C = \epsilon_0 w t / d$. Let a be the unit cell size; the a serves as scaling factor and all the other parameters (l, d, w, t) are proportional to a . The expression of L and C show that both scale as a . This gives an important relation between the resonance frequency and scaling factor; $\omega_0 = c \sqrt{d / (w l^2)} \propto a^{-1}$ where $c = 1 / \sqrt{\epsilon_0 \mu_0}$ speed of light in vacuum. A crude but useful relation as resonance frequency is inverse to size of SRR !!

This is crude as the reasoning above we based on ‘perfect metal’ having infinite electron density. In this case current excited by the external field is carried by electrons with zero velocity!!



In reality since metal has finite electron density and hence a finite plasma frequency, there must be finite electron velocity proportional to current i in the loop. Hence there is additional kinetic inductance L_{kin} that accounts for the total kinetic energy of the moving electrons. As we make size smaller the L_{kin} increases we say $L_{kin} \propto a^{-1}$. Result is the magnetic resonance is

$$\omega_0 = 1 / \sqrt{(L + L_{kin})C} = 1 / \sqrt{a^2 + \text{const}}$$

$$LC \propto (a)(a) = a^2; \quad L_{kin} C \propto (a)^{-1}(a) = \text{const}$$

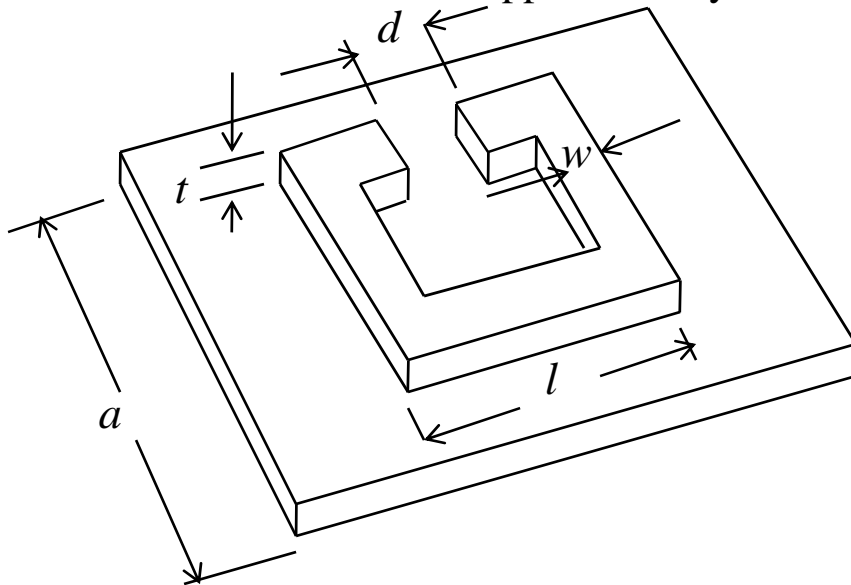
Saturation effect in SRR

$$\omega_0 = 1 / \sqrt{(L + L_{kin})C} = 1 / \sqrt{a^2 + \text{const}}$$

The const is the origin of the saturation effect of SRR, is due to product of L_{kin} and C . The simulations show the SRR cannot go beyond 350-375 THz corresponding to about 800 nm.

Klein MW et al, (2006) ; Single split ring resonator at optical frequency; limits of size scaling; Opt. Lett 31: 1259-1261

As we approach dimension of nm say 100 nm, 50 nm etc. we are approaching skin depth dimensions of the noble metals; which is approximately 50 nm at optical (visible) wavelengths.



Magnetic Plasmon Resonance and Geometric LC (i.e. GLC) resonance

For uW region of G Hz artificial magnetic element SRR or double SRR providing $\mu < 0$, the metal used is considered as perfect conductors, as because skin depth in the G Hz is of order of μm and the size of the structures are mm. The strong artificial magnetic response is achieved by operating the SRR in the vicinity of the LC resonance of the SRR-and the technique can be extended to T Hz and mid IR. In these regions of frequency also metals can be approximated as perfect conductors, as skin depth is small as compared to size of SRR. Therefore the frequencies of these LC resonance are determined entirely by SRR geometry but not by the e.m. property of the metals. In accordance with this the SRR response is enhanced by some particular ratio of radiation λ to the structure size a . Therefore, we refer the LC resonance of SRR as Geometric LC Resonance. (GLC)

The situation drastically changes in the optical part of the spectrum, where thin (sub-wavelength) metal components behave very different when their size becomes less or comparable to the skin depth. The skin depth is of the order of 50 nm for noble metals in optical frequency regime. Here in this frequency range the “electrical surface plasmon resonance” (SPR) occurs due to collective electron oscillation in metal structures (nevertheless this gives rise to various plasmon-enhanced optical phenomena). This plasmonic nature of the e.m. response in metals for optical/mid-IR frequencies is the main reason why the original method of GLC resonances in uW/mid-IR range is not extendable to higher frequencies.

Thus Magnetic plasmon resonance occur in the nanostructures much smaller or comparable size than in optical wave-length.

Theory and mathematics of Magnetic plasmonic resonance in next class

Objective of meta-magnetic structure at optical frequency

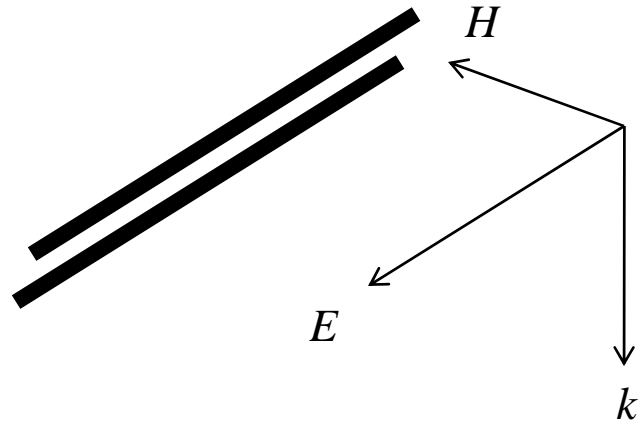
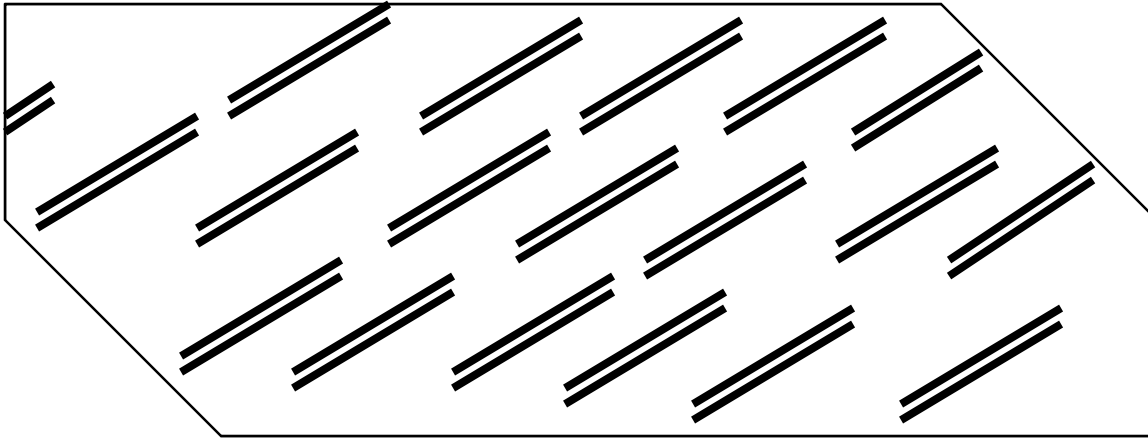
Therefore objective is to find a structure that fulfills

1. Compatibility with known nano-fabrication methods.
2. Magnetic resonance be excited by external magnetic field
3. Negligible saturation effect for high frequency operation

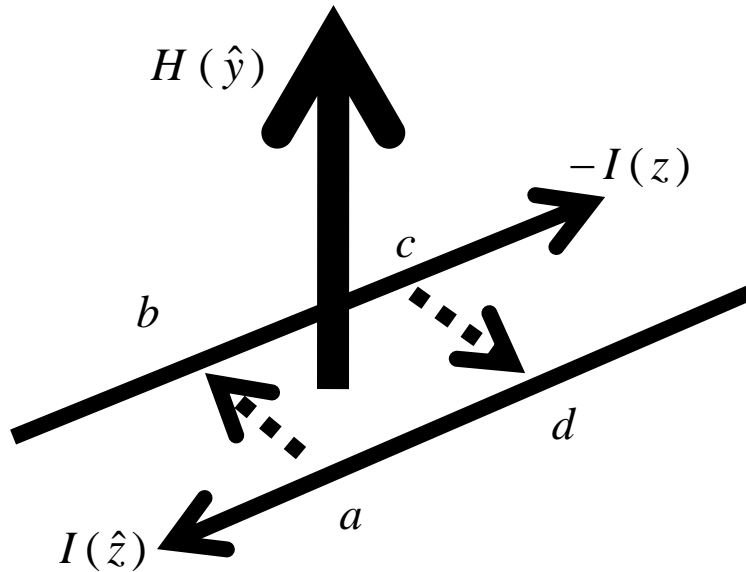
More advanced artificial elements for magnetic meta-material include array of pairs of metallic rods, plates, or strips. Each of these structures is capable of supporting a principal eigenmode with an anti-symmetric current distribution in the coupled system. The optical diamagnetic response in pair of metal rods was predicted in 2002

Podolsiy VA, Sarychev AK, Shalev VM (2002); Plasmon modes in metal nano-wires in left handed materials. J of Non-Linear Opt. Phys Mater 11: 65-74

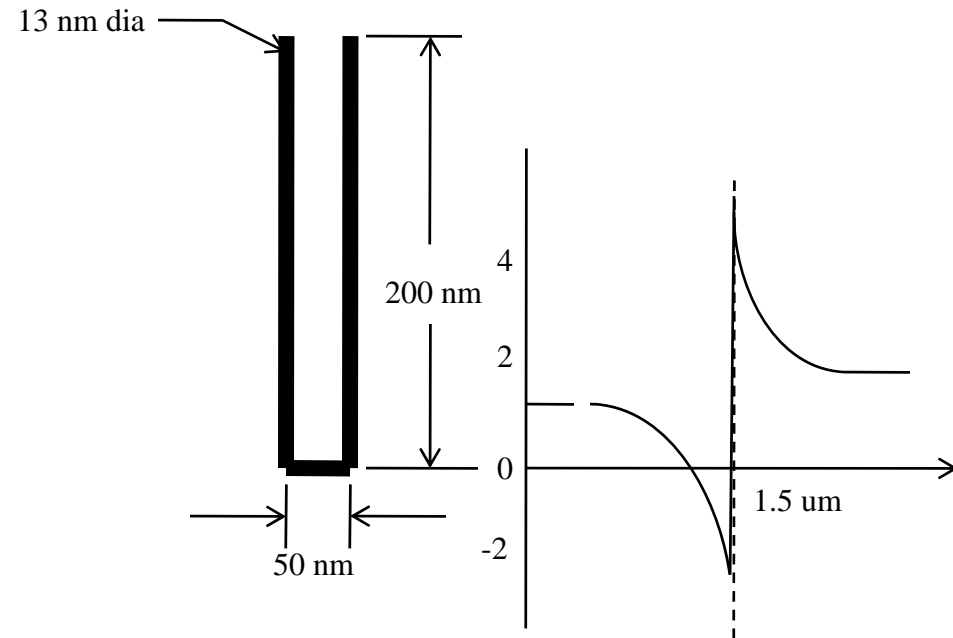
Composite consists of pair of metallic nanorods for magnetic response at optical frequencies



Magnetic plasmon resonance in nano-antenna structure (pair of nano wires)



Current in two parallel metal nano-wires excited by external magnetic field the displacement currents closing the circuit –generates a magnetic dipole



Magnetic plasmon resonance in silver nano-antenna, at frequency $\lambda = 1.5 \mu\text{m}$ with silver di-electric function as

$$\epsilon_m(\omega) \cong \epsilon_b - \frac{\omega_p^2}{\omega^2(1 - i\omega_\tau/\omega)}$$

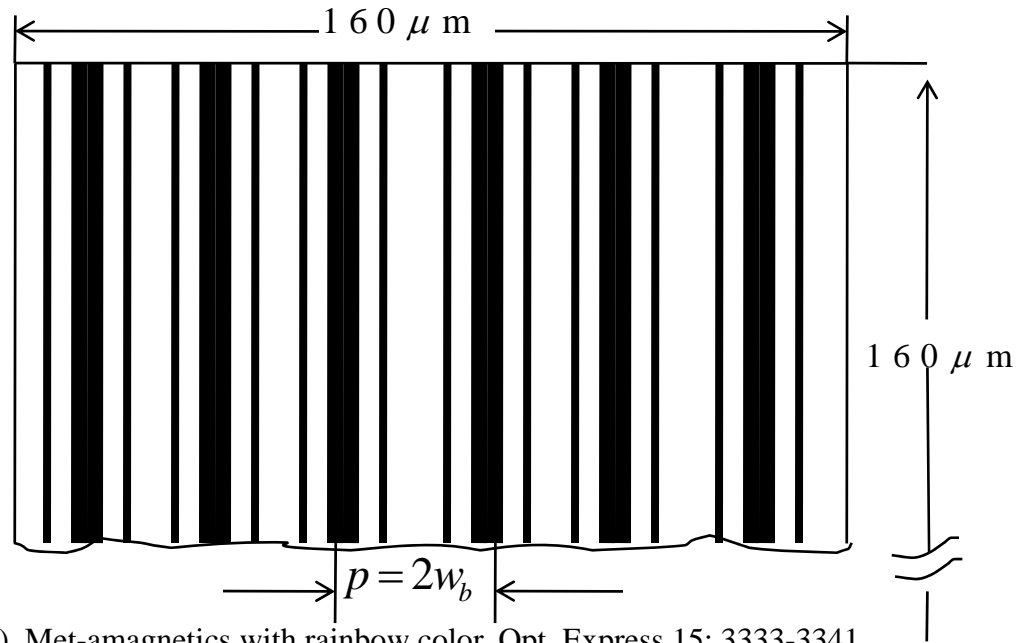
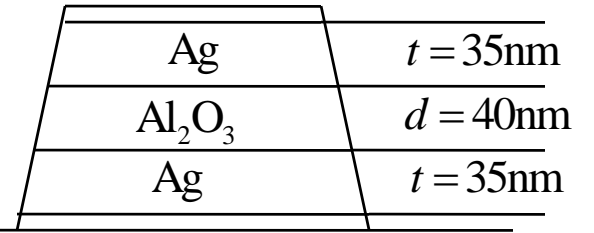
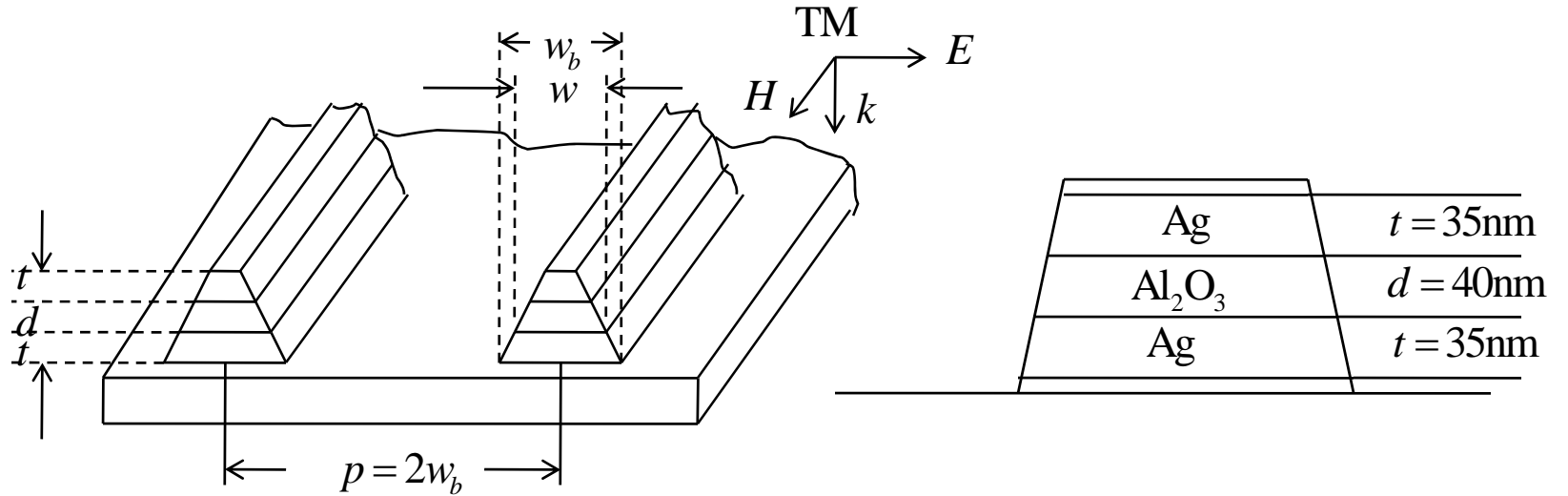
$$\epsilon_b = 5; \quad \omega_p = 9.1; \quad \omega_\tau = 0.02$$

For example at $\lambda = 1.5 \mu\text{m}$

$$\epsilon'_m \approx -120; \quad \epsilon''_m / |\epsilon'_m| \approx 0.025$$

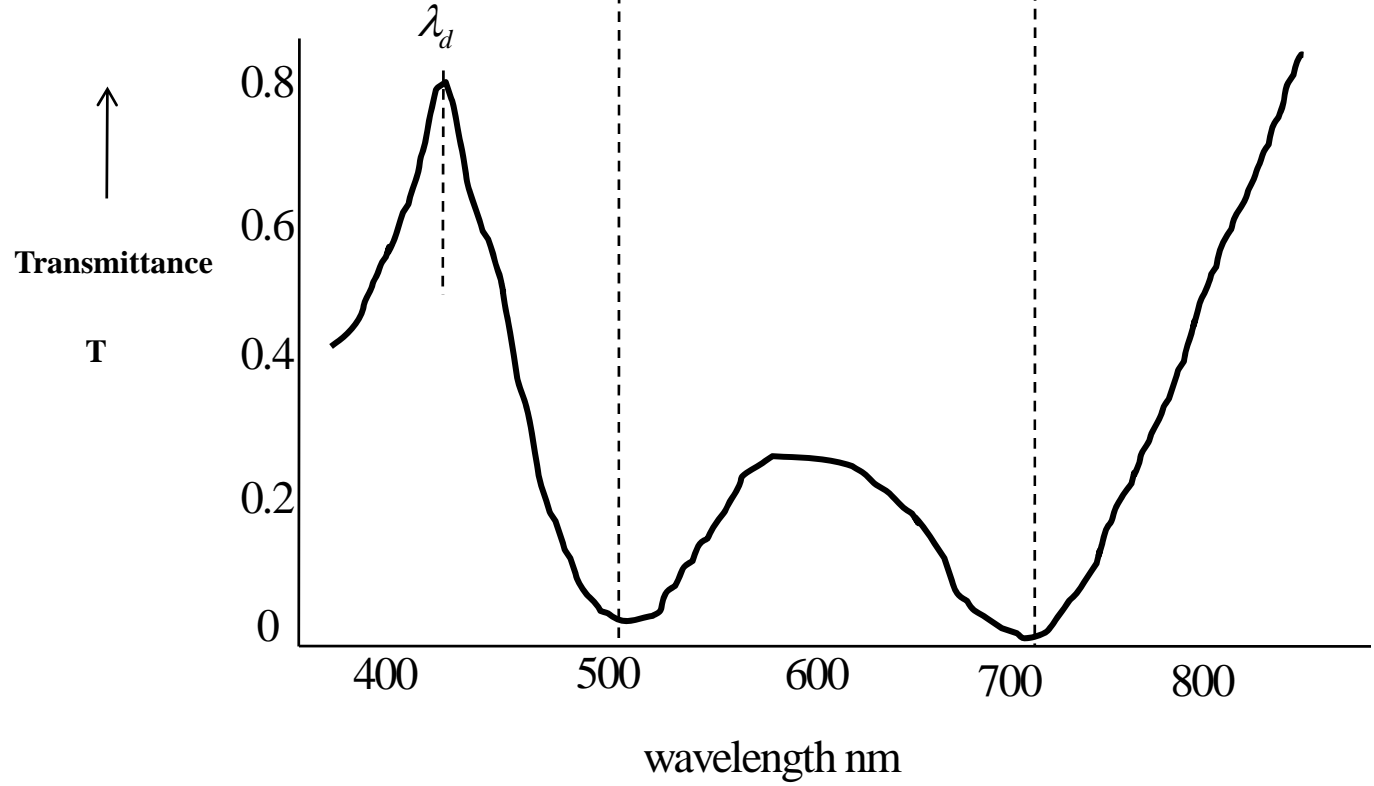
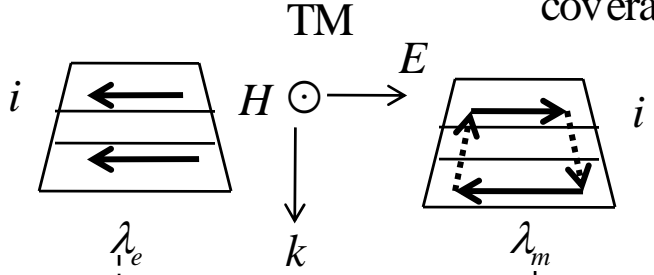
Volume fraction $p = 0.3$

Magnetic meta-atom with coupled nano-strips



The T spectra of coupled nano-strips

$w_b = 164\text{nm}$; $w = 118\text{nm}$; $p = 300\text{nm}$
 coverage% = 55



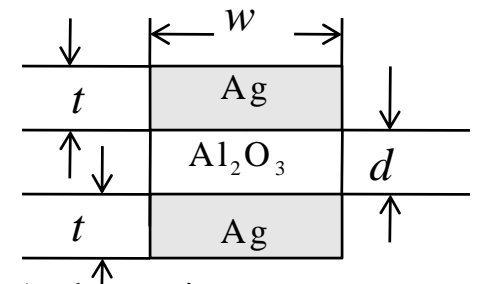
Magnetic & electric resonances in the T spectra of coupled nano-strips

Under TM excitation magnetic resonance is around λ_m , resulting from an anti-symmetric current flow in upper and lower Ag strips, which results from a circular current giving a magnetic dipole response. This magnetic response is major feature that we shall pursue in the coupled nano strip structure. In addition to the anti-symmetric mode, the strip pair of Ag also supports a symmetric current mode at λ_e resulting in electric dipole response. These two resonances induce two minima in T spectra (obviously) will correspond to maxima in R spectra (not shown).and will have corresponding large absorption at these two points; λ_m and λ_e ; which is natural for metal dielectric composite structures.

The T spectra also show a sharp turn back at a shorter wavelength λ_d ; this characteristic wavelength shows a indicates the diffraction threshold and serves as fingerprint of such grating like structure. For 1D grating with periodicity p a diffraction channel gets formed whenever the wavelength λ reaches below diffraction threshold given by $\lambda_{d,j} = n_s p / j$ where j is positive integer, and n_s is the refractive index of the grating substrate (here glass $n_s = 1.52$). When the wavelength $\lambda < \lambda_{d,j}$ strong distortion in transmission spectra is present and substantial optical power gets transferred to grating scattering.

Puscasu I et al; (2001); Modeling parameters for the spectral behavior of infra red frequency selective surfaces; Appl. Opt. 40:118-124.
Wood RW (1935) Anomalous diffractive grating; Phys Rev 48: 928.

Analytical formulas for magnetic nano-strips



We have to follow ‘cavity’ model approach

We have $\lambda_m > 2n_d w$ where n_d is refractive index of spacer (Al_2O_3), the cavity resonant wavelength λ_m are described by

$$\varepsilon'_m(\lambda_m) = 1 - \frac{n_d^2}{t\kappa} \left[1 + \coth\left(\frac{d\kappa}{2}\right) \right]; \quad \kappa = \sqrt{\left(\frac{\pi}{w}\right)^2 - \left(\frac{2\pi n_d}{\lambda_m}\right)^2}$$

Where ε'_m is the real part of the metal permittivity. The above equation has no analytical solution, thus First order approximation for coth is used to solve this with $d\kappa/2 \ll 1$; in addition the second term in The square root can be neglected assuming $2\pi n_d \ll \lambda_m$. The Drude model for silver $\varepsilon'_m = 5 - (\lambda^2 / \lambda_p^2)$ having plasma wavelength $\lambda_p = 134.6 \text{ nm}$, using these we write above approximation as

$$\lambda_m = \lambda_p \sqrt{4 + \frac{n_d^2 w}{\pi t} + \frac{2 n_d^2 w^2}{\pi^2 t d}}$$

Above also states that scaling down the w gives shorter λ_m , reducing the thickness t makes λ_m more

For large dimensions of the coupled strips $w^2 \gg \lambda_p^2$ and $td \gg \lambda_p^2$; we get resonant wavelength that is independent of t and d only depends on geometric parameter w : $\lambda_m \approx 2n_d w(1 + \lambda_p^2 / 4\pi^2 td) \approx 2n_d w$. This case is natural solution for basic cavity mode of e.m. cavity with characteristic size w . This is also case of micro-wave case of GLC resonance where it is not depending on plasmonic metal property.

Cavity model derivation in next class with magnetic plasmonic resonance

Approaching ideal electric or magnetic dipole as we go towards optical frequency

As we have observed that the size of the meta-atoms in the case of optical frequency approach the skin-depth order, and thus the metallic property at that frequency be taken into account for the desired magnetic plasmonic resonance. In case of the GHz meta-atoms SRR the e.m. wave did not penetrate the metal, whereas in optical frequency the e.m. wave does penetrate the metal as the skin depth ordered dimensions need be.

If electric field enters the metal and gives polarization the charge separation is of the order of skin depth, thus an electric dipole gets formed

$$\begin{array}{c}
 \leftarrow w \rightarrow \\
 \boxed{+ \rightarrow -}
 \end{array}
 \quad p_e \quad w \sim 50\text{nm} \quad p_e = Nq_e w; \quad \lim w \rightarrow 0$$

Similarly a loop is formed for the current, en-circling an infinitesimal small area gives rise to magnetic dipole

$$\begin{array}{c}
 i \\
 \text{---} \rightarrow \text{---} \\
 \text{---} \leftarrow \text{---}
 \end{array}
 \quad p_m \quad p_m = iA; \quad \lim A \rightarrow 0$$

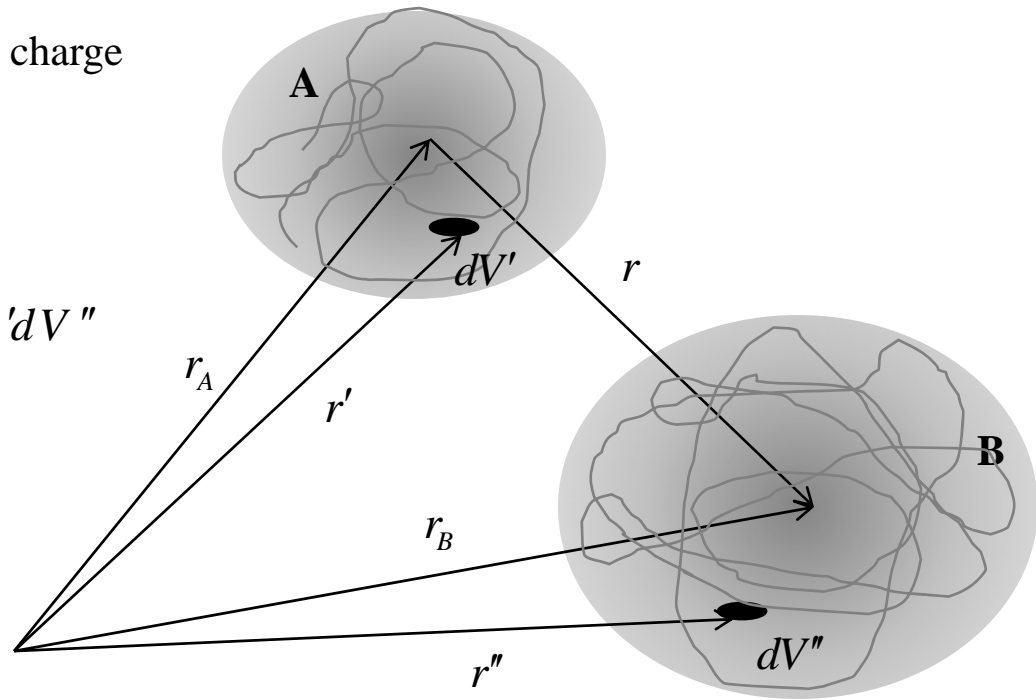
We can thus study the interactions of various electric dipoles and magnetic dipoles in meta-atoms of meta material to get resonance states

Interaction potential of two charge densities

So far we saw interaction of nano-metric (nano-particles) system with its macroscopic environment. Here we will try and review the interaction potential between two meta-atoms or meta-molecules. These considerations are important for understanding of delocalized excitations, energy transfer between meta-particles and collective phenomena. We assume that internal structure of the meta-particles are not affected by the interactions –therefore there is no ‘charge’ transfer and bond formation.

The interaction potential between charge densities A and B is

$$V_{AB} = \frac{1}{4\pi\epsilon_0} \iint \frac{\rho_A(r')\rho_B(r'')}{|r' - r''|} dV' dV''$$



Potential of two charge densities and moments

Zero order moment $q_A = \int \rho_A(r') dV'$ $q_B = \int \rho_B(r'') dV''$ is total charge

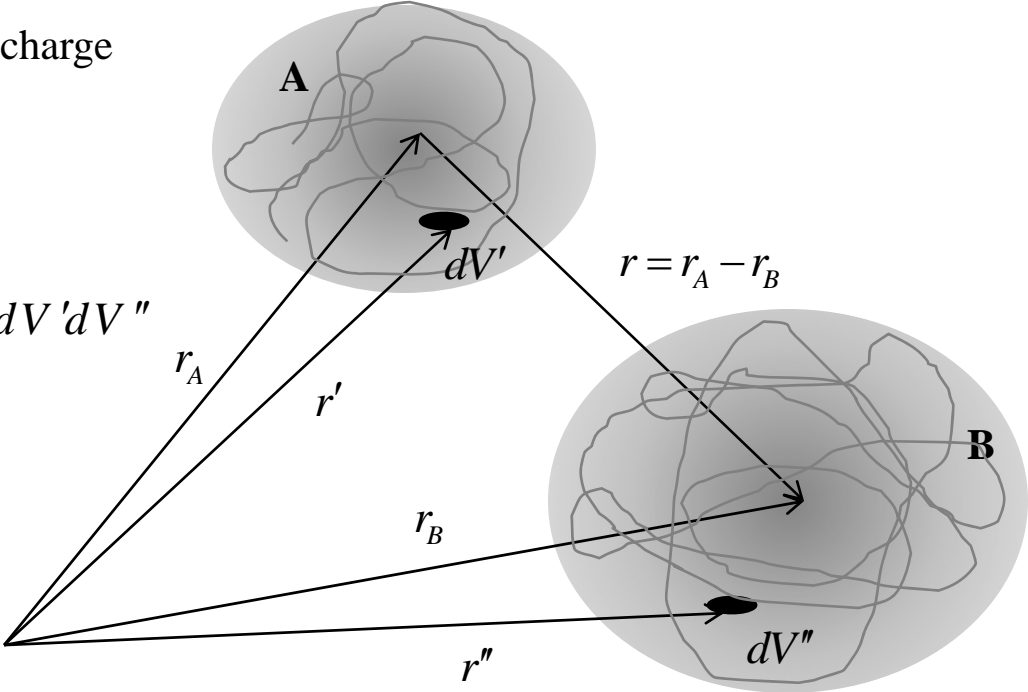
First order moment $p_A = \int \rho_A(r')(r' - r_A) dV'$ $p_B = \int \rho_B(r'')(r'' - r_B) dV''$ is dipole moment

Second order moment $\vec{Q}_A = \int \rho_A(r') \left[3(r' - r_A)(r' - r_A) - |r' - r_A|^2 \right] dV'$ is Quadra-pole moment

The interaction potential between charge densities A and B is

$$V_{AB} = \frac{1}{4\pi\epsilon_0} \iint \frac{\rho_A(r') \rho_B(r'')}{|r' - r''|} dV' dV''$$

We have to expand this in terms of defined multi-poles



Multi-pole expansion for potential function

$$\begin{aligned}
 V_{AB} &= \frac{1}{4\pi\epsilon_0} \iint \frac{\rho_A(r') \rho_B(r'')}{|r' - r''|} dV' dV'' = \frac{1}{4\pi\epsilon_0} \iint \frac{\rho_A(r') \rho_B(r'') dV' dV''}{|(r' - r_A + r_A) - (r'' - r_B + r_B)|} \\
 &= \frac{1}{4\pi\epsilon_0} \iint \frac{\rho_A(r') \rho_B(r'') dV' dV''}{|r - [(r'' - r_B) - (r' - r_A)]|} \quad \text{used is } r_A - r_B = r
 \end{aligned}$$

Use

$$\begin{aligned}
 |r - x|^{-1} &= (r^2 - 2r \bullet x + x^2)^{-\frac{1}{2}} = \frac{1}{r} \left(1 + \frac{1}{r^2} [x^2 - 2r \bullet x] \right)^{-\frac{1}{2}} \\
 &= \frac{1}{r} + \frac{r \bullet x}{r^3} + \frac{3(r \bullet x)^2 - r^2 x^2}{2r^5} + \frac{5(r \bullet x)^3 - 3r^2 (r \bullet x) x^2}{2r^7} + \dots
 \end{aligned}$$

$$x = [(r'' - r_B) - (r' - r_A)]$$

We get interaction potential as, in terms of defined monopole, dipole, and Quadra-pole moments etc

$$V_{AB}(r) = \frac{1}{4\pi\epsilon_0} \left[\frac{q_A q_B}{r} + \frac{q_A p_B \bullet r}{r^3} - \frac{q_B p_A \bullet r}{r^3} + \frac{r^2 (p_A \bullet p_B) - 3(p_A \bullet r)(p_B \bullet r)}{r^5} + \dots \right]$$

Interpretation of terms

$$V_{AB}(r) = \frac{1}{4\pi\epsilon_0} \left[\frac{q_A q_B}{r} + \frac{q_A p_B \cdot r}{r^3} - \frac{q_B p_A \cdot r}{r^3} + \frac{r^2 (p_A \cdot p_B) - 3(p_A \cdot r)(p_B \cdot r)}{r^5} + \dots \right]$$

The first term is charge-charge interaction. The next two terms are charge-dipole interaction potential that requires at least one particle to be carrying net charge, and this potential is very short distance as compared to charge-charge interaction (a long range interaction) as it decays with r^{-2}

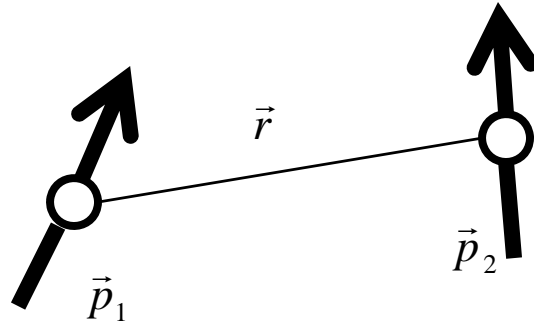
The fourth term is interaction potential of dipole-dipole interaction. This is the most important one among the neutral particles. This terms gives rise to van der Walls forces. This is very short range as it decays with r^{-3} and strongly depends on the dipole orientation. We emphasize here that this potential V_{AB} only accounts for interaction mediated by near field of the two dipoles.

Our meta atoms or meta particles be it electric or be it magnetic are charge neutral, so the interaction potential between two nano-meta-particles of meta-material is

$$\begin{aligned} V_{AB}(r) &= \frac{1}{4\pi\epsilon_0} \left[\frac{r^2 (p_A \cdot p_B) - 3(p_A \cdot r)(p_B \cdot r)}{r^5} + \dots \right] \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{p_A \cdot p_B}{r^3} - \frac{3(p_A \cdot r)(p_B \cdot r)}{r^5} \right] \end{aligned}$$

Dipole-Dipole coupling

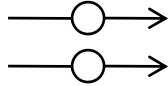
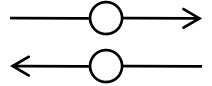
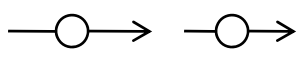
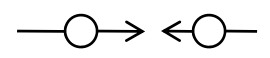
Two dipoles (electric or magnetic) with moments p_1 and p_2 interact at center to center distance r , the quasi-static interaction potential (sometimes also referred to energy) V is



$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{p_1 \cdot p_2}{r^3} - \frac{3(p_1 \cdot r)(p_2 \cdot r)}{r^5} \right)$$

$$V = \gamma \frac{p_1 \cdot p_2}{4\pi\epsilon_0 r^3}$$

This is purely for side-by-side i.e. transverse or end to end i.e. longitudinal coupling

Transverse coupling	$\gamma = +1$ 	$\gamma = -1$ 
Longitudinal coupling	$\gamma = -2$ 	$\gamma = +2$ 

Why interactions between meta-atoms/particles are needed for overall study ?

For practical applications of meta-material, three dimensional or even bulk structures are needed. Since the size of the meta-material constituents and hence the unit cells are much smaller than the wavelength of the incident light, the lateral and vertical finite spacing will inherently lead to ‘strong’ interaction between neighboring meta-material elements.

Liu N, Guo H, Fu L, Schweizer H, Kaiser S, Giessen Harald, Phys. Status Solidi; B 2007, 244, 1251-1255

C. Rockstuhl, T Zentgraf, H Guo, N Liu, C. Etrich, I. Loa, K Syassen, J Kulh, F. Lederer, H Giessen; Appl. Phys. B. 2006 84, 219

N Liu, H C Guo, L W Fu, S Kaiser H Schweizer, H Giessen; Nat Mater 2008, 7, 31-37.

J F Zhou, T Koschny, M Kafesaki, C M Soukoulis, Phy Rev B 2009 80, 035109.

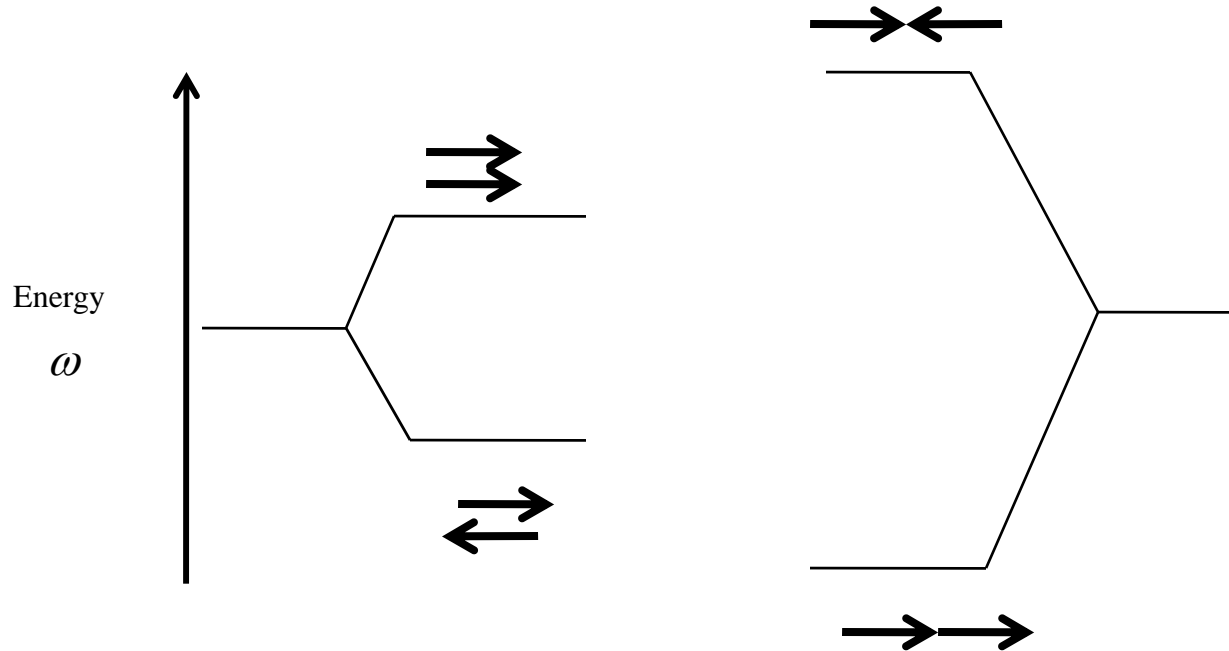
As a result the optical properties can be changed substantially compared to an individual meta-material element. This is analogous to solid-state physics where the electronic properties of solids can vary dramatically from those of individual atoms. For example carbon, where the optical spectra of individual atoms differ strongly from those of graphite or diamond. This example also points to a fact that the arrangement of the unit cell in the lattice of the solids is crucial for resulting properties.





N W Ashcroft, N D Mermin in solid state physics, Brooks/Coole Thomson Learning Singapore 2003

We limit to a first order approximation of only dipole-dipole interaction of meta-atoms/particles in Bulk meta-material; although higher order multi-pole interactions are possible.

Energy levels of two dipole coupling

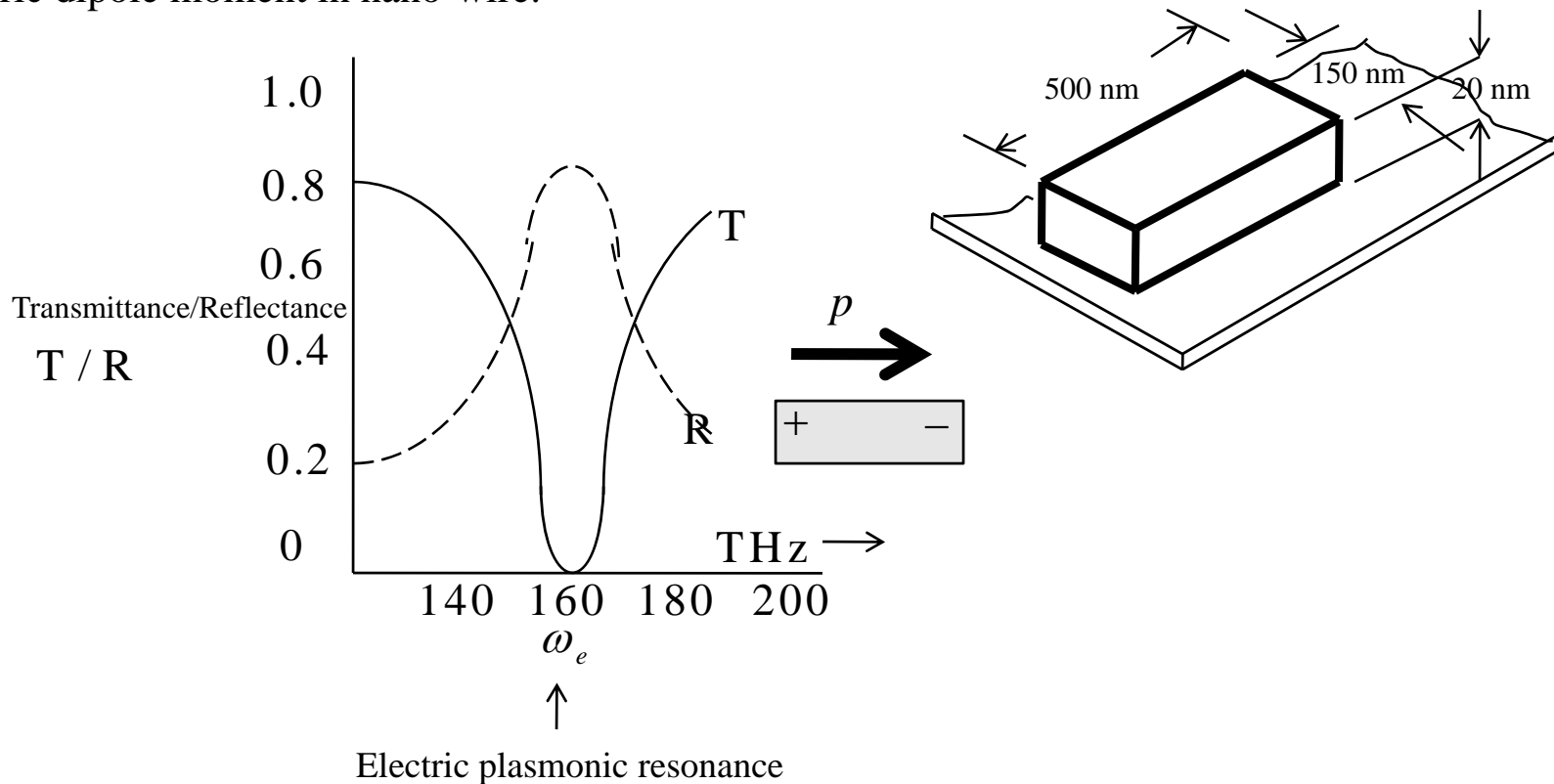
Energy is related to frequency $E \sim \hbar \omega$. Attraction is lower energy than repulsion; example laterally coupled electric/magnetic asymmetric dipoles attracts each other and thus decreasing the restoring force and leading to a lowering of resonance frequency.



		Repulsion
		Attraction

Single metallic nano-particle in optical illumination

A gold nano wire of 500 nm by 150 nm by 20 nm residing on a glass substrate and is illuminated by light polarized linearly along longer axis of the wire at normal incidence. A resonance is observed around 160 THz (1870 nm) in the spectrum. Associated with this resonance is the excitation of an electric dipole moment in nano-wire.

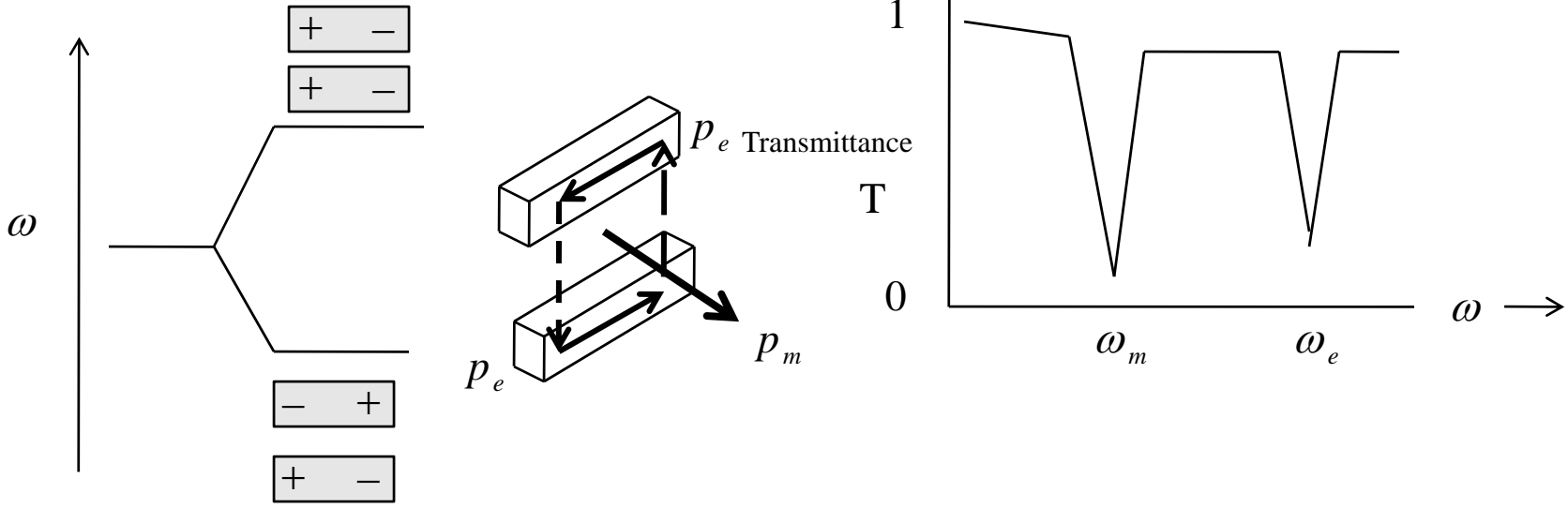


U Kreibig, M Vollmer, Optical properties of metal clusters, Springer Berlin 1995

U Kreibig, P. Zacharias, Z Phys, 1970, 231; 128-143

A magnetic meta-atom by coupled metallic nano-wire pairs

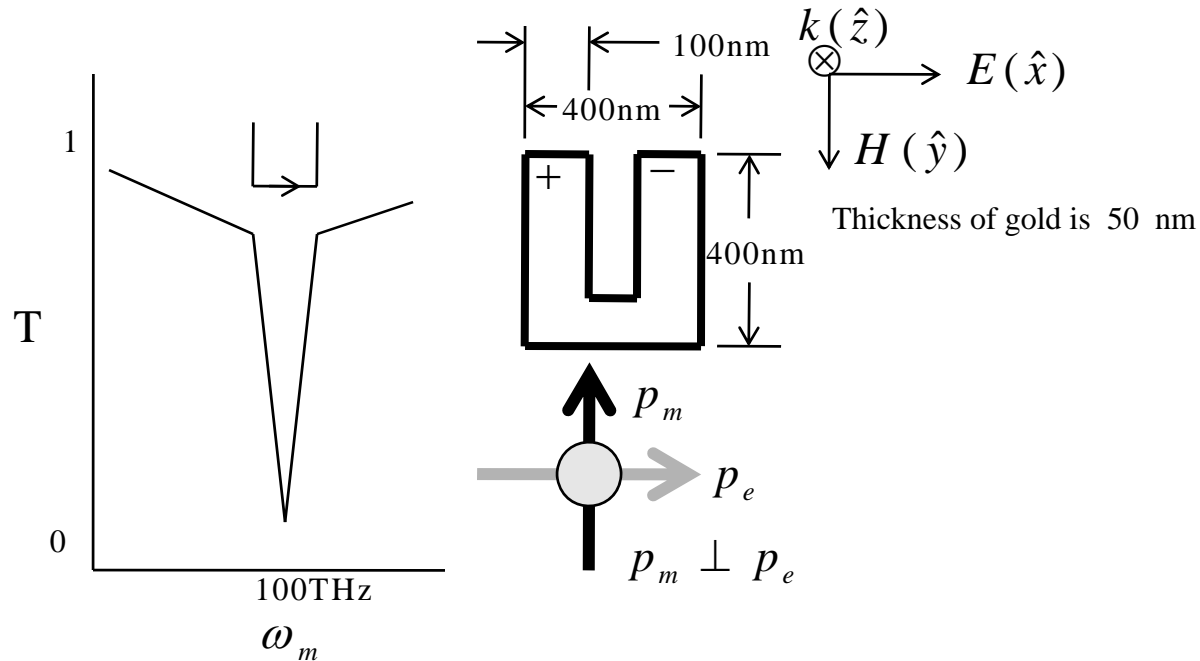
Consider two gold nano-wires with a finite separation, they are strongly coupled because of proximity. The interaction can lift the bare plasmonic mode of individual nano-wire and lead to two new modes as a result of plasmonic hybridization, one with symmetric alignment of the two electric dipoles and another one with anti-symmetric alignment. The anti-symmetric mode of the two nano-wires is termed as a ‘magnetic-resonance’ as anti-symmetric currents in the two nano-wires together with the displacement currents between the two nano-wires (i.e. in the dielectric separation) leads to a resonant excitation of the magnetic dipole moment, thus giving magnetic response in the system. Therefore the two nano-wires stacked in this fashion can act as magnetic atom, which is fundamental building block of magnetic meta-material at optical range.



G Dolling, C. Enkrich, M Wegener, C M Soukoulis, S. Liden, J F Zhou, Opt. Lett. 2005, 30, 3198-3900
 W S cai, U K Chettiar, H K Yuan, V M Shalaev, Opt-Express, 2007, 15, 3333-3341.

Split Ring resonator a magnetic meta-atom

This is a fundamental block for artificial magnetism or a magnetic meta-atom. When a linearly polarized light with its electric hand E incident along the gap bearing side of SRR at normal incidence, electric dipole like plasmons can be excited in the entire SRR, thus giving rise to magnetic dipole moment perpendicular to SRR plane.

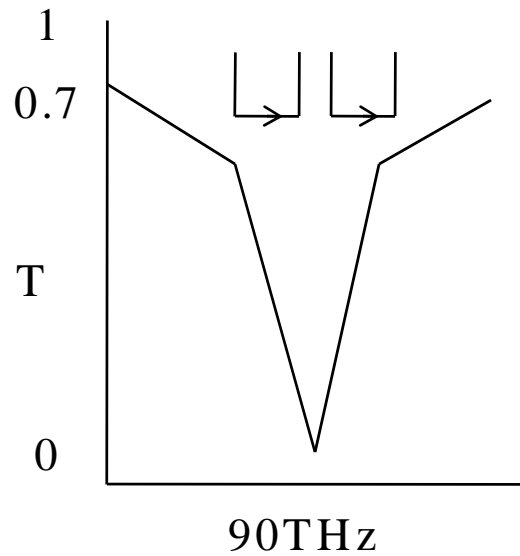
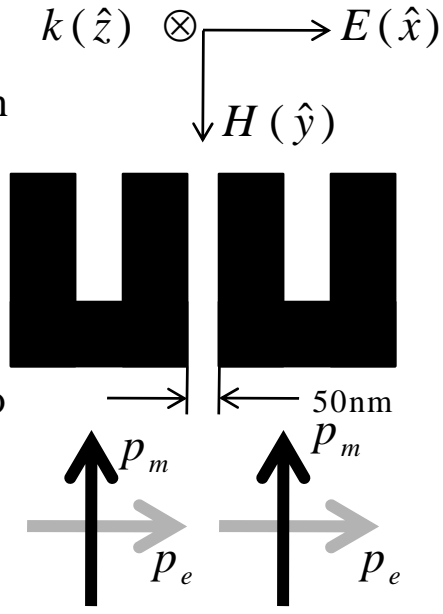


When two such SRR are arranged in different configurations e.g. next to each other or above each other same type of coupling rules as in case of p_e apply to p_m . The coupling behavior is more complex than in case of two metallic nano-particles, as of the fact in SRR the both electric as well as magnetic coupling should be taken into account. Also it is not clear which coupling mechanism is dominant.

It is possible to switch off the electric coupling and retain only magnetic coupling

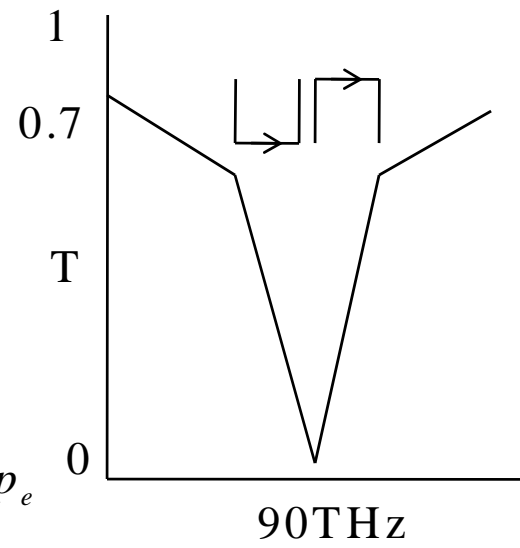
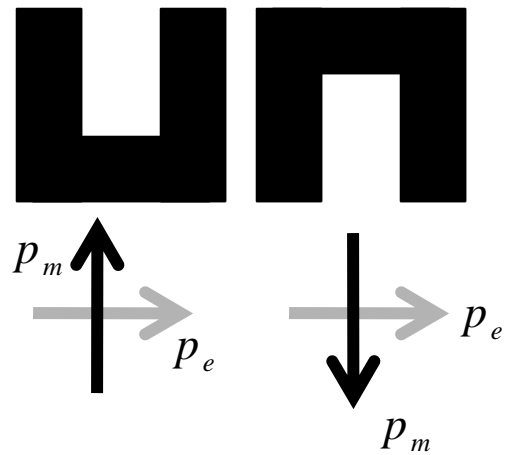
Lateral coupling of SRR with zero degree rotation and 180 degree rotation

There is no phase retardation between two SRR as a result they are symmetrically excited, the spectrum of this show one resonance, at the resonance frequency the two p_e are aligned parallel and as are the two magnetic dipoles p_m



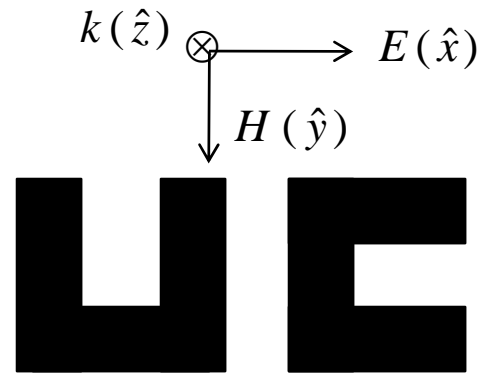
Tilted incidence of light would cause symmetry breaking and will introduce a certain lateral phase shift between two SRR thus making anti-symmetric modes weakly observable.

Similarly the 180 degree orientation depicts a single resonance, with two p_m aligned anti-parallel, and the two p_e aligned parallel.

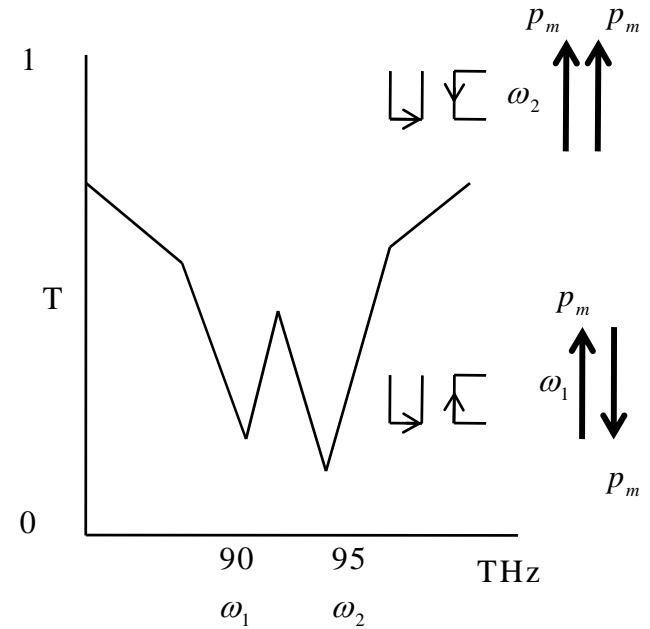


Lateral coupling of SRR with 90 degree rotation

The circulating currents in the right SRR cannot be directly excited by the incident light, because of polarization of E along the arm. At the resonance the mutual inductance between the two results in excitation from left SRR to transfer to the right by inductive coupling



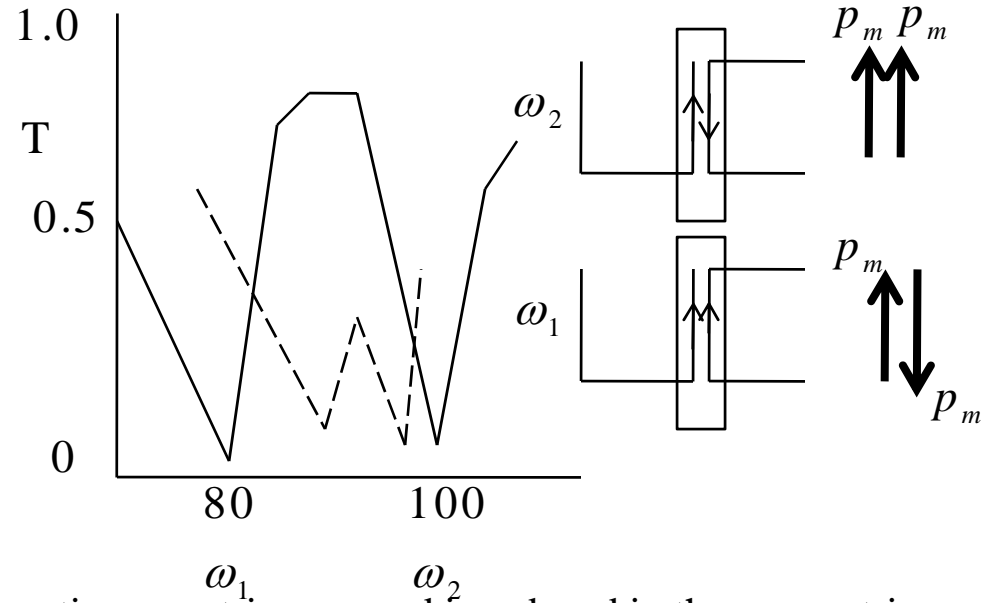
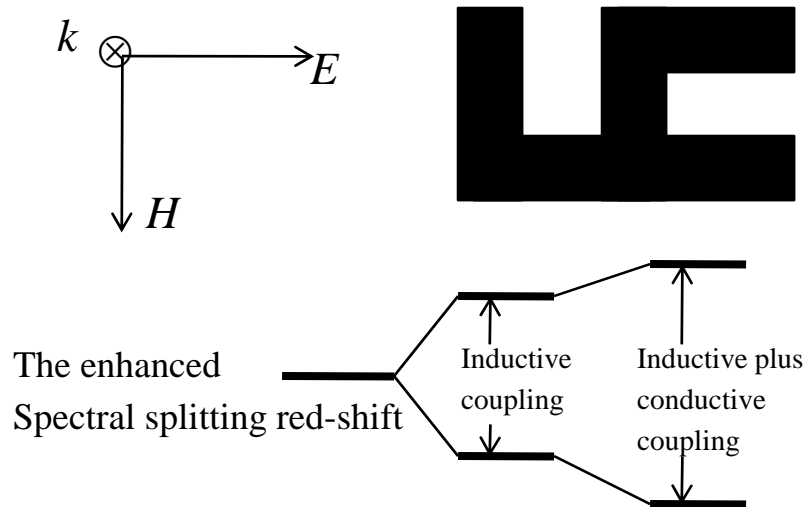
this introduces also a phase retardation between the two. The fact that electric dipoles excited in the two SRR are perpendicular and that interaction gets thus cancelled. As a result the coupling between the two magnetic dipoles play a key role in splitting of spectra, with two magnetic resonances.



In analogy to the states of two simple atoms hybridized into molecular orbital we can term resulting coupled system as “split-ring-molecule” in which two split ring “atoms” are coupled inductively because of structural asymmetry. The two magnetic dipoles are aligned anti-parallel and parallel at the lower and higher resonance respectively. This complies with hybridization picture of two transversely coupled dipoles. In anti-symmetry mode the N-pole and S-poles of the magnetic dipoles attract each other and thus leads to lower (energy, potential) frequency, while in symmetry mode poles with same sign repel leading to higher resonance.

Joining two SRR with 90 degree orientation

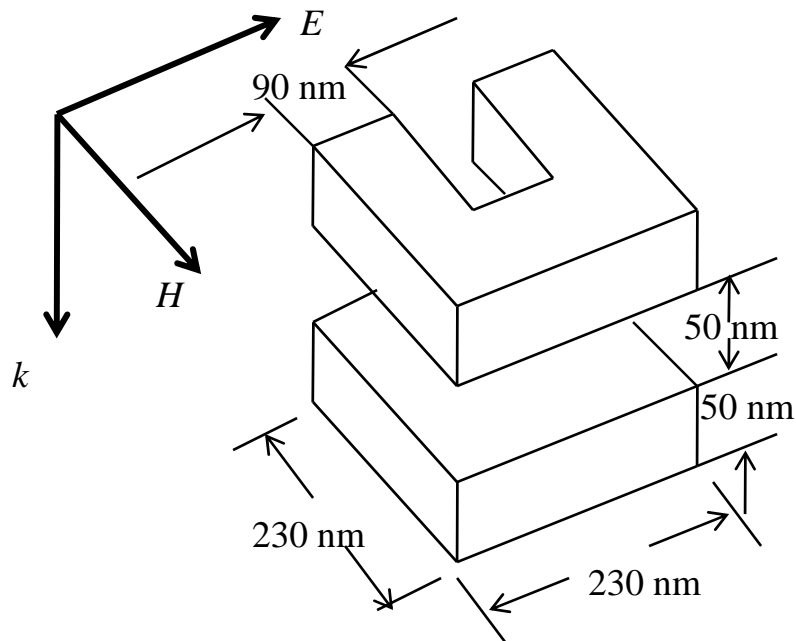
The em coupling strength can 90 degree rotated SRR can be altered by changing the relative distance. The two SRR can be physically connected to effectively improve the coupling strength. Then from The earlier inductive coupling we go to “conductive-coupling”. The spectral splitting that is directly related to em coupling strength is enhanced



Current distribution in the joining link is enhanced in anti-symmetric case and is reduced in the symmetric case. Consequently in addition to enhanced inductive coupling due to reduced distance, conductive coupling from the common conduction currents aids further increasing the coupling strength between two SRR-for the asymmetric mode. On the other hand in case of symmetric mode the currents from connecting part aid oppositely-as a result the interaction of SRR remains predominantly inductive-hence this mode does not show red shift (the second resonance is rather blue shifted from 95 to 100 THz); while the first resonance is red-shifted from 90 earlier to 80 THz now)

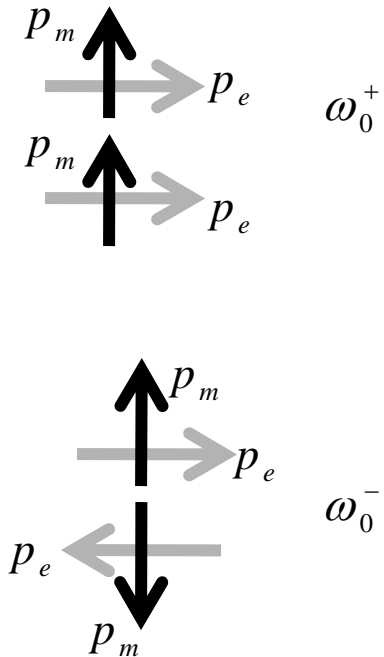
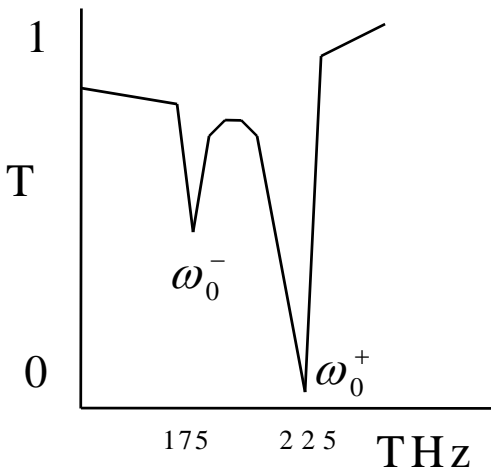
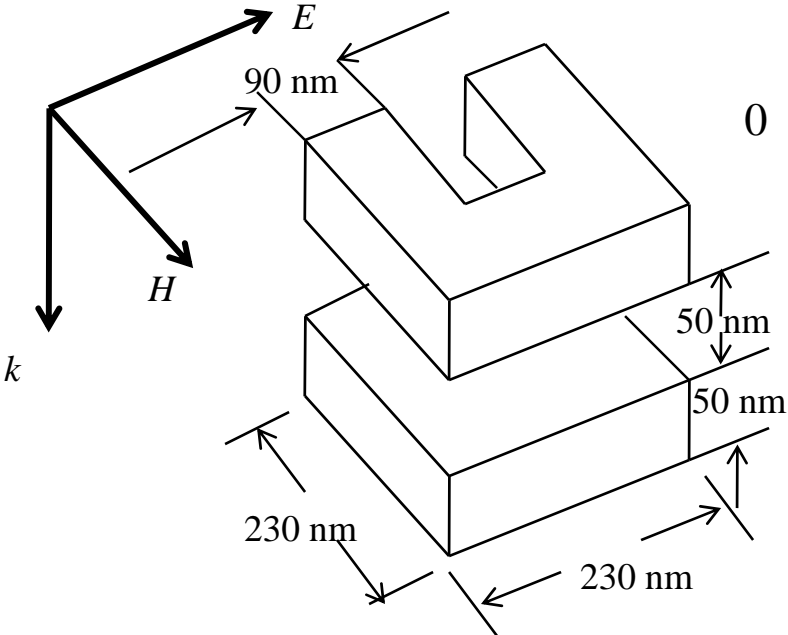
Vertical coupling of SRR (Stereo-Meta-material)

We term these structures as stereo-meta-material as analogous to stereo-isomers in chemistry, where the Atoms are arranged in molecules with different 3D-arrangements. Each unit cell has two SRR of identical geometry, but the two are stacked vertically in space with different twist angles. The optical property Of these stereo meta-material can be altered substantially by altering twist angles between the two split-ring 'atoms'. This is due to variation in interaction particularly how the magnetic and electric inter-actions are dependent on the spatial arrangements of these SRR.



Zero degree Vertical coupling of SRR (Stereo-Meta-material)

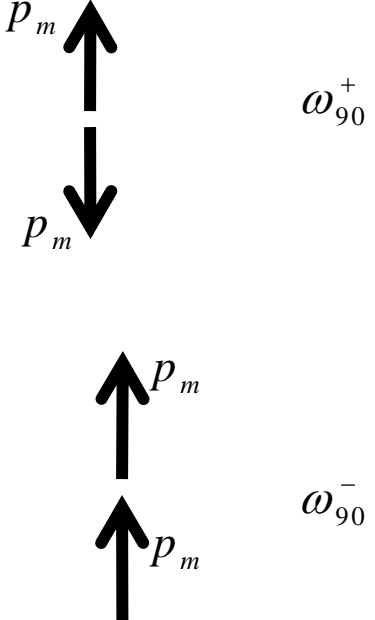
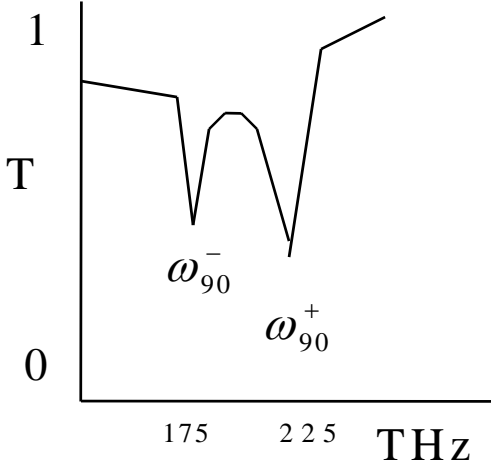
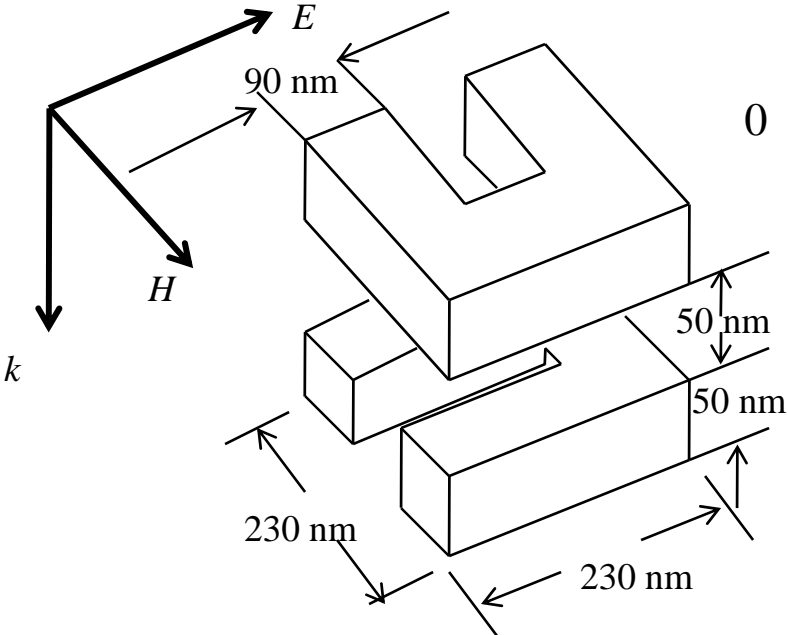
Two resonances are observed ω_0^- and ω_0^+ . The electric dipoles excited in two SRR oscillate anti-phase and in-phase for these resonances respectively, thereby giving rise to two induced dipole magnetic dipole moments; aligned anti-parallel at lower resonance and parallel at higher resonance.



In ω_0^- mode, the p_e are attractive while P_m are repulsive in ω_0^+ mode, the p_e are repulsive while p_m are attractive thus here electric and magnetic interaction counter each other, however, the electric interaction determined by longitudinal dipole interactions dominates.

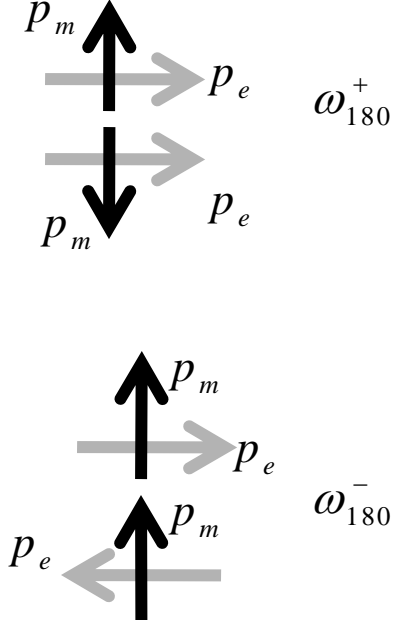
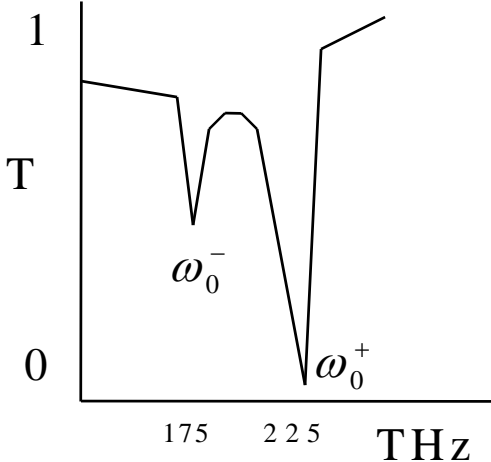
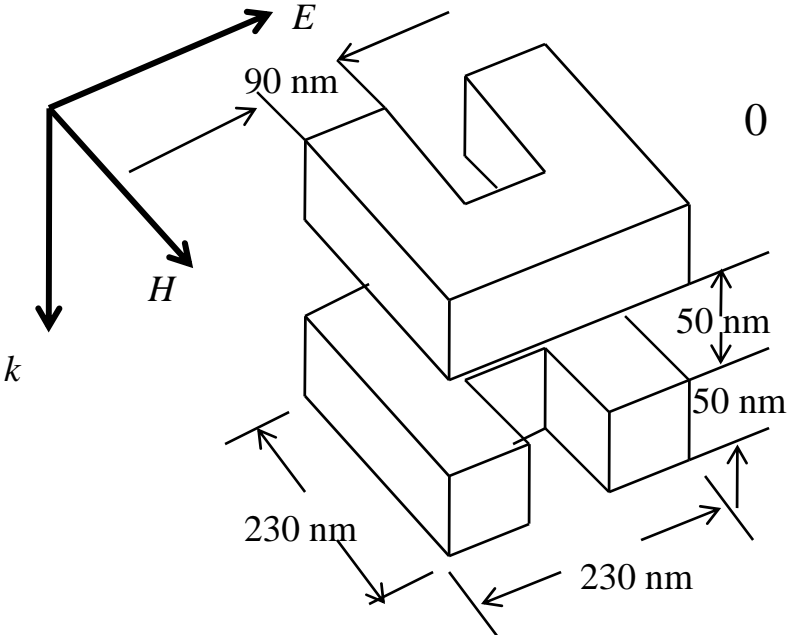
90 degree Vertical coupling of SRR (Stereo-Meta-material)

Current in the bottom SRR cannot be excited by direct incident light; nevertheless for this coupled system, at the resonance frequency, excitation from upper SRR is transferred inductively to the lower; leading to the formation of new plasmonic mode ω_{90}^- and ω_{90}^+ ; with no electric dipole interaction, as a result only the longitudinal magnetic dipole interactions determine these two resonances.



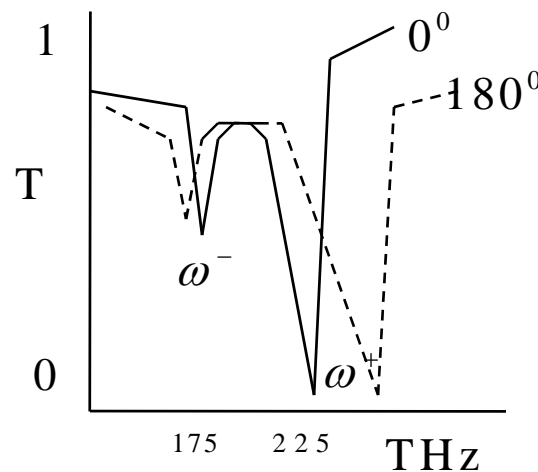
180 degree Vertical coupling of SRR (Stereo-Meta-material)

The interaction between the two SRR results in two new plasmonic modes ω_{180}^- and ω_{180}^+ ; these resonances are associated with the excitation of the electric dipoles in two SRR oscillating in anti-phase and with in-phase. The two resulting P_m are aligned parallel and anti-parallel. In essence the transverse p_e and longitudinal P_m , contribute positively in this resulting in greater spectral splitting indicating larger coupling strength.



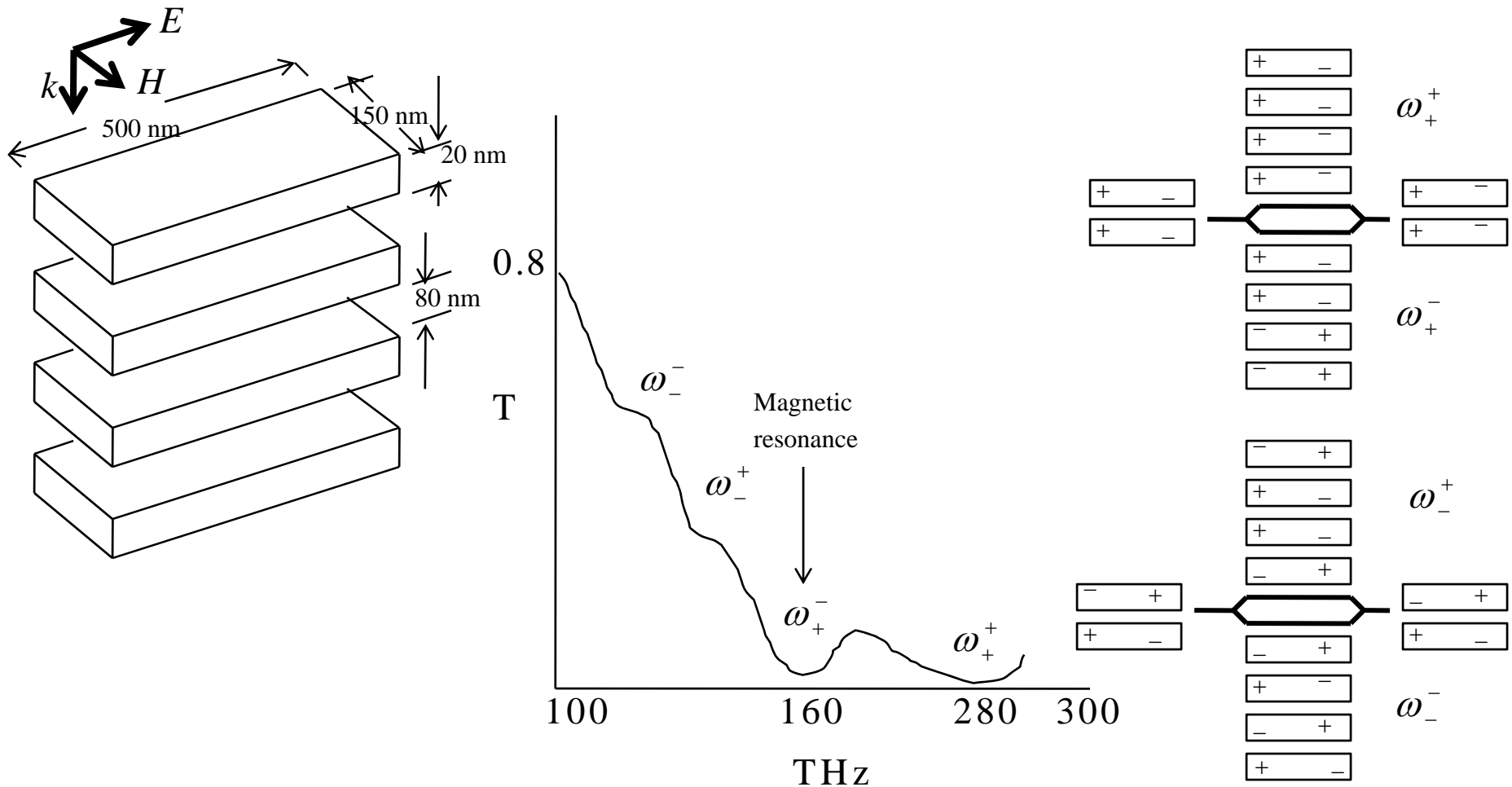
Unequal resonances in 0 and 180 degree (Stereo-Meta-material)

The 180 degree orientation gives larger spectral splitting as the couplings aids; larger red shift for lower and larger blue shift for higher resonance. It is worth mentioning that electric coupling plays a key role in 0 degree and 180 degree SRR. In both cases the electric dipole in the two SRR oscillate anti-phase at lower resonances ω_0^- ; ω_{180}^- . Such resonances are not easily excited by light, that is they are sub radiant in character



On the other hand at the higher resonances ω_0^+ ; ω_{180}^+ the electric dipoles in two SRR oscillate in phase. Such resonances can strongly couple to light, that is they are super radiant. As a result the ω^- are much less pronounced in width and magnitude w.r.t. the ω^+ .

Three dimensional meta-material stacked nano-wires of gold (noble metals)



Four resonances are observed. Straightforward to understand the resonant behavior by regarding the system as two coupled nano-wire pairs.

Four resonances in the coupled four nano wires

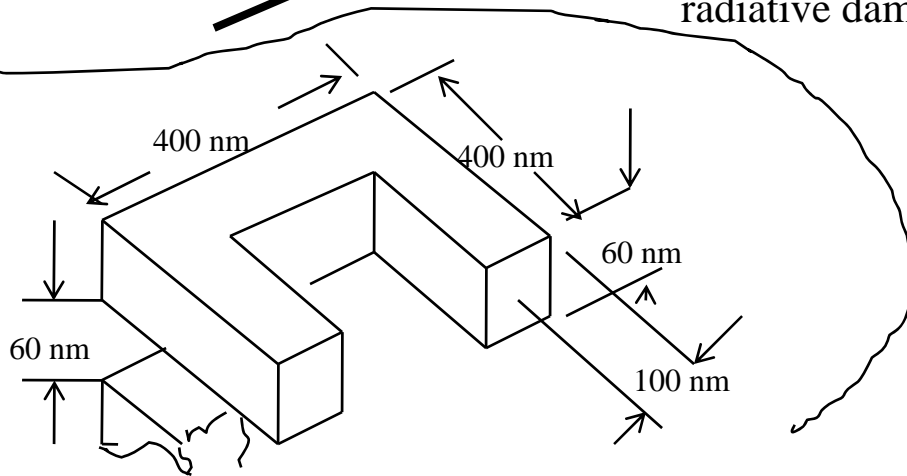
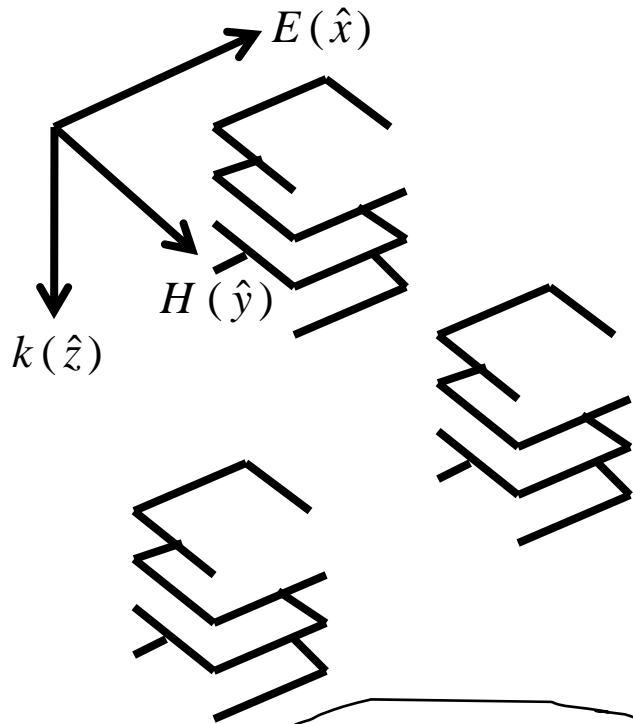
The ω_+^+ and ω_+^- modes actually results from the interaction of the symmetric modes of the two nano-wire pairs. In case of the ω_+^+ modes, the charge oscillations inside the four wires are all in-phase, and thus it is highly repelling case of the restoring force, all repelling. This leads to the ω_+^+ mode having the highest frequency among the four modes. Similar to the anti-symmetric mode in the case of two nano wires, the ω_+^- can be treated as magnetic resonance.

Simultaneously the generation of the ω_-^- and ω_+^- modes results from the coupling of the anti-symmetric modes of the two nano-wire pairs. The ω_-^- has the lowest resonance frequency as it has charge oscillations inside each nano-wires out-of-phase w.r.t . neighbor wires; thus having restoring force all attractive. Similarly the ω_-^- and ω_+^- modes which would be dark at the quasi-static limit, can be excited by light in a real system as a consequence of phase retardation.

In principle a coupling also exists between the symmetric and anti-symmetric modes of the two nano-wire pairs, but we ignore that due to larger frequency separation and thus lower coupling intensity.

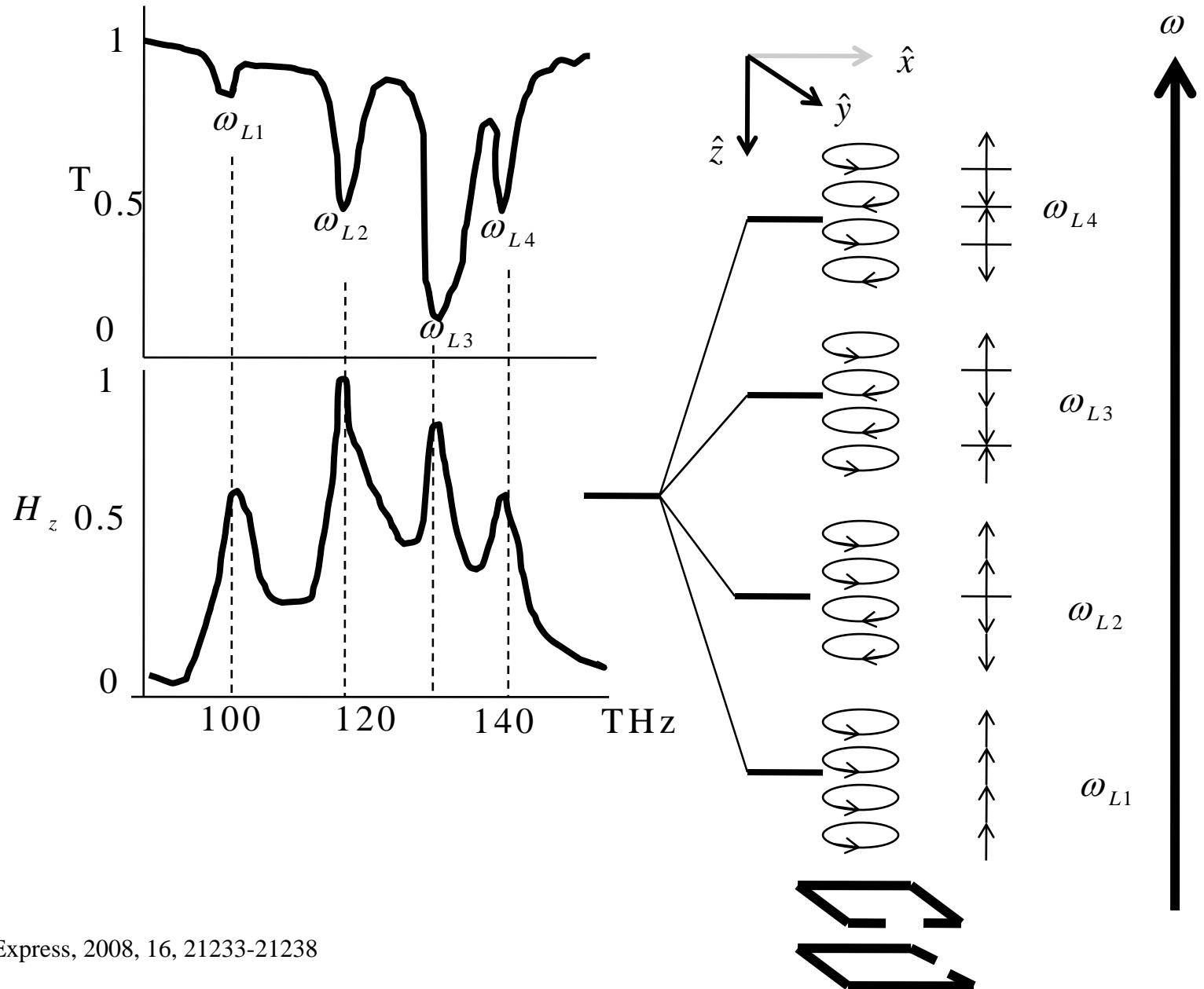
Stacked four SRR meta-material (90 degree rotated)

The electrical dipoles of next neighbor do not interact as of orthogonality, and we are only left with magnetic dipoles of the neighboring SRR to interact, and the coupling is longitudinal. We are getting thus four resonances. As the examination of the $-z$ - component of H field shows the lowest resonant frequency is the one with all magnetic dipoles are aligned in parallel. As analogy to solid state physics we might call this state as “ferromagnetic”; we might expect “optical ferromagnetism”. But actually ferromagnetism exhibits a hysteresis, which is associated with phase transition and it remains in place after the external H is turned off. But in the optical case no magnetism remains after femto-seconds of light being turned off. but in this case, because of radiative and non radiative damping

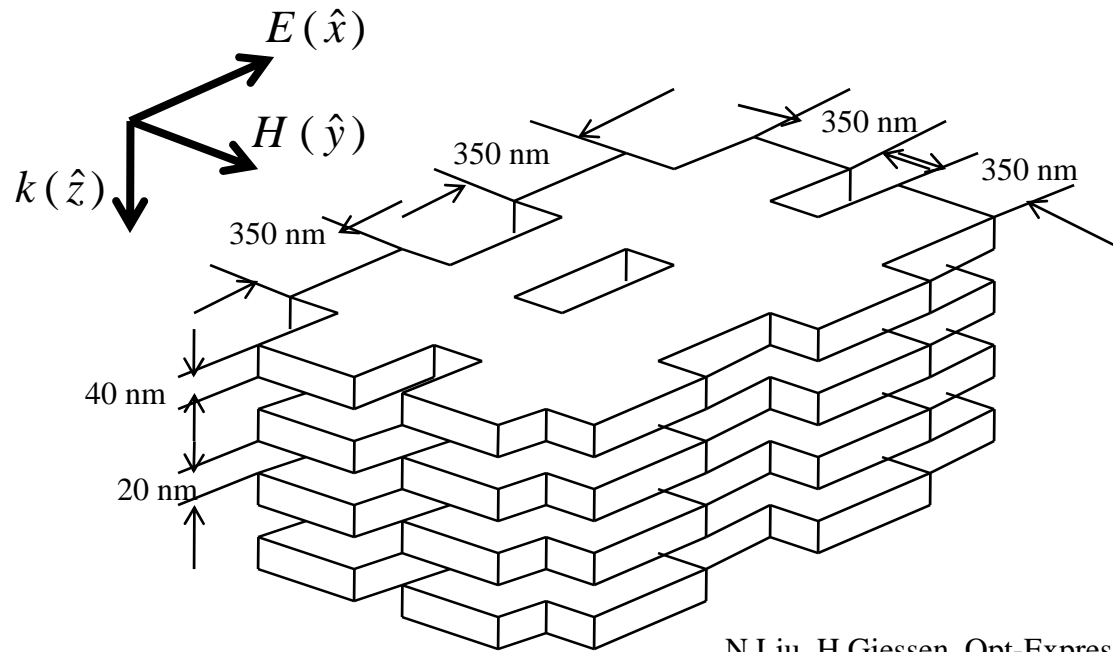


Ferromagnetism is a quantum phenomena a and for spin exchange Pauli's exclusive principles apply. In our case this is purely classical fields (classical-spin) with no Pauli's exclusive principle. Thus this parallel arrangement of magnetic moments in the ground state be termed as “optical superparamagnetism”

Four resonances due to interaction of four magnetic dipoles



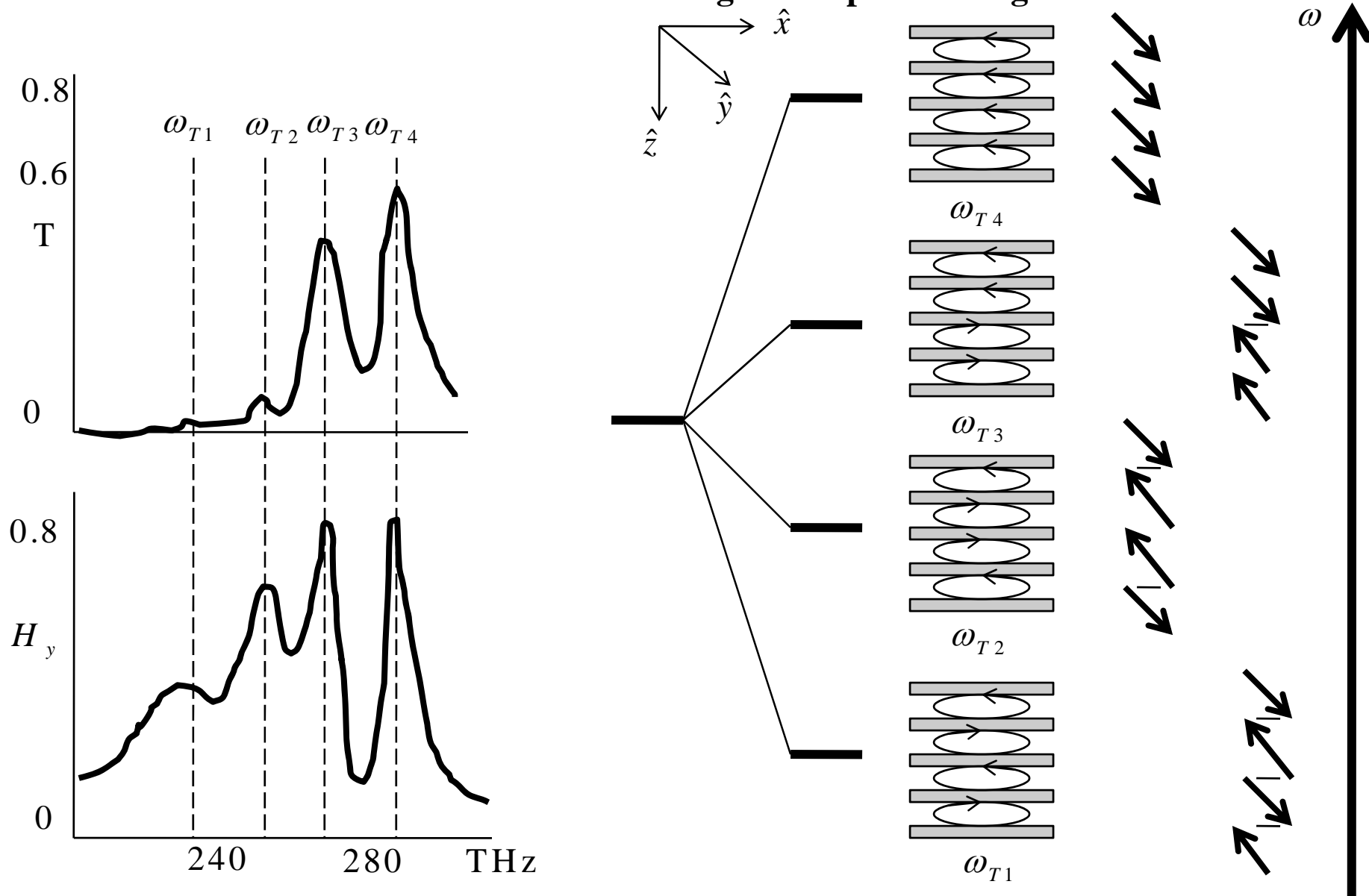
Stacked fish net meta-material



N Liu, H Giessen, Opt-Express, 2008, 16, 21233-21238

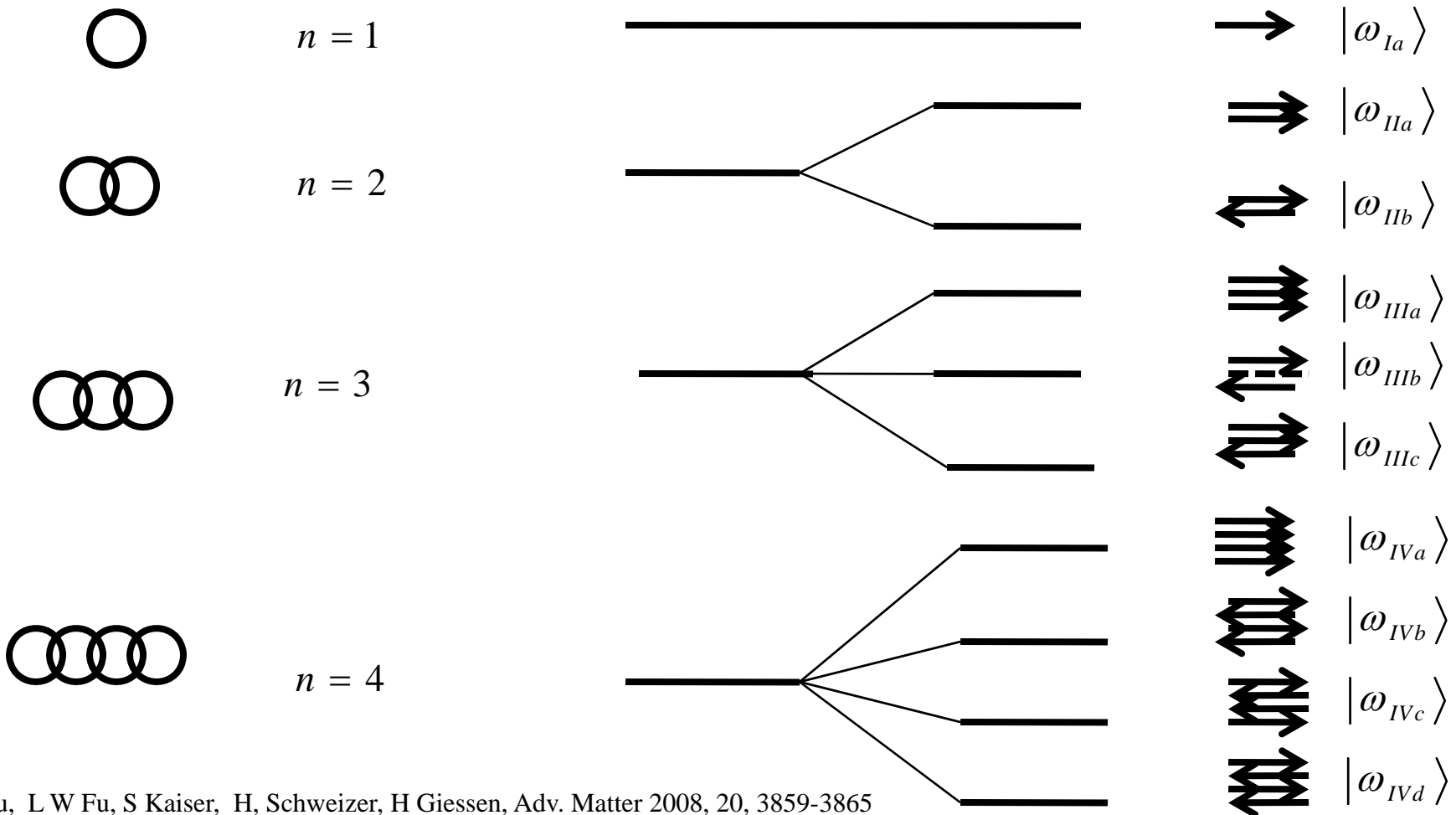
Transverse coupling between the magnetic dipoles can be realized in a 3D stacked fishnet meta-material. The fishnet layers of gold are surrounded by air, and the wire widths in both directions are designed to be equal for polarization independence. In the transmitted spectrum (T) four resonances are observed with a large enhancement of localized magnetic field H_y with probe placed inside the gaps. The incident light excites current loop in each wire pair resulting in magnetic dipole moment. These magnetic dipoles are transversely coupled thereby leading to the formation of four new modes associated with different symmetry. Each resonance is associated with the excitation of magnetic dipole moment inside the four gaps between each pair of gold wires-in y-direction aiding the H thus enhancing transmissions.

Four resonances due to interaction of four magnetic dipoles aiding the H and T .

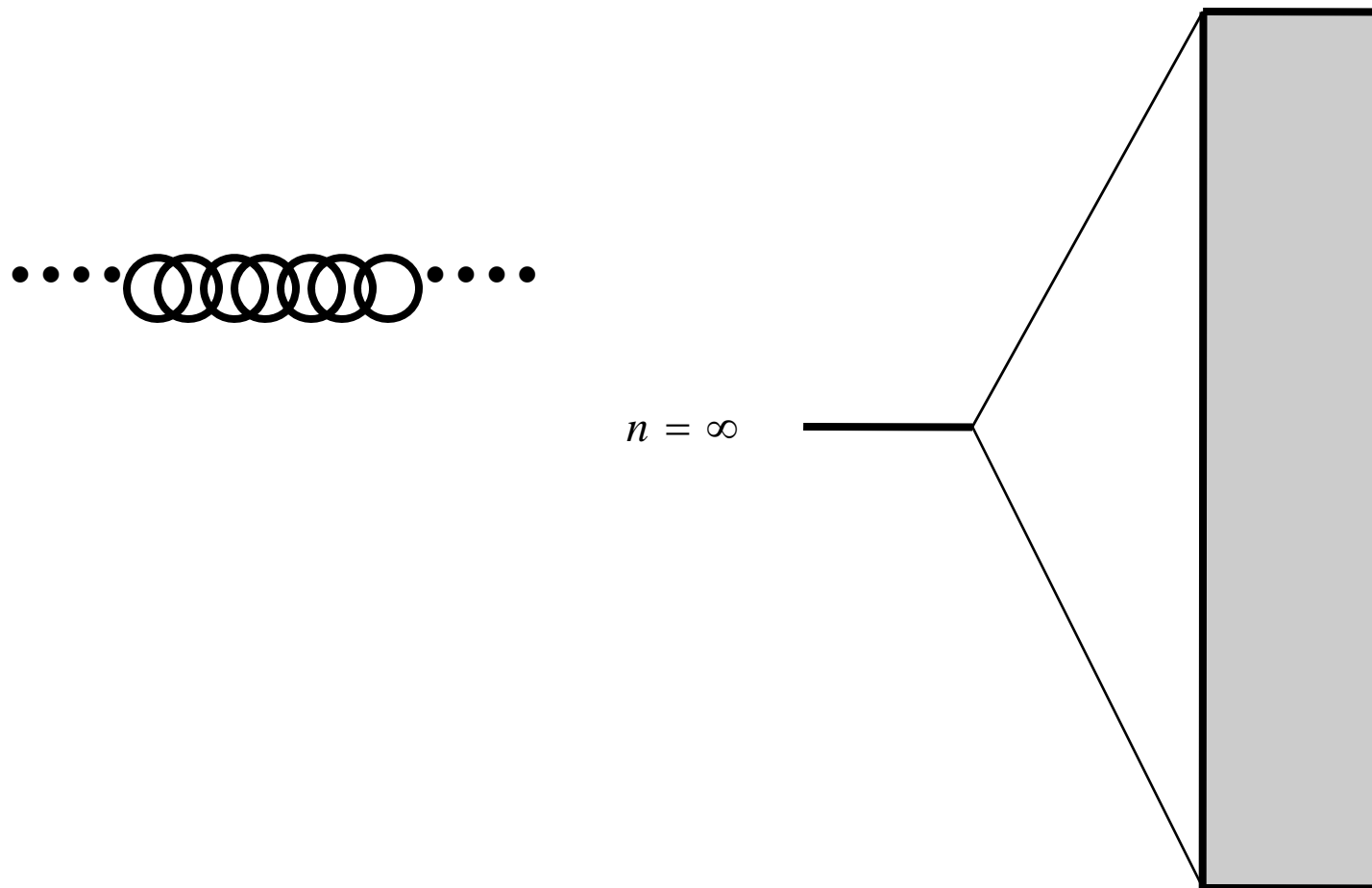


Number of meta-material elements vis-à-vis spectral frequency level

Intuitively a spectral band consisting of different frequency levels will be formed when more and more such meta-material elements are included (just like case in solid state physics). The bandwidth is given by the original coupling strength of two nearest neighbors which is similar to a tight binding model.



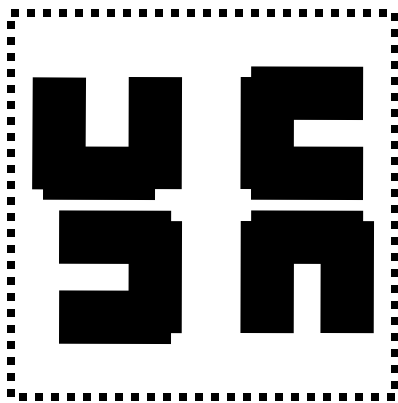
Formation of resonance band



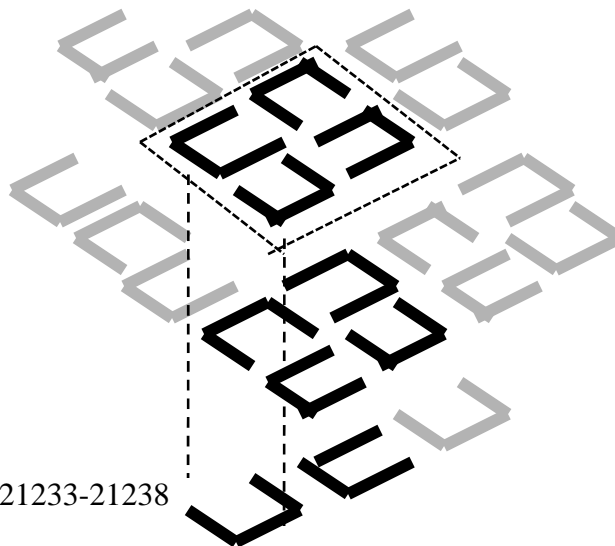
The formation of the resonance band arising from the coupling between magnetic atoms. Two atoms form a molecular states by hybridization which emerge into bands when many atoms are coupled

Complex three-dimensional meta-materials

Combining the concept of longitudinal and transverse coupling we can arrange meta-material magnetic dipole in 3D. In this design the combination of longitudinal and transverse coupling is unique in the sense that it allows for the study of purely magnetic dipole-dipole interaction in a meta-solid in the lateral as well as vertical direction. (considering only nearest neighbor interaction) . The planer unit cell is shown consisting of four SRR at an angle 90 degree to relative neighbor.



Longitudinal coupling can be introduced as a complication to the 2D system by subsequently stacking the planer unit cell with each layer Twisted by 90 degree. The coupling strength of the longitudinal as well as transverse coupling can be controlled by altering the lateral and vertical separation of the constituent SRR.



A 3D magnetic meta-solid results. also is termed as “photonic spin crystal” future challenge is to understand complicated spectra of such magnetic systems and analyzing their collective modes which are analogous to spin waves and magnons in solid state physics.

Getting artificial magnetism from high dielectric permittivity material

We till now discussed meta-magnetic by using 'plasmonic resonances' to introduce asymmetric current modes in sub wave length metal structure. We can generate artificial magnetism that relies on Mie resonance in the sub wave length dielectric resonators. Sub wave length particles with very high positive dielectric permittivity support strong resonances with a large displacement currents, which can cause a strong magnetic field induced by contra-directional displacement currents. Moreover a strong dielectric constant implies a small wave length inside high permittivity region.: thus physical size of the resonator can be many times smaller than the free space wave length. This situation justifies the treatment of the system as a macroscopically homogeneous medium and use the effective medium theories, to describe interaction with external wave.

Materials with very high dielectric permittivity at microwave to mid, to far IR are Ferroelectric ceramics ($\text{Ba}_x\text{Sr}_{1-x}\text{TiO}_3$ BST) in uW region, LiTaO_3 ; TlBr ; TlC ; SiC in mid and far IR.

When placing the sub wave length resonators together to form meta-material it is not essential to follow Any particular lattice symmetry as the Mie resonances is localized; the critical quantities are the geometry (size and shape) of individual rods or spheres, which determine the Mie resonance

O' Brien S, Pendry JB (2002), Photonic band-gap effect and magnetic activity in dielectric composites; J Phys Condens Matter, 14:4035-4044

Huang KC, Povinelli ML et al (2004), Negative effective permeability in polaritonic- photonic crystals; Appl Phys Lett, 85:543-545

Spitzer WG, et al (1959), Infrared properties of hexagonal silicon carbide; Phys. Rev 113: 127-132.

Taubner T et al, (2006), Near field microscopy through a SiC superlense. Science 313:1595.

Our study is to have meta magnetic via plasmonic theory-and we shall be dealing on that in details

Conclusions

(Plasmonic) Meta-materials consists of metallic nano-structures, which allow the possibility to tailor their optical properties. Incident light excites 'coherent oscillations of free electrons' which leads to localized particle plasmon resonances. The resonance frequency depends on the size shape and most importantly the dielectric function of the metal as well as dielectric function of the surroundings.

The shape of the individual meta-atoms can vary substantially: from simple spheres, ellipsoid, wires, split ring resonator SRR, meshes, fishnet etc.

The magnetic activity in optical regime cannot be got by scaling the micro-wave SRR as the size of the meta-atoms/particles become comparable to the skin depth dimension, thus concept of magnetic plasmon resonances be developed-which is responsible for magnetic dipole moment, at optical Frequency.

The overall bulk behavior comes via interaction of individual dipoles excited at nano-particle level, giving hybridized states of resonances (as in solid state physics).

The basic underlying theory to get meta-magnetics is of Magnetic Plasmonic Resonance and Cavity mode along with severe losses at optical visible regime-what we are dealing in next class in detail.

End of part-5