Left Handed Maxwell Systems In Optical Regime

PART-4

Electrical Meta-Material-in optical regime

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Few salient points

The electric response of a medium is described using electric permittivity ε , the main purpose of the design of electric-meta-material is to create ‘artificial’ metal-dielectric composites that has designed value of the permittivity- ε

These materials are also called ‘artificial-dielectric’ because the serve as macroscopic analogue of natural dielectric-except that the atoms or molecules are artificially structured (called meta-atoms/molecules)

One of the most popular artificial dielectric is “rodded-medium” where periodically spaced lattice of metallic rods is embedded in other dielectric medium; has been of use in microwave region extensively. The rodded medium is also called wire-grid or wire mesh which emulates a plasma similar to “Drude-metal”, and the plasma frequency can be tuned by varying the geometrical parameters of the wire-array, was rediscovered by Sir John Pendry.

Importance of “artificial-dielectric” the electric meta-material at optical frequencies

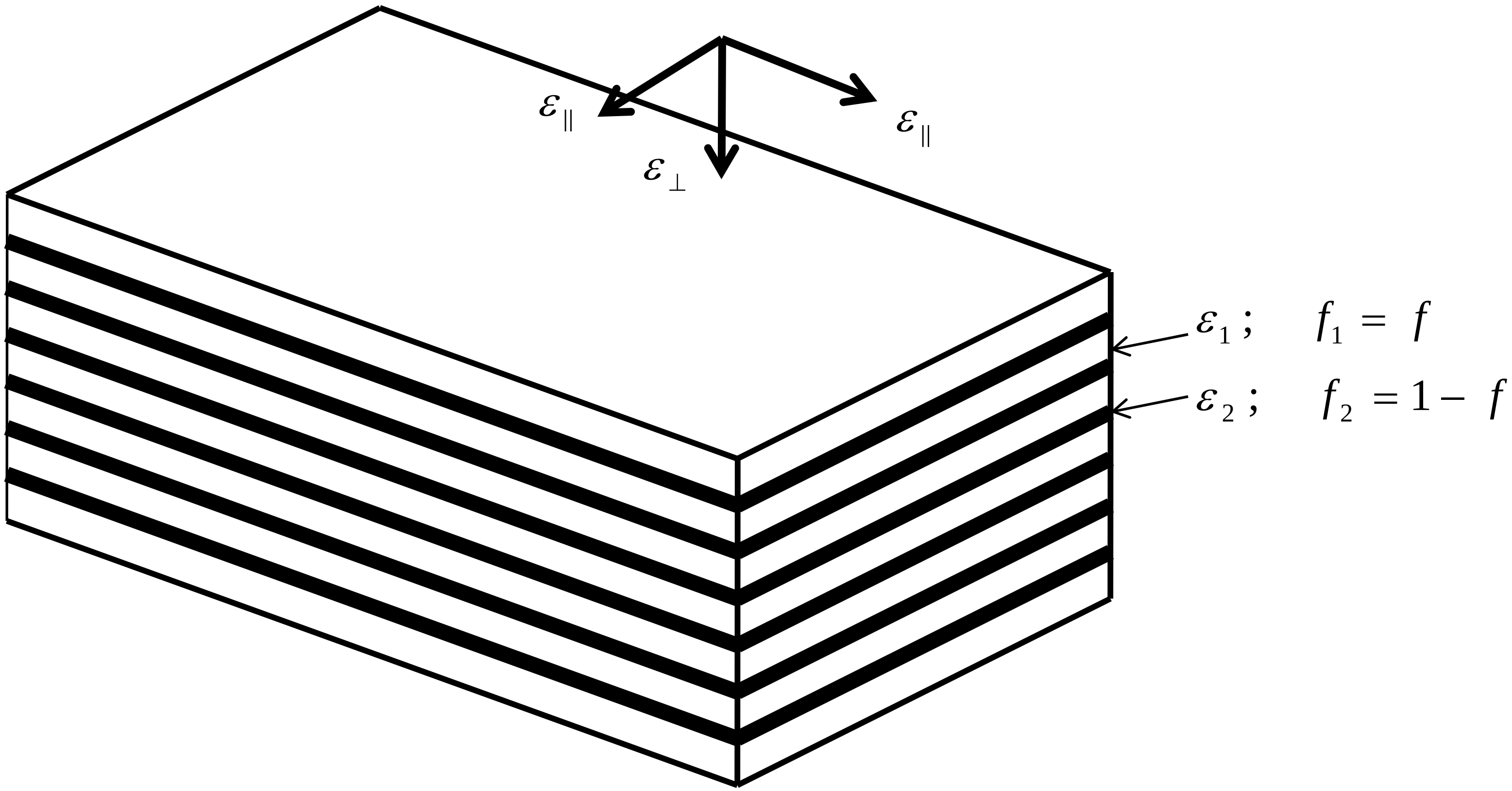
The optical artificial dielectric are (undoubtedly) within the category of the plasmonic (or photonic) meta--material, although they might not be so novel as to symbolize a meta-material research!

Various artificial dielectric both periodic and disordered serve as the bases for a more complicated meta--material structures and meta-material devices (as negative indexed material, optical hyper lens)

These artificial dielectrics are particularly necessary when the desired device requires a gradient in the Material property-example of a structure with varying permittivity required for “cloaking-device”.

Stratified Metal-Dielectric Composites & it's Optical Properties

Periodic layer composite of two isotropic constituent material with ε_1 and ε_2 respectively; with volume fill factor f and $(1-f)$



In this system there are two principal situations (1) when *E* field is directed parallel to the planer interface or (2) *E* is directed perpendicular to the planer interface. When the thickness of the layers is much less than the wave-length of shining light; this stratified system acts as a meta-material and can thus be described by equivalent (effective) parameters ε_{eff} ; $\varepsilon_{\parallel}$ & $\varepsilon_{\perp}$

Electric response of stratified meta-material to a parallel polarized E field

Now we should calculate response to the principal polarization of E field via evaluating $\varepsilon_{\parallel}$ & $\varepsilon_{\perp}$

Basic constituent equation (regardless of polarization of E field) is : $D_i = \varepsilon_i E_i$; where the subscript i denotes medium 1 or 2 or e for effective. This basic constituent relation holds for each constituent layer, as well as whole composite.

For a incident wave polarized parallel to the interfaces of the layer, the electric field E must be continuous across the boundary between the layers, i.e. $E_1 = E_2 = E_e$

The effective electric flux density (displacement) D_e is taken as the volume-averaged sum of the flux density of the two constituents $D_e = f_1 D_1 + f_2 D_2 = f_1 \varepsilon_1 E_1 + f_2 \varepsilon_2 E_2 = \varepsilon_{\parallel} E_e$

Applying the above continuity condition we have; effective permittivity for E directed parallel to the interface

$$\varepsilon_{\parallel} = f_1 \varepsilon_1 + f_2 \varepsilon_2$$

This is weighted mean (average)

Electric response of stratified meta-material to a perpendicular polarized E field

The situation gets changed when the E field is perpendicular polarized; in this case the electric flux density (displacement) is continuous at the boundary (normal component); while the average (effective) E field is the weighted average with fill fractions

$$D_i = \varepsilon_i E_i \qquad D_1 = D_2 = D_e \qquad \varepsilon_e = \varepsilon_\perp \qquad E_e = f_1 E_1 + f_2 E_2$$

$$\varepsilon_\perp = \frac{\varepsilon_1 \varepsilon_2}{f_2 \varepsilon_1 + f_1 \varepsilon_2}$$

The two effective permittivity $\varepsilon_\parallel$ & $\varepsilon_\perp$, can be got through Bruggeman EMT formula with shape-effect

Refer part-3 of lecture series :http://pdfcast.org/pdf/left-handed-maxwell-systems-in-optical-regime-part-3-effective-medium-approximations-for-optical-met

$$f_1 \frac{\varepsilon_1 - \varepsilon}{\varepsilon_1 + \kappa\varepsilon} + f_2 \frac{\varepsilon_2 - \varepsilon}{\varepsilon_2 + \kappa\varepsilon} = 0$$

The screening of external field by the medium is by κ, this factor is infinity when all the boundaries of the composite are parallel to the E field. Put $\kappa \to \infty$, in EMT, as following steps

$$f_1(\varepsilon_1 - \varepsilon)(\varepsilon_2 + \kappa\varepsilon) + f_2(\varepsilon_2 - \varepsilon)(\varepsilon_1 + \kappa\varepsilon) = 0$$

$$\kappa\left\{ f_1\left[\frac{\varepsilon_1\varepsilon_2}{\kappa} + \varepsilon\varepsilon_1 - \frac{\varepsilon\varepsilon_2}{\kappa} - \varepsilon^2\right] + f_2\left[\frac{\varepsilon_1\varepsilon_2}{\kappa} + \varepsilon\varepsilon_2 - \frac{\varepsilon\varepsilon_1}{\kappa} - \varepsilon^2\right]\right\} = 0 \qquad \text{put} \quad \kappa \to \infty$$

$$\varepsilon\left[\left(f_1\varepsilon_1 + f_2\varepsilon_2\right) - \left(f_1 + f_2\right)\varepsilon\right] = 0 \qquad (f_1 + f_2) = 1 \quad \varepsilon_\parallel = \varepsilon = f_1\varepsilon_1 + f_2\varepsilon_2 = 0$$

The other extreme is when E is perpendicular to the boundary, then we have zero screening $\kappa = 0$ putting in EMT we get

$$f_1\left[1 - \frac{\varepsilon}{\varepsilon_1}\right] + f_2\left[1 - \frac{\varepsilon}{\varepsilon_2}\right] = 0 \qquad f_1 + f_2 = 1 \qquad \varepsilon\left[\frac{f_1}{\varepsilon_1} + \frac{f_2}{\varepsilon_2}\right] = 1; \quad \varepsilon = \varepsilon_\perp = \frac{\varepsilon_1\varepsilon_2}{f_2\varepsilon_1 + f_1\varepsilon_2}$$

Generalization for several composites of stratified meta-material

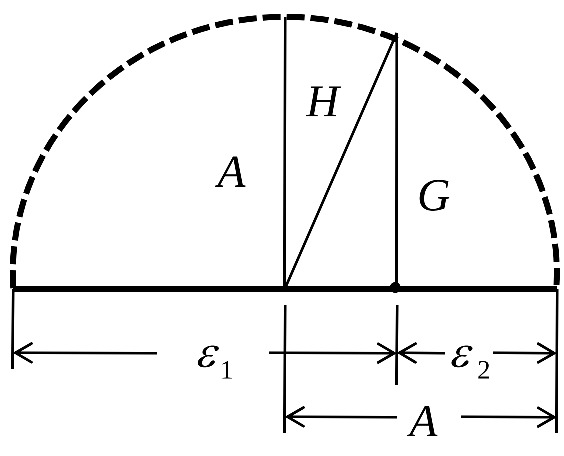
From the previous discussion we can generalize the expression for the effective permittivity for periodic layered stratified system consisting of many more materials, for the two principal polarization as:

$$\varepsilon_{\parallel} = \sum_i f_i \varepsilon_i \qquad \text{Weighted - mean} \qquad \varepsilon_{\parallel} = f_1\varepsilon_1 + f_2\varepsilon_2$$

$$\frac{1}{\varepsilon_{\perp}} = \sum_i \frac{f_i}{\varepsilon_i} \qquad \text{Weighted - HarmonicMean} \qquad \varepsilon_{\perp} = \frac{\varepsilon_1\varepsilon_2}{f_1\varepsilon_2 + f_2\varepsilon_1}$$

with $\sum_i f_i = 1$

The parallel response is weighted mean of the constituents, and the perpendicular response is the weighted ‘harmonic’ mean of the constituents



$$\varepsilon \in \Re^{+}; \quad f = 0.5$$

$$A = \frac{1}{2}\varepsilon_1 + \frac{1}{2}\varepsilon_2 = \varepsilon_{\parallel}$$

$$H = \frac{\varepsilon_1\varepsilon_2}{\frac{1}{2}\varepsilon_2 + \frac{1}{2}\varepsilon_2} = \varepsilon_{\perp}$$

$$G = \sqrt{\varepsilon_1\varepsilon_2}; \quad G^2 = AH$$

Lower & Upper bounds of effective permittivity of composite-Wiener bounds

The expression of $\varepsilon_{\|}$ & $\varepsilon_{\perp}$, provide the upper and lower bounds for the effective permittivity of the composite; as because, the E field cannot be screened more than full screening with $\kappa = 0$ or less than 'zero' screening for $\kappa = \infty$. These limits or bounds are called Wiener bounds

Let us take a metal dielectric composite, with metal $\varepsilon_1 = \varepsilon_1' + i\varepsilon_1''$ with volume filling $f_1 = f$ and dielectric $\varepsilon_2 = \varepsilon_2' + i\varepsilon_2''$; with filling factor $f_2 = 1 - f$. When we change the filling factor f from 0 to 1, the effective permittivity obviously be moving from point ε_2 to point ε_1

The permittivity is in complex plane the *x*-axis is real part of permittivity and the *y*-axis is the imaginary part of the permittivity. Thus the ε_1 and ε_2 are two isolated points in this 'complex-plane'.

When varying the filling fractions, the low screening bound given as $\varepsilon_{\|} = f_1\varepsilon_1 + f_2\varepsilon_2$ moves along straight line connecting the two points $(\varepsilon_1', \varepsilon_1'')$ and $(\varepsilon_2', \varepsilon_2'')$ on this complex plane.

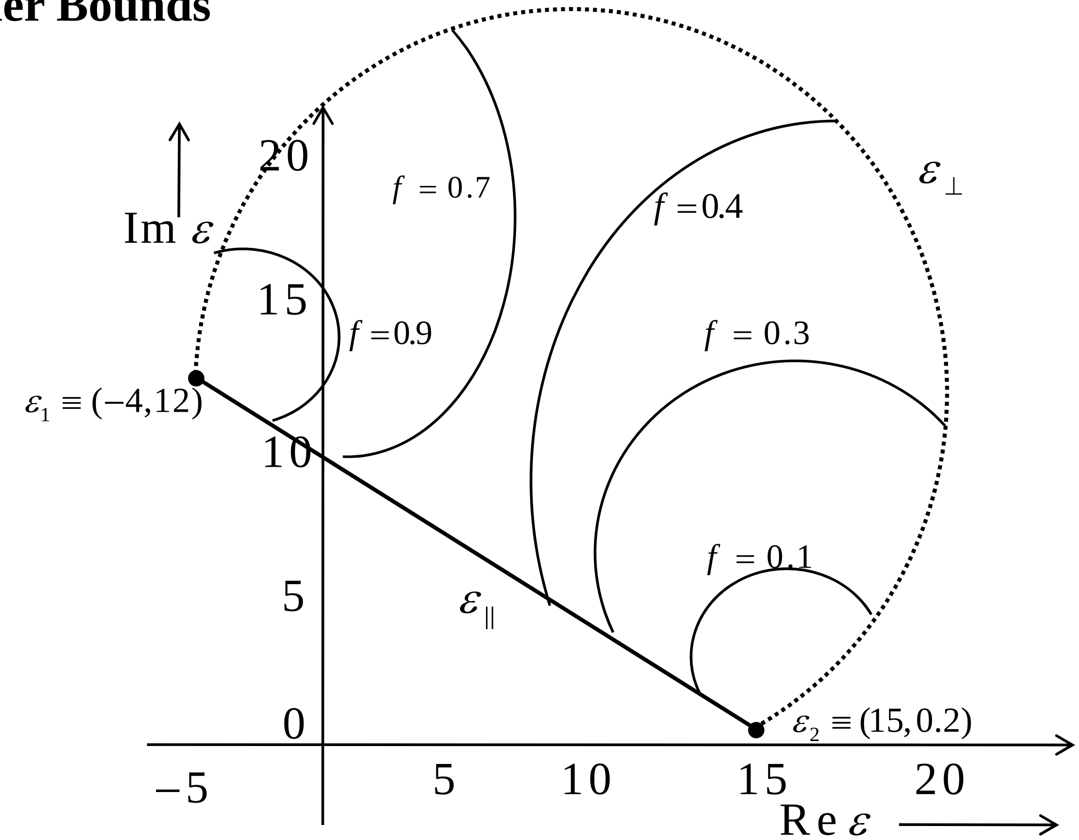
While the high screening bound $\varepsilon_{\perp} = \dfrac{\varepsilon_1\varepsilon_2}{f_2\varepsilon_1 + f_1\varepsilon_2}$, is an arc of circle passing through $(\varepsilon_1', \varepsilon_1'')$ and $(\varepsilon_2', \varepsilon_2'')$ centre as origin.

Wiener), (1912) Die Theorie des Mischkorpers fur das Feld der stationaren Stromung Abh Math-Phys Klase Koniglich Sachsichen Des Wiss : 32: 509-604

Aspen DE (1982), Local-field effects and effective medium theory-a microscopic perspective; Am J Phys: 50:704-709.

Aspen DE (1982), Optical properties of thin film, Thin solid films 89:249-262

Wiener Bounds



$\varepsilon_1 = -4 + 12i$ at $\lambda = 600$nm

$\varepsilon_2 = 15 + 0.2i$ at $\lambda = 600$nm

Titanium - Silicon

When the metal filling fraction f changes from 0 to 1, the effective permittivity $\varepsilon_{\parallel}$ for the parallel polarization varies from point $\varepsilon_2 \equiv (15, 0.2)$ to the point $\varepsilon_1 \equiv (-4,12)$ in the straight line; while the permittivity in the perpendicular direction $\varepsilon_{\perp}$ moves from point ε_2 to point ε_1 in an arc (dashed line). For a fixed metal milling f , (say $f = 0.3$) , the shape dependent EMT i.e. $f \frac{\varepsilon_1 - \varepsilon}{\varepsilon_1 + \kappa\varepsilon} + (1 - f)\frac{\varepsilon_2 - \varepsilon}{\varepsilon_2 + \kappa\varepsilon} = 0$ defines a thin arcs (curves) indicating all the possible values of the effective permittivity of the composite regardless of microstructure of the medium. The curves lies in between the two bounds.

About Wiener Bounds-to get epsilon zero and epsilon infinity meta-material

The Wiener Bounds provide us with the accessible range of the effective permittivity in a composite. By the use of layered stratified metal-dielectric composites the effective permittivity of this artificial di- -electric can reach very interesting values that are difficult to access in a conventional bulk material.

As an example the dielectric constant $\varepsilon_{\parallel}$ can be zero when $f_1 / f_2 = -\varepsilon_2 / \varepsilon_1$ is fulfilled for a low loss constituents. The other polarization i.e. $\varepsilon_{\perp}$ can approach infinity for $f_1 / f_2 = -\varepsilon_1 / \varepsilon_2$

In the complex plane diagram, when the $\mathrm{Im}\,\varepsilon_1$ and the $\mathrm{Im}\,\varepsilon_2$, are small, the two points corres- -ponding to $f = 0$ and $f = 1$ both lie very close to the *x*-axis, and thus circular arc extends to ∞ large values for $\varepsilon_{\perp}$.

These extreme values of $\varepsilon = 0$; $\varepsilon = \infty$ have great potential in development of photonic nano- -devices where sub wavelength nano-particles with appropriate ε used as lumped elements working a t optical frequencies . In such devises or circuits epsilon infinity material are needed as conducting wires for ‘optical’ displacement currents while epsilon-zero media serving as insulator to isolate each functional element. Also with choice of appropriate filling fraction, it is possible to create highly an- -isotropic media with $\varepsilon_{\parallel} \approx 0$, and $\varepsilon_{\perp} \approx \infty$ simultaneously, such as in super-lens. With dedicated control of the spatially varying thickness combinations the stratified metal-dielectric structure can be used for designing cylindrical ‘optical-cloaks’.

Wire array ‘Rodded Media’ Electric Meta-material

We have used rodded media in our micro-wave meta-material design; here we apply the same to the optical-frequencies, and note down the differences in the approach. Such rodded ‘artificial dielectrics’ wire composites have been shown to as ‘dilute-plasma’ that create negative electric response with controllable strength.

Refer: LHM Lecture 1-8

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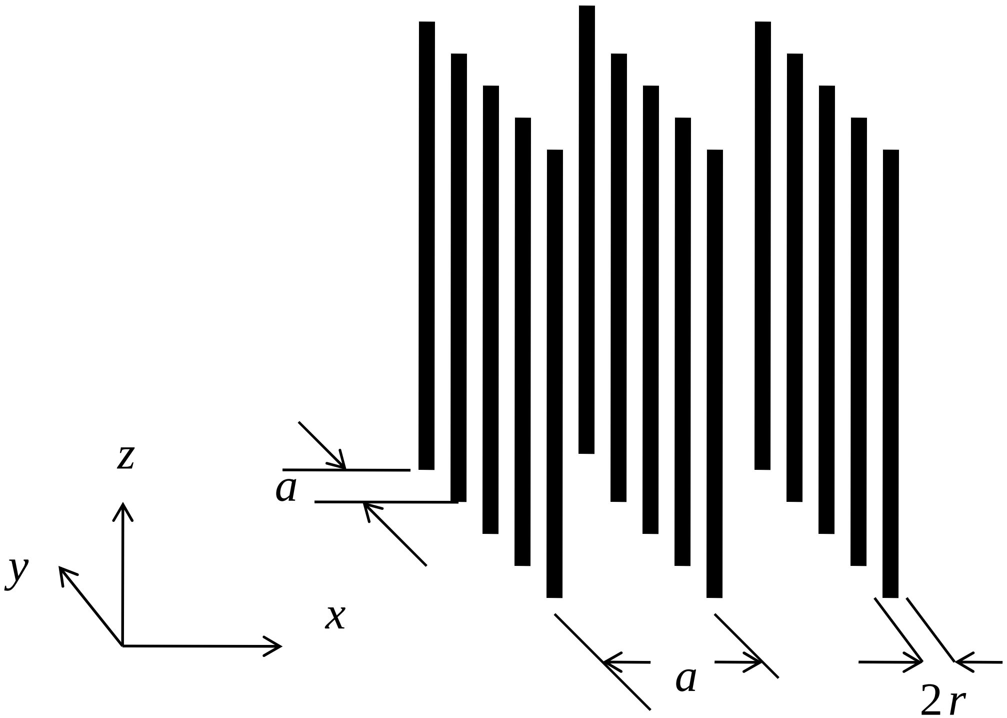
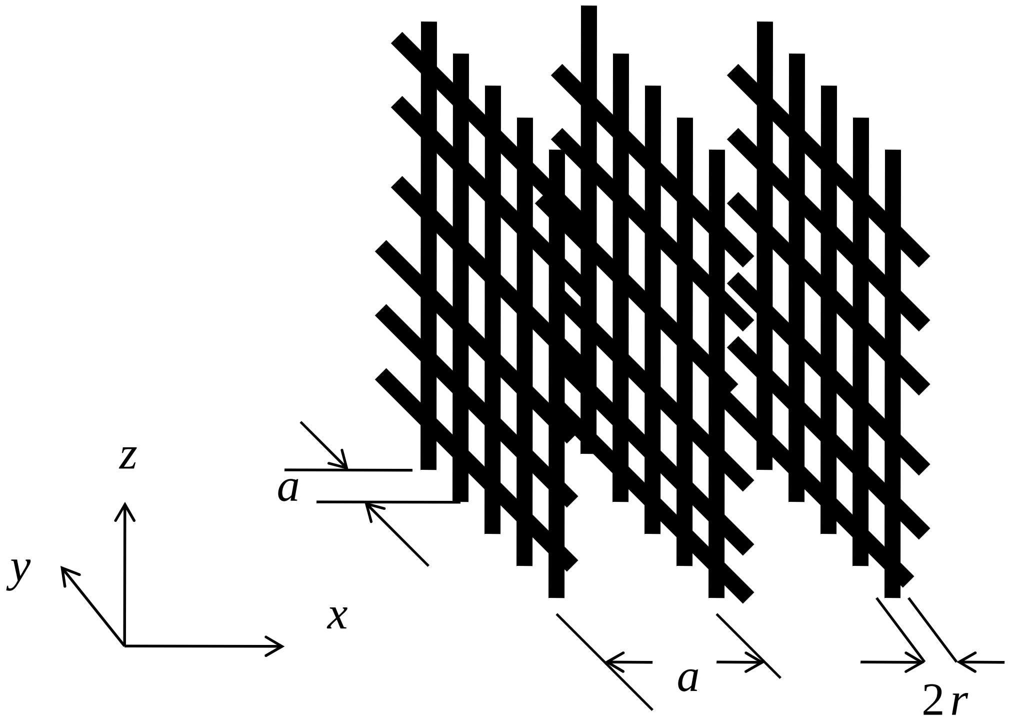
2D-Rodded Medium wire-array



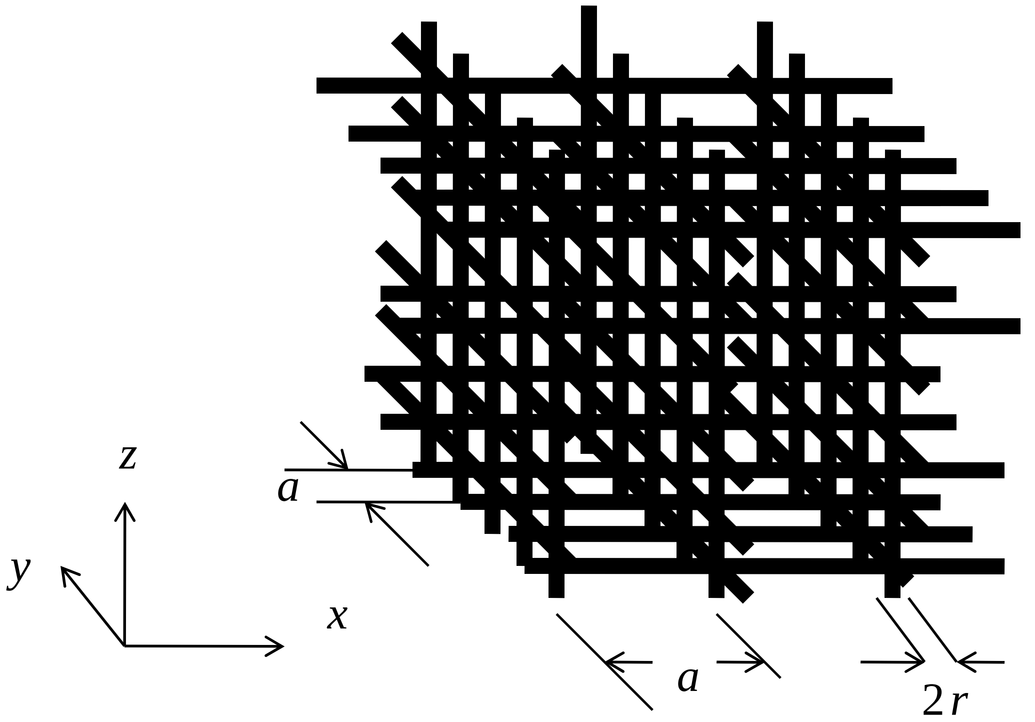
A two-dimensional wire-array for *z*- polarized electric field

3D-Rodded Medium wire-mesh



A three-dimensional lattice wire-mesh for any electric field polarized within *y*-*z* plane

3D-Rodded Medium wire-grid



A three-dimensional ‘quasi-isotropic’ wire-grid

Review of rodded-medium

The rodded medium displays plasmonic response in certain frequency band depending upon geometrical parameter like wire diameter $2r$ and unit cell length a. When lattice constant $a \ll \lambda$, this wire system of rodded medium is treated as electric meta-material because the e.m. radiation fails to resolve the wires, which are sub wavelength in size for this condition.

We simplify our discussion with 2D rodded medium, (which however is an isotropic); but we are interested in the polarization where the plane wave is propagating *x-y* plane with E field polarized along the thin-wires z direction. Therefore we denote $\varepsilon(\omega)$, to denote the element $\varepsilon_{zz}(\omega)$ in 3×3 permittivity tensor

From Drude we get $$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}$$; for metals.

As opposed to a real Drude metal, where ω_p and Γ has real physical meaning, here for rods as effective Drude model for wire-array, ω_p and Γ are closely tied to the geometry of wire as well as metal properties. Hence for getting the effective dielectric function, $\varepsilon_{eff}(\omega)$ for an electric meta--material the essentiality is to have 'approximate' ω_p and Γ, based on the known quantities.

We know plasma frequency in bulk metal depends on the density and mass of the free electrons; and this Is situated near UV or visible region for good (noble) conductors. Here in the rodded medium the ω_p Is substantially reduced relative to that of pure bulk metal.

Reduction in plasma frequency in rodded medium

First the effective electron density gets diluted by a factor $\pi r^2 / a^2$ because the free electrons are restricted within physical boundaries of wires.

Secondly we have increased 'effective mass' of electron. This results as the currents in the wire (because the E field drives it) excites a 'magnetic field'. From the direct consequence of the Lenz's law, the self inductance possessed by these metallic rods, acts to oppose any rate of change of currents. This reluctance to the rate of change of currents makes the carriers of currents i.e. the free electrons gain inertia-movement becomes lethargic; thus apparently the mass of electrons have increased. Note that notion of 'effective electron mass' should be understood here in the context of the described 'self-inductance'

These two factor reduces the plasma frequency

Plasma frequency of bulk metal $\omega_{pm}^2 = \dfrac{Ne^2}{\varepsilon_0 m}$ gets changed to $\omega_p^2 = \dfrac{N_{eff} e^2}{\varepsilon_0 m_{eff}}$ for rodded medium

$$N_{eff} < N \qquad m_{eff} > m \qquad \omega_p < \omega_{pm}$$

The effective number density and effective mass of electron in rodded medium

The effective electron density in the wire array is taken in a volume-averaged manner as:

$$N_{eff} = N\frac{\pi r^2}{a^2}$$

: N is the actual electron density in the pure metal

The modification in the electron mass is due to the self-inductance. We have to apply the concept Of generalized (canonical) momentum (in quantum mechanics). A charged particle with the static mass m, and charge q, the overall momentum is $p = mv + qA$; and A represents 'vector-potential'.

Therefore, the effective-mass from the self inductance is related to magnetic vector potential as

$$m_{eff} = \frac{eA}{v} \qquad m_{Total} = m + m_{eff} \approx m_{eff}$$

For a mean electron velocity v and free electron density N in the metallic wires the current flow in one wire is $\pi r^2 eNv$ .This current (z direction) gives rise to a magnetic-field H around the wire, with strength decreasing as distance from wire R ; as: $H = (\pi r^2 eNv)/(2\pi R)$

This is just Ampere's law. This circular magnetic field H is curl of vector potential directed z .

$H = \nabla \times A / \mu_0$. Although vector potential is not uniquely defined, depends on gauge choice.

Our assumption of $r << a$ as well as lattice symmetry gives a A as: $A = \frac{\mu_0 r^2 eNv}{2}\ln\left(\frac{a}{R}\right)$

We get effective electron mass as: $m_{eff} = \frac{\mu_0 r^2 e^2 N}{2}\ln\left(\frac{a}{r}\right)$. Since wires are almost from perfect conductors, the flow of electrons is

bound to the surface, it is fair estimate that electrons feel $A(r)$. Thus $R = r$, in above

The effective plasma frequency of rodded medium

$$\omega_p^2 = \frac{N_{eff} e^2}{\varepsilon_0 m_{eff}} = \frac{2\pi c^2}{a^2 \ln(a/r)} \qquad c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

The above description is by Sir John Pendry

Modified plasma frequency estimations are

Shalaev and Sarychev

$$\omega_p^2 = \frac{2\pi c^2}{a^2 \left[\{\ln(a/r\sqrt{2})\} + \{\pi/4\} - \{3/2\} \right]}$$

Maslovski

$$\omega_p^2 = \frac{2\pi c^2}{a^2 \left[\ln(a^2/\{4ar - 4r^2\}) \right]}$$

Amitesh Kumar, Arijit Majumder, Sougata Chatterjee, Subal Kar, Shantanu Das

$$\omega_p^2 = \frac{2\pi c^2}{\int_0^{a/2} dx \ln\left[\frac{(a/2)^2 + x^2}{r\{2\sqrt{(a/2)^2 + x^2} - r\}} \right]}$$

Calculation of effective damping for rodded medium

To complete the Drude approximation, there is damping term to be determined, for the rodded medium. This Γ term gives losses in medium. The damping constant Γ vanishes for the wires with perfect conductor and the permittivity is thus for that rodded medium would be $\varepsilon_{eff}(\omega) = 1 - \omega_p^2 / \omega^2$. However, the finite conductivity $\sigma < \infty$ for real metals results in ohmic losses in the 'thin-wires'; which can be Introduced by modifying the inductance of the wire with an added imaginary part. From the circular magn- -etic field expression (appearing due to induced electric currents in wire due to application of E), that is H as obtained $H = (\pi r^2 eNv)/(2\pi R)$; the calculation of (self) inductance of thin wire is

$$L = \mu_0 \frac{\int_r^{a/\sqrt{\pi}} H dR}{\pi r^2 eNv} = \mu_0 \frac{\int_r^{a/\sqrt{\pi}} \frac{\pi r^2 eNv}{2\pi R} dR}{\pi r^2 eNv} = \frac{\mu_0}{2\pi} [\ln R]_r^{a/\sqrt{\pi}} \simeq \frac{\mu_0}{2\pi} \ln\left(\frac{a}{r}\right) \quad \text{for} \quad r \ll a$$

The upper limit of integration is equivalent radius of square lattice i.e. $r_{equil} = a/\sqrt{\pi}$

Compare this above with obtained expression i.e. $\omega_p^2 = (2\pi c^2)/[a^2 \ln(a/r)]$ gives

$$\omega_p^2 = \frac{1}{\varepsilon_0 a^2 L}$$

For a metal with finite conductivity we replace above obtained L with $L^* = L - \frac{1}{i\omega\sigma\pi r^2}$

Some manipulation with complex inductance to get Γ the loss factor or damping

Start with perfect conductor which gives us effective conductivity as $\varepsilon_{eff} = 1 - \dfrac{\omega_p^2}{\omega^2}$

For a real L of wires we have $\omega_p^2 = \dfrac{1}{\varepsilon_0 a^2 L}$; if we replace by complex inductance we get

$$(\omega^*)_p^2 = \frac{1}{\varepsilon_0 a^2 L^*} = \frac{1}{\varepsilon_0 a^2 \left(L - \dfrac{1}{i\pi r^2 \sigma\omega} \right)} = \frac{1}{\varepsilon_0 a^2 L + i\dfrac{\varepsilon_0 a^2}{\pi r^2 \sigma\omega}} = \frac{1}{\dfrac{1}{\omega_p^2} + i\dfrac{\varepsilon_0 a^2}{\pi r^2 \sigma\omega}} = \frac{\omega_p^2}{1 + i\dfrac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma\omega}}$$

We use this plasma frequency in the lossless Drude metal and get

$$\varepsilon_{eff} = 1 - \frac{(\omega^*)_p^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2 \left(1 + i\dfrac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma\omega} \right)} = 1 - \frac{\omega_p^2}{\omega \left(\omega + i\dfrac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma} \right)}$$

Comparing with $\varepsilon_{eff} = 1 - \dfrac{\omega_p^2}{\omega(\omega + i\Gamma)}$

We get $\Gamma = \dfrac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma}$

From here we see that for $\sigma \to \infty$ a perfect conductor wire loss $\Gamma \to 0$

Effective permittivity of the rodded medium

The effective permittivity of embedded wires in vacuum

$$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_p^2}{\omega\left(\omega + i\,\dfrac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma}\right)}$$

If the wire array is embedded in a host medium with permittivity ε_h , instead of vacuum; then we get

$$\varepsilon_{eff}(\omega) = \varepsilon_h - \frac{\omega_p^2}{\omega\left(\omega + i\,\dfrac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma}\right)}$$

Since the effective plasma frequency in a wire medium can be tuned by adjusting the medium’s geometry the spectral region of desired permittivity values can be engineered to occur practically at any frequency range from the micro-wave region to the optical range. Using metals made into an array of thin wires, the plasma frequency ω_p of the medium can be reduced by several orders of magnitude. This what we did to get epsilon negative value, for our negative refractive index experiments at X-Band and Ka-Band

The damping constant is $$\Gamma = \frac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma}$$

Example of G Hz wire-array meta-material epsilon negative case and plasma frequency

Parameters for 2D wire-array

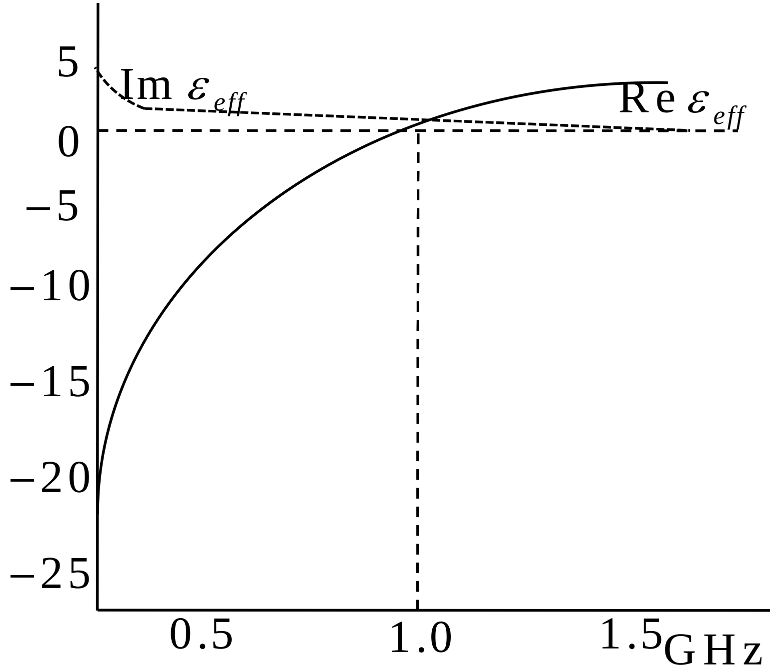
$$r = 5\mu\text{m}; \quad a = 40\text{mm}; \quad \text{silver}; \quad N = 5.8\times10^{28}\,\text{m}^{-3}; \quad \sigma = 6.3\times10^{7}\,\text{Sm}^{-1}$$

According to $N_{eff} = N\pi r^2/a^2$ the electron density is reduced by eight order of magnitude

As per $m_{eff} = (\mu_0 r^2 e^2 N/2)\ln(a/r)$ is $2.1\times10^{25}\,\text{kg}$ 'heavier' than silver atom

These two effective parameters gives reduction in plasma frequency as from $\omega_p = e\sqrt{N_{eff}/\varepsilon_0 m_{eff}}$ we get six orders reduction $\omega_p = 2\pi\times1.0\,\text{GHz}$

The damping constant from $\Gamma = \varepsilon_0 a^2\omega_p^2/\pi r^2\sigma$ is $0.018\omega_p$



This wire array system works as transparent dielectric above 1 G Hz, where effective permittivity is positive and has negative effective permittivity below this plasma frequency and thus blocks the e.m. radiation. A plasmonic high pass filter

Some points about G Hz electric meta-materials

The wires in the rodded media described need not comprise of pure metals. Instead a dielectric frame forming the lattice and coated with metals will work; provided the coating thickness is greater than the metal's skin depth at the frequency of operation. (especially while working with gold, silver etc)

Our example of wire-array is well with in our 'effective medium approximation' limit; and thus we can call it safely meta-material, because the lattice constant a is about $1/10^{th}$ of the free space wave length at 1 G Hz (i.e. 0.3 m) . The metal filling fraction in the wire array is only few ppm. (Such a low concentration of metal is enough to produce a dramatic change in effective dielectric function!).

This feature is not unexpected if we think of rodded medium in terms of effective medium approximations Bruggeman EMT etc. When the E field is polarized along the wires , the field experiences a maximum screening as indicated by near zero depolarization factor. Therefore a small concentration of metals is sufficient to produce substantial contribution to overall dielectric function.

In addition, the permittivity component normal to the wire direction should have a value identical to that of host, because wires and E field experiences negligible interaction when they are perpendicular

Wire-array at optical frequency

$$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_p^2}{\omega\left(\omega + i\,\dfrac{\varepsilon_0 a^2 \omega_p^2}{\pi r^2 \sigma}\right)}$$

The above formula for effective permittivity for rodded medium is based on the assumption that the wires are made from quasi-perfect conductor with a bulk permittivity approaching infinity. The finite conductivity in metals give rise to a damping constant Γ, of the effective medium. While the effective plasma frequency $\omega_p^2 = N_{eff} e^2 / \varepsilon_0 m_{eff} = 2\pi c^2 / a^2 \ln(a/r)$ plus the other formulas seems to be independent of metal properties. This assumption is only justified for e.m. wavelengths longer than the IR range, where the magnitude of the negative permittivity in noble metals is very large.

In visible frequencies the metal permittivity following a Drude model $\varepsilon_m = 1 - \omega_{p,m}^2 / \omega^2$ has a limited magnitude, and the effective plasma frequency ω_p of the wire medium is necessarily dependent on the plasma frequency of the bulk metal, and is proportional to volume fraction of wires.

$$\omega_p^2 = \left(\frac{\pi r^2}{a^2}\right)\omega_{p,m}^2$$

This is quite intuitive as the ω_p of the wire media should approach towards the bulk value $\omega_{p,m}$ when the filling factor approaches unity.

Silveirinha MG (2006) Nonlocal homogenization model for periodic array of epsilon negative rods; Phys Rev E 73: 046612

Use of wire-array at optical frequency

At the optical frequency the wire-array have been used in wide variety of structures and devices.

In the optical negative indexed meta-materials , arrays of metal wires and their analogues are the Mainstream choice for creating controllable negative epsilon background, which is necessary in most negative index media.

The wire media is also to construct meta-materials with ultra low refractive indices of less than unity. Such ultra low index band corresponds to frequencies slightly higher than the effective plasma frequency.

Schwartz BT, Piestun R, (2003) , Total external reflection from meta-materials with ultra low refractive index; J Opt. Soc. Am B 20 : 2448-2453

Schwartz BT, Piestun R, (2004); Waveguiding in air by total external reflection from ultra low refractive index meta-material; Appl Phys Lett 85 : 1-3

Rodrguez-Esquerre VF, et al (2005); Power splitters for wave guides composed by ultra low refractive index metallic nanostructures; Appl Phys Lett 87 : 091101

Wire-media and arbitrary polarization and spatial dispersion-advanced topic

We assumed ideal polarization for the wire-medium discussion; the E field being parallel to the wires. for the excitation E field in arbitrary direction (orientation) the wire-mesh may support ‘additional’ modes, that cannot be accurately characterized by the ‘local-permittivity’.

In such situations the electric response of the medium becomes ‘non-local’ with spatial dispersion, which means that the induced polarization is dependent upon not only the point-wise field values but also on the field variations at the other points in space.

Belov PA, et al; (2003) , Strong spatial dispersion in wire media in the very large wavelength limit; Phys Rev B 67: 113103
Shapiro MA , et al (2006); Spatial dispersion in meta-material with negative dielectric permittivity and its effect on surface waves. Opt. Lett. 31: 2051-2053.
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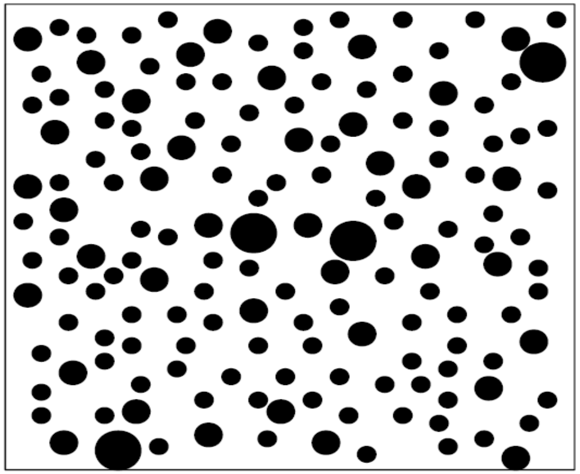
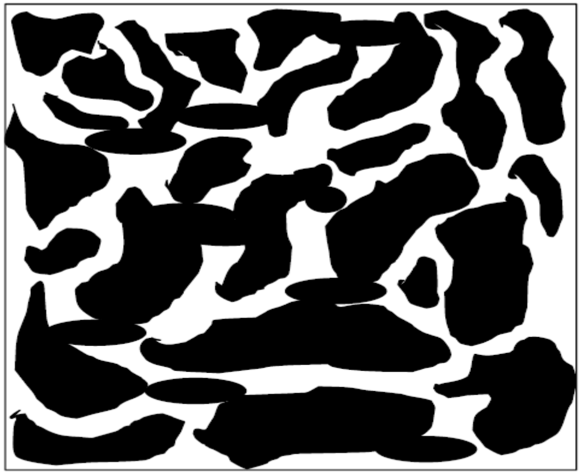
Random (non-periodic) metal inclusions a disordered meta-material

Random metal-dielectric composites can be regarded as disordered meta-materials with very interesting optical properties differing markedly from the bulk materials.

Random metal-dielectric composites is fabricated by range of deposition techniques, including thermal evaporation, electron beam evaporation and sputtering as well as electroplating. Due to the nature of these depositions techniques, majority of random metal-dielectric composites are confined to thin film or coating on two or three dimensional substrate. Therefore in most cases such random-composites are treated as two-dimensional system.





During these deposition process first small isolated islands nucleate first , with additional deposition the metal concentration increases and islands grow and coalesce forming irregular shaped clusters of fractal geometry.

Percolation films

The percolation threshold implies more than merely an insulator-to-metal transition. When the filling fraction of the metal components approaches the percolation threshold , a modest alteration of the film morphology will induce a drastic change in the optical response.

Percolation films are characterized by fractal geometry that will appear similar at different scales.
A percolation metal-dielectric film is formed by cluster of all sizes, from the size of individual particle to the ‘infinite’ fractal cluster that spans the whole film.

Such morphological feature results in creation of ‘localized’ plasmonic resonances, sometimes called “hot-spots” , across a broad e.m. spectrum.

Formation of localized plasmonic resonances ‘hot-spot’ in percolation film

At the optical frequencies noble metals exhibit very high conductivity with a very small loss factor, as determined by the imaginary part of the metal’s permittivity. Each grain metal grain cluster can be described by inductance L connected in series to a resistance R . The dielectric gaps in turn acts as capacitance C . By virtue of this description the composite films can be stated to be RLC circuit. If we represent l the size of the metal and dielectric grains, both the L and C should scale proportional to the size l . The frequency at which the effective RLC circuit resonate is $\omega_r = 1/\sqrt{LC} \propto l^{-1}$
Therefore, smaller cluster resonate at high frequency, while larger cluster resonate at low frequency.

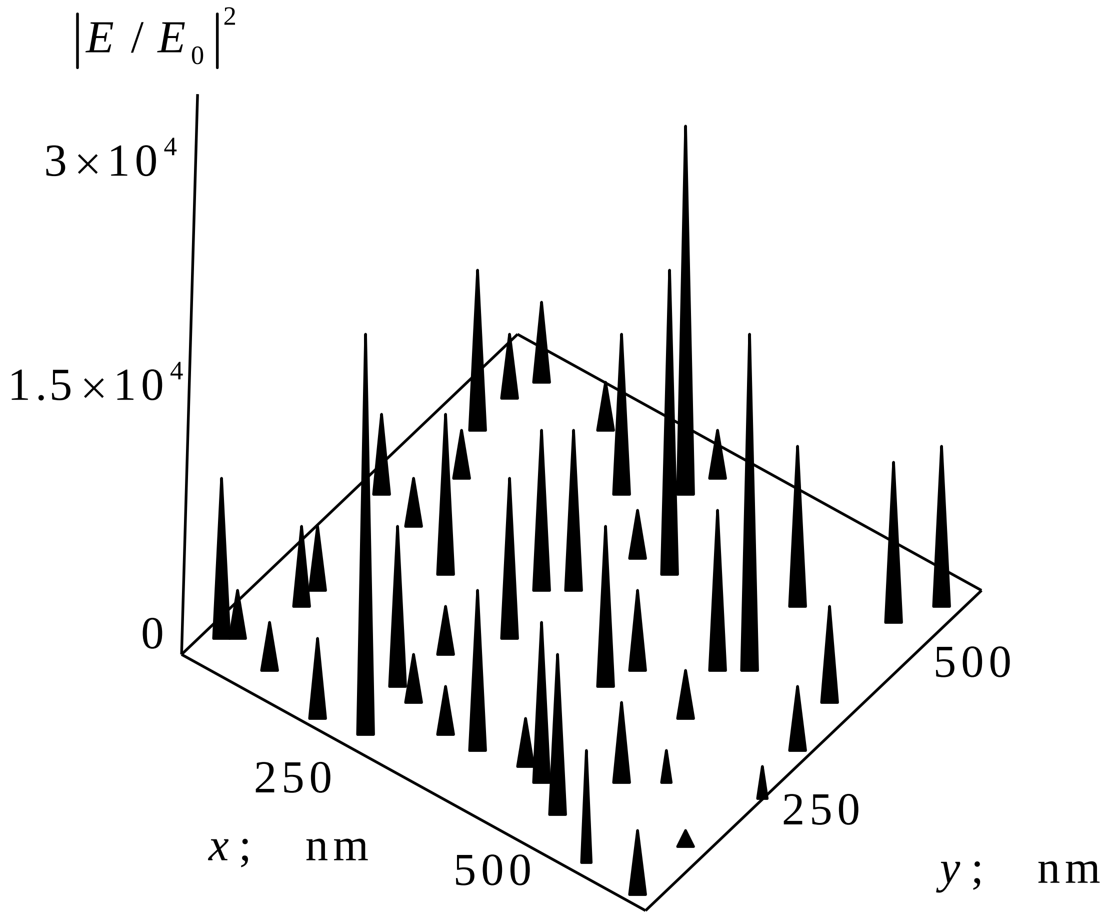
Thus, metal-dielectric film is a collection of resonating RLC circuit, with R, L and C taking random values. The geometrical disorder in this composites determines these random values and give rise to resonance frequencies covering a very wide spectral range from UV to mid IR.

The light-induced plasmon modes in percolation metal-dielectric films can lead to dramatic enhancement of optical responses in wide spectral range. The creation of e.m. ‘hot-spot’ in these percolation film is a phenomena within the broad category of Anderson’s localization (localization of electron wave--function without diffusion in semiconductors with a certain high degree of randomness of impurities).

There are giant local fields in these films, enhanced by 10E5 concentrated in nano-meter sized area.

Anderson PW, (1958), Absence of diffusion in certain random lattices , Phys Rev 109: 1492-1505.

Spatial distribution in metal-dielectric (silver-silica) percolation film at an arbitrary wavelength (1500nm)



Sarychev AK, Shalaev VM (2000), Electromagnetic field fluctuations and optical nonlinearlties in metal-dielectric composites, Phys Rep 335: 276-371

Genov DA, Sarychev AK, Shaleav VM, (2003), Plasmon localization and local field distribution in metal-dielectric films, Phys Rev E, 67: 056611

Conclusions

The importance of the metal-dielectric composite to make artificial dielectric function at the optical frequencies is important aspect of the meta-material applications in the optical regime. A careful treatment thus is needed to get the effective dielectric properties with these developed theories in this direction.

End of part-4