Left Handed Maxwell Systems
In Optical Regime

PART-2
Metals in Optical Region its Electrodynamics

Shantanu Das
RR&PS
Reactor Control Division, B.A.R.C. Mumbai-400085
shantanu@barc.gov.in

Few salient points

Conventionally metals are used as mirrors and optical thin films in optical system

In meta-material (optical or for microwave ranges) metals are used in the ‘unit cell’.

The sharp contrast between the optical response of metals and that of the dielectrics is essential in making meta-atoms functional elements.

Common features from everyday observation tell us that at optical frequencies (i) metals are opaque (unless the metal is thinner than the skin depth-which is of order of few nano-meter at visible range), and (ii) the metals are highly reflective, most light impinging on the metal surface are reflected. Both these (i) and (ii) originate from the behavior of electrons in the metal.

Since the energy levels of metals do not have any forbidden gap, consequently photon energy from any electromagnetic radiation is enough to excite an electron to a higher level. Since the empty electron states are continuously available light of all frequency that enters into a metal (which is not easy) can be absorbed Within a tiny propagating length, usually < 100 nm. Many metals like silver and aluminum have bright Silvery appearance when exposed to sunlight, as they are ‘highly’ reflective across the entire visible spectra While some metals like gold copper are colored as the high frequency of the light are not reflected, and What we see is containing predominant colors in the yellow and red.

All these phenomena can be explained by dielectric function of metals.

Optical Properties of Metals

Similar to study of the dielectrics most of the mechanism behind light-metal interaction are ascribed to dispersion in permittivity function (i.e. frequency dependency of dielectric function of metals). The start point is the constitutive equations as below; plus the behavior of the free electron in the *E* field.

$$D = \varepsilon_0 E + P = \varepsilon_0 (1 + \chi_e) E = \varepsilon_0 \varepsilon_r E$$

$$B = \mu_0 (H + M) = \mu_0 (1 + \chi_m) H = \mu_0 \mu_r H$$

The e.m. response of a metal is largely dictated by ‘collective movement’ of the free electrons within the metal crystalline structure-(plasma). As in case of the dielectric we have taken a model of Lorentz oscillator, for an electron; we can extend the same harmonic oscillator model to metals, in this case the of Metals the electrons are free to move about in the metal lattice, ‘without restoring force or the spring constant’. This is also a Drude model of free electron.

Drude model for dielectric function for metals

The Drude model links electric properties of material with behavior of electrons (or holes)

Consider a typical electron denoted by $x = x(t)$ the deviation for equilibrium position

External electric field $E = E(t)$. The motion of bound electron with mass m, charge q, friction or damping as $m\Gamma$ and spring constant as $m\omega_0^2$ is

Recall $m(\ddot{x} + \Gamma\dot{x} + \omega_0^2 x) = qE$ Spring constant $m\omega_0^2$ vanishes

Fourier transforming the above we get

$$m(-\omega^2 - i\omega\Gamma)\tilde{x} = q\tilde{E}$$

Gives position function in frequency as $\tilde{x}(\omega) = \frac{q}{m}\frac{\tilde{E}}{(-\omega^2 - i\omega\Gamma)}$

Dipole moment of typical electron $p(\omega) = \tilde{p} = q\tilde{x}$

Polarization and the dielectric function from Drude model

There are *N* typical electrons per unit volume, then polarization *P* is

$$\tilde{P} = Nq\tilde{x} = \varepsilon_0 \chi_e \tilde{E}$$ also we have $$\tilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}}{(-\omega^2 - i\omega\Gamma)}$$ gives

Dielectric susceptibility as $$\chi_e(\omega) = \frac{Nq^2}{\varepsilon_0 m} \frac{1}{(-\omega^2 - i\omega\Gamma)} = \frac{\omega_p^2}{(-\omega^2 - i\omega\Gamma)}$$

$$\omega_p^2 \triangleq \frac{Nq^2}{\varepsilon_0 m}$$ Defines Bulk (volume) Plasma frequency

Recall the constitutive equation as

$$D(\omega) = \varepsilon_0 E(\omega) + P(\omega) = \varepsilon_0 \{1 + \chi_e(\omega)\} E(\omega) = \varepsilon_0 \varepsilon(\omega) E(\omega)$$

$$\varepsilon - 1 = \chi_e = \frac{\omega_p^2}{(-\omega^2 - i\omega\Gamma)}$$

Therefore the dielectric function is

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{(\omega^2 + i\omega\Gamma)}$$

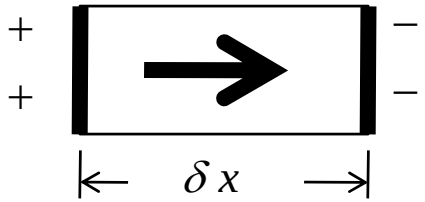
$$= 1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2} + i \frac{\omega_p^2 \Gamma}{\omega(\omega^2 + \Gamma^2)}$$

Plasma frequency

If the electrons in a plasma are displaced from a uniform background of positive charges, the E fields will built up in such a direction as to restore the neutrality of plasma by pulling the electrons back to their original positions. The free electron of metal behave as plasma.

Because of their inertia the electrons will overshoot and oscillate around their equilibrium position with a characteristic frequency called ‘plasma-frequency’.

A rough derivation is as follows:



Consider a unit area of plasma having number density of *N* electrons per unit volume. If *e* is electron charge and δx is the charge separation from background, when *E* field is applied then, from Gauss law (total charge enclosed in volume is equal to electric field times ε_0

$$E = Ne(\delta x)/\varepsilon_0$$

The plasma will oscillate at ω_p ; thus consider small amplitude oscillation as $\delta x = \delta x_0 e^{-i\omega_p t}$

There will be force balancing as

$m\frac{d^2(\delta x)}{dt^2} = -eE = -\frac{Ne^2(\delta x)}{\varepsilon_0}$ substituting the displacement expression from above we get

$-\omega_p^2 m = -Ne^2/\varepsilon_0$ Bulk (volume) Plasma frequency is $\omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$

Critical (plasma) wavelength for metals

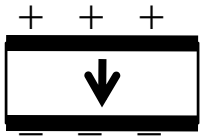
The critical wavelength λ_p below which the metals become transparent is $\lambda_p = 2\pi c / \omega_p$ & above which they are opaque and highly reflective. Table gives figures for alkali metals

Metal	Li	Na	K	Rb	Cs
$(\lambda_c)_{obs}$	2050 A	2100 A	3150 A	3600 A	4400 A
$(\lambda_c)_{cal}$	1500 A	2100 A	2900 A	3200 A	3600 A
N_{eff} / N	0.54	1.00	0.85	0.79	0.67

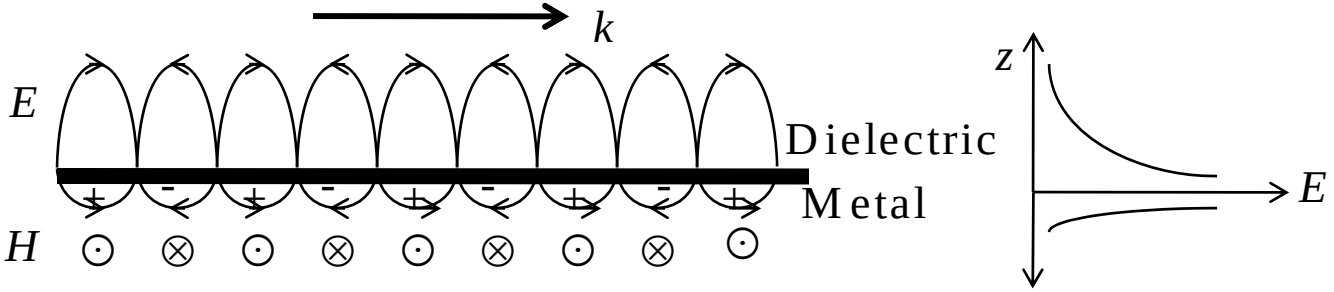
Plasma waves (plasmons or plasmon polariton)

Metals are expected to allow for electron density waves; and these waves are called plasmons.

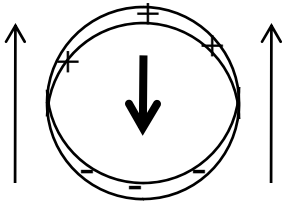
Bulk Plasmon: Metals allow for e.m. wave propagation above plasma frequency and they become transparent, and these bulk wave with volumetric plasma frequency is bulk plasmon (also called bulk plasmon polariton)



Surface plasmon: At the surface of dielectric-metal interface, sometimes called SPP surface plasmon polariton. Plasmon confined to surface that can interact with light to form propagating SPP. The SPP is strong localized field, and a TM wave *H* perpendicular to *k*



The confinement effect in SPP results in resonant SPP modes in metal nano particles



Drude model parameter for Gold

Correction for $\omega \gg \omega_p$ $\qquad \varepsilon(\omega) = \varepsilon_\infty - \dfrac{\omega_p^2}{(\omega^2 + i\omega\Gamma)}$

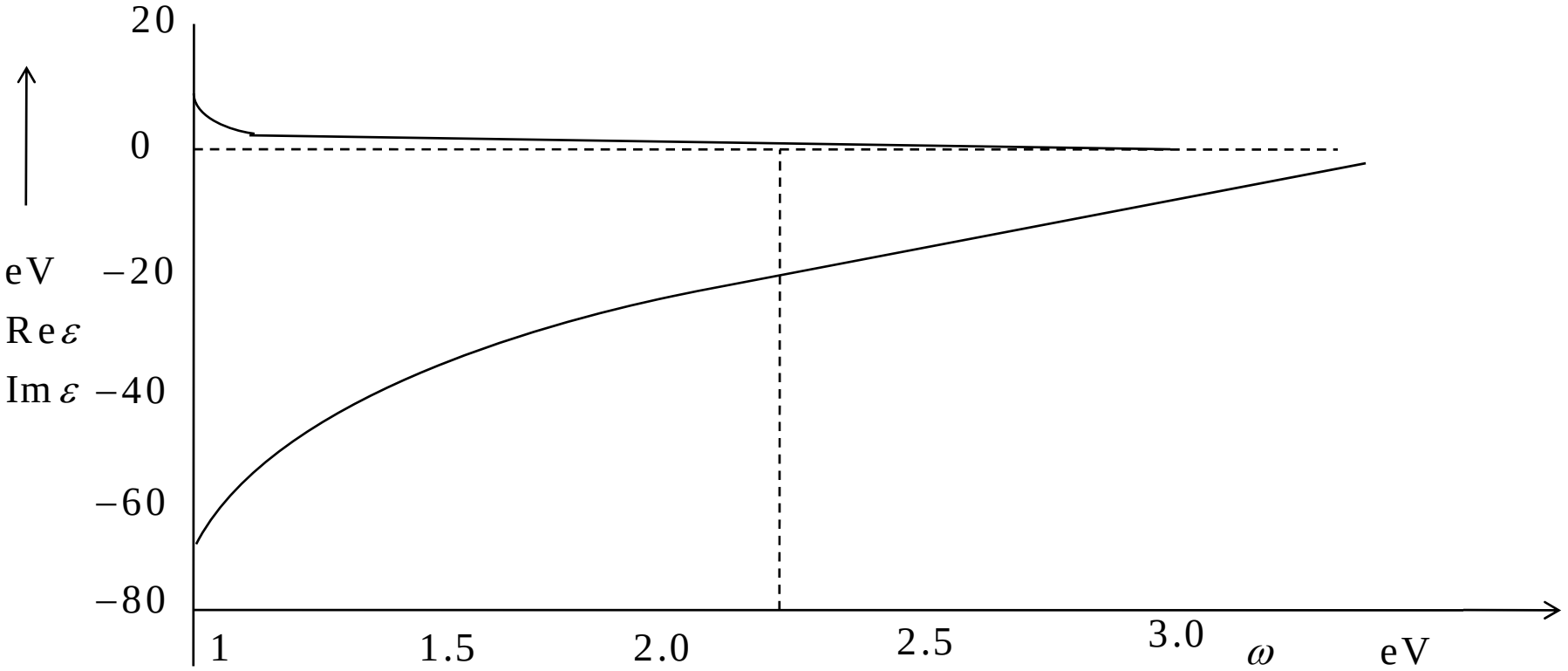
Determined by Johnson & Christy 1972

$\varepsilon(\lambda = \infty) = \varepsilon_\infty = 9.5 \qquad \hbar\omega_p = 8.95\text{eV} \qquad \hbar\Gamma = 0.069\text{eV}$

With these parameters the Drude model fits optical measurement well for $\hbar\omega < 2.25\text{eV}$ (green)

The refractive part of ε can be large and negative while absorptive part is small

This allows SPP (Surface Plasmon Polariton)



Discussions

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{(\omega^2 + i\omega\Gamma)} = 1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2} + i\frac{\omega_p^2\Gamma}{\omega(\omega^2 + \Gamma^2)}$$

Above we got frequency dependent dielectric function for metals. The ω_p is the volume plasma frequency at which the density of the 'electron gas' in metal oscillates, is $\omega_p = \sqrt{Nq^2/\varepsilon_0 m}$ Note that the damping constant which represents collision rate is absolutely necessary to give the imaginary part in $\chi_e(\omega)$ or $\varepsilon(\omega)$. The inverse of damping constant i.e. Γ is the mean electron collision time- $\tau = \Gamma^{-1}$. Therefore the damping constant Γ is related to the electron mean free path *l*, and the Fermi velocity by $\Gamma = v_F / l$. Although Fermi velocity does not enter the Drude model It will be used to evaluate the 'size effect dependence' on the dielectric function.

Metal	ω_p (eV)	$\omega_p(10^{15} s^{-1})$	Γ (eV)	$\Gamma(10^{15} s^{-1})$	$v_F(10^6 ms^{-1})$
Silver	9.2	14.0	0.021	0.032	1.4
Gold	9.1	13.8	0.072	0.11	1.4
Copper	8.8	13.4	0.092	0.14	1.6
Aluminum	15.1	22.9	0.605	0.92	2.0

Need modification in Drude model

The Drude model though gives elegant & concise treatment for noble metals; it has to be modified to adequately reflect reality. From the table of noble metals we observe that, “with almost same plasma frequency about 9 eV, the gold and silver appears different when exposed to visible light”. Since gold is “yellowish” in color what happens to green and blue portions of the light? Drude model does’nt say this!

It is not unexpected (that we need some modification to Drude, at optical frequencies); since we have only accounted for free electrons, but not all the electrons in metal are free. Although the behavior of noble metals is dominantly governed by free electrons, the contributions of the bound electrons should be taken also in the visible range, as we are working at high frequency (that means high photon energies).

Drude model for metals describe the behavior of electrons in only the outer atomic orbital 5s, 6s, 4s for Silver gold copper respectively. At these optical frequencies (high energy photons), there are ‘inter-band transitions’ from 5d to 6sp state in gold. For silvery colored metals, like silver or alkali metals these inter-band transition occurs well beyond the visible spectra.

The influence of inter-band transitions need be thus supplemented to the Drude model.

Inter-band transition inclusion in Drude model for metal's dielectric function

The contribution of the inter-band transition of the bound electrons to the dielectric function is very similar to the corresponding resonances in dielectric material, as we derived

$$\varepsilon(\omega) = 1 + \sum_{j=1,2} \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

Considering only ω_2 , we write the dielectric function at inter-band transition as

$$\varepsilon_{i,b}(\omega) = 1 + \frac{\omega_1^2}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

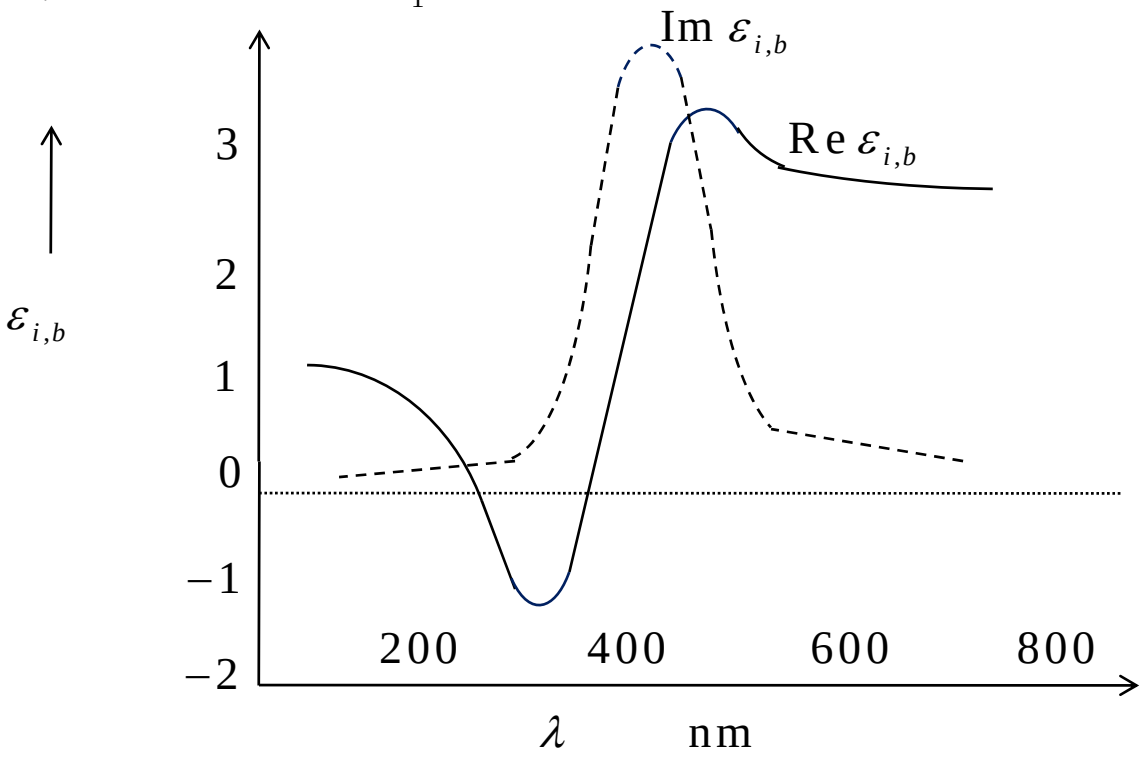
Where ω_0 denotes oscillation frequency of a bound electron under an applied electric potential, and the ω_1 and γ are related to number density and the damping of bound electrons respectively.

The overall dielectric function of the metal is thus contains Drude term for free electrons plus inter-band term for the bound electrons at optical frequency range

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega) = \varepsilon_{i,b}(\omega) + 1 - \frac{\omega_p^2}{\omega^2 + i\Gamma\omega}$$

Example of inter-band transition in gold

We plot $\varepsilon_{i,b}(\omega)$ for gold with an inter-band transition in visible region based on formula for $\varepsilon_{i,b}$ The central frequency ω_0 of resonance is 2.8 eV, corresponding to wave-length of 450 nm. Taken value of $\gamma = 0.6\text{eV}$ and $\omega_1 = 3\text{eV}$.



Inter-band transition of bound electrons indeed exerts large influence on the dielectric properties of gold at visible region. The dielectric function of gold is distorted, moving towards positive ε direction (more dielectric like) associated with increased loss for blue-green light. Thus blue-green light is strongly absorbed by gold leaving predominantly the yellow light reflected back

More impact of inter-band transition & modified Drude dielectric function for metal

A noticeable feature of inter-band transition is that it has a 'non-zero' value of dielectric function for metals even at longer wave-lengths than that of resonance wave-length.

$$\varepsilon_{i,b}(\omega) = 1 + \frac{\omega_1^2}{\omega_0^2 - \omega^2 - i\gamma\omega}$$

$$\varepsilon_{i,b}(0) = 1 + \frac{\omega_1^2}{\omega_0^2} > 0$$

$$\varepsilon_{i,b}(\lambda \to \infty) = \varepsilon_\infty > 0$$

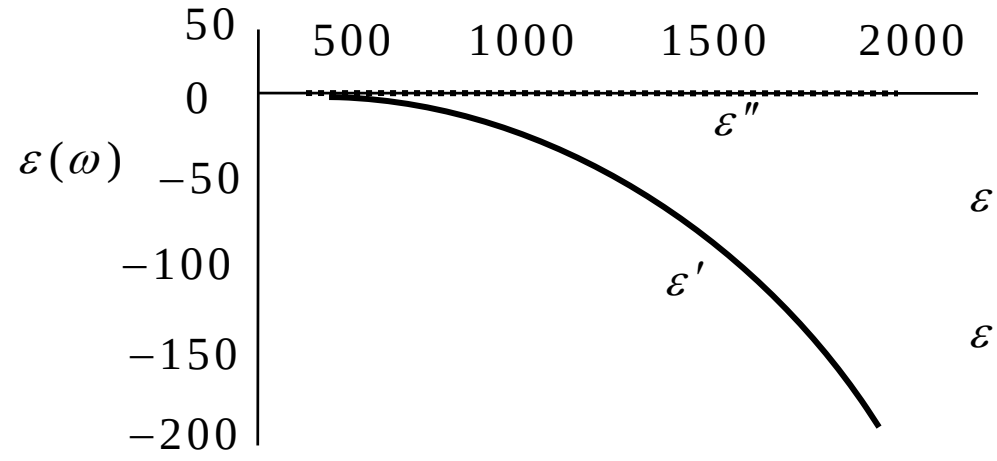
Therefore, while working at longer wave-lengths away from inter-band resonance, (IR for gold, and visible plus IR for silver), we can therefore replace the frequency dependent $\varepsilon_{i,b}(\omega)$ term by a constant offset ε_∞

It is important to note that due to complicated band structures of bound electrons there are several or multiple inter-band transitions, in the UV or deep UV region. Hence usually ε_∞ is a sum a after taking all pertinent transitions into account. Finally the modified Drude model taking into all inter-band transitions is:

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\Gamma\omega} = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + \Gamma^2} + i\frac{\omega_p^2\Gamma}{\omega(\omega^2 + \Gamma^2)}$$

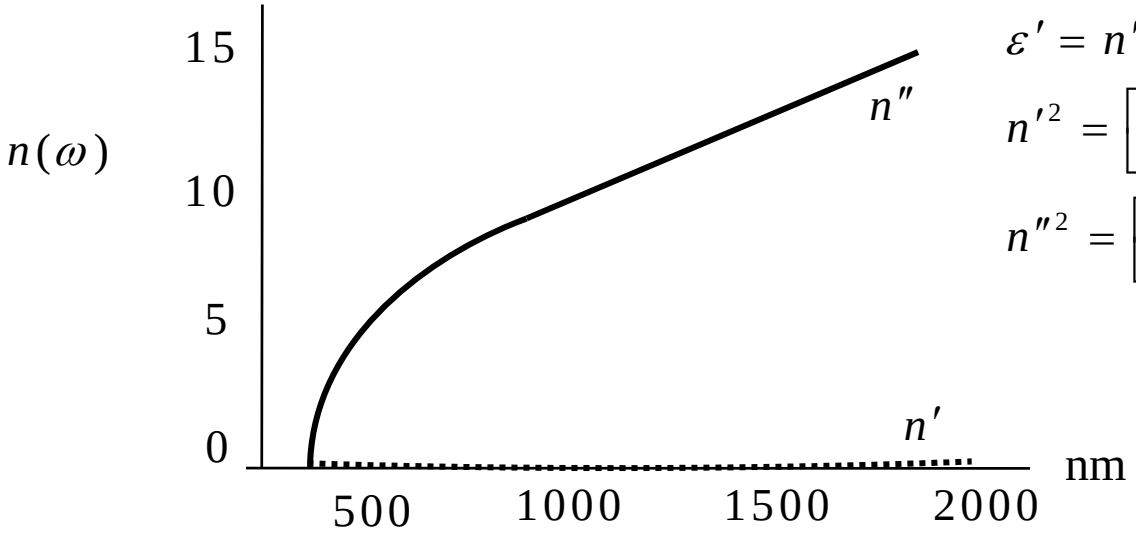
The empirical values of ε_∞ silver and gold are 5 & 9 respectively. The Γ^2 can be neglected as at optical frequency $\Gamma < \omega$.

Drude model for dielectric function & refractive index of silver at optical regime visible and near IR range



$$\varepsilon'(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + \Gamma^2} \approx \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}$$

$$\varepsilon''(\omega) \approx \frac{\omega_p^2 \Gamma}{\omega^3}$$



$$\varepsilon' = n'^2 - n''^2; \quad \varepsilon'' = 2n'n''$$

$$n'^2 = \left[\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}\right]/2$$

$$n''^2 = \left[-\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}\right]/2$$

Observations regarding metal at optical frequency

As opposed to dielectric materials (which gets characterized by +ve permittivity at optical frequency) the real part of the dielectric function of the metals (rather noble metals) is distinctively –ve. A negative value of $\varepsilon'(\omega)$, says that, the free electrons in metal oscillate out of phase w.r.t. driving *E* field. Thus most of the incident photons gets reflected at the metal-dielectric interface surface. Also at the optical frequency $\varepsilon'' \le |\varepsilon'|$ holds for the dielectric function of the metals; therefore the refractive index has pronounced imaginary part while the real part of refractive index is almost zero (slight +ve).

From the Fresnel's equation we recall the reflection coefficient is $(n_1 - n_2)/(n_1 + n_2)$. This reflection coefficient must be unity when n_1 is real and n_2 is purely imaginary, i.e. obvious as ratio of two complex conjugate is unity $(a - ib)/(a + ib) = 1$. As a result, at the boundary between a dielectric (say air) and noble metal (say silver), almost all the incident light gets reflected (as long as incident wavelength is substantially longer than that of inter-band transitions.

Another phenomena results from large n'' that is light can penetrate through a very thin layer of the noble metals, and the skin depth which is inverse of the absorption coefficient $\alpha = 4\pi n'' / \lambda_0$ is of the order of 50 nm at the optical frequencies.

Interaction of light with nano-sized metal –the size effect correction in Drude model

While studying the interaction of light with the nano sized metals (or metal dielectric composites), the properties of metals (as obtained from Drude model) needs to be further modified due to size restriction

The damping parameter Γ is to be modified as the size of metal goes down to tens of nanometers. The Γ is collision rate related to electron's mean free path l in metal (as $\Gamma = v_F / l$). This l is the mean free path of electron for the unrestricted size of metal.

When the length scale of the continuous metal portion of the meta-material unit cell is comparable or smaller than the l , the electron's movement is therefore further gets limited by the physical boundary, of the metal structure, and effective mean free path is reduced as:

$$\frac{1}{l_{eff}} = \frac{1}{l} + \frac{1}{R}$$

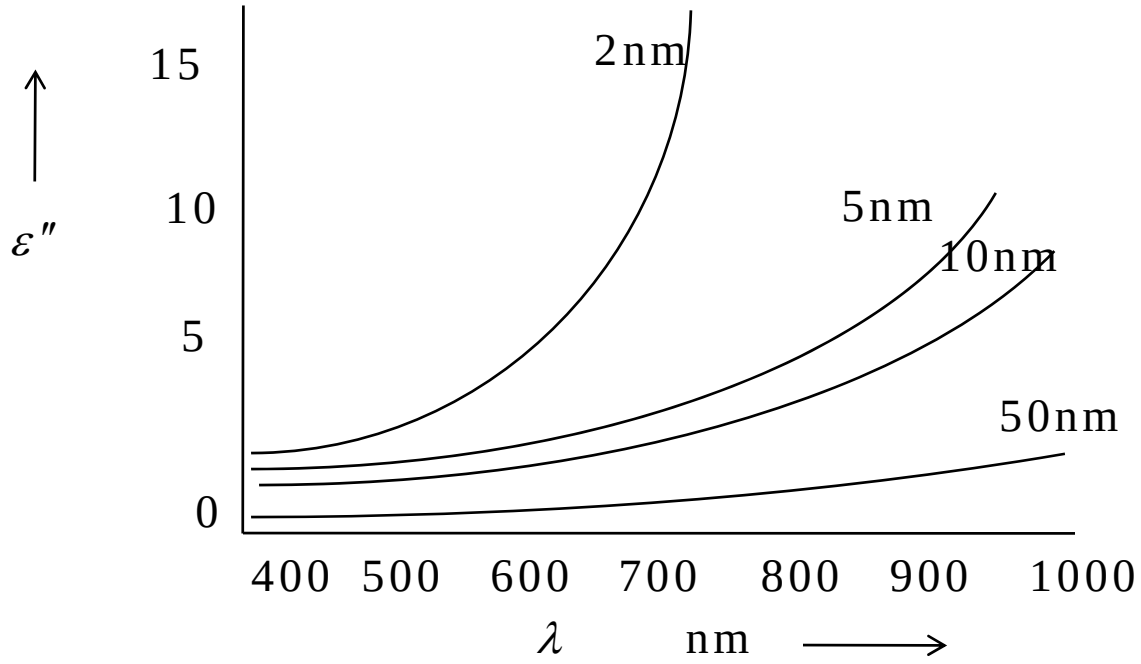
Where R is the size of the metal particle and l_{eff} is effective (size-limited) mean free path of the electrons. From where we get size-limited damping constant as

$$\Gamma_{eff} = \Gamma + a\frac{v_F}{R}$$

The factor *a* is on the order one depends on geometry specifics

The effect of size

Since the real part of the dielectric function of the metal is only marginally related to damping, while the imaginary part is proportional to Γ , the $\varepsilon''(\omega)$ is effected much by size-factor. Thus when the size of the metal (silver) is in tens of nanometers or smaller, the ε'' is substantially larger than the bulk value. This part should be of consideration in optical meta-material designs, metal nano strips, fish-net



Paul Drude, German physicist, 1863-1906

Electrical conductivity from Drude

Typical charged particle motion in electric field is $m(\ddot{x} + \Gamma\dot{x} + \omega_0^2 x) = qE$

Electric current density is $J = Nqv = Nq\dot{x}$, Fourier transforming we get $\tilde{J} = Nq(-i\omega)\tilde{x}$

Recall $$\tilde{x}(\omega) = \frac{q}{m}\frac{\tilde{E}(\omega)}{(\omega_0^2 - \omega^2 - i\omega\Gamma)}$$

Ohm's law is $\tilde{J}(\omega) = \sigma(\omega)\tilde{E}(\omega)$

Therefore conductivity expression is:

$$\sigma(\omega) = \frac{Nq^2}{m}\frac{-i\omega}{\omega_0^2 - \omega^2 - i\omega\Gamma}$$

A material with $\sigma(0) = 0$ is an electrical insulator, it cannot transport dc currents

A material with $\sigma(0) > 0$ is an electrical conductor

In metal, free charged particles (electrons) we set $\omega_0 = 0$, giving $\sigma(\omega) = \frac{Nq^2}{m}\frac{1}{\Gamma - i\omega}$

$$\frac{\sigma(\omega)}{\sigma(0)} = \frac{1}{1 - i\omega/\Gamma} \qquad \sigma(0) = \frac{Nq^2}{m\Gamma} \qquad \sigma(0) > 0$$

Note the dc conductivity is always positive.

Conduction currents in metals

For a general case we find that $\tilde{J} = \left[\frac{\sigma(0)}{1 - i(\omega / \Gamma)} \right] \tilde{E} = \sigma(\omega)\tilde{E}$

For $\omega = 0$, we have static conductivity or dc conductivity as $\sigma_0 = \sigma(0) = Nq^2 / m\Gamma$

For a very low frequency $(\omega / \Gamma) \ll 1$, the dynamic conductivity $\sigma(\omega)$ is purely real and electrons follow the electric field *E*.

As the applied frequency is increased, the inertia of the electrons introduce a phase-lag in the electrons response to the field, and dynamic conductivity is complex.

At very high frequency $(\omega / \Gamma) \gg 1$ the dynamic conductivity is purely imaginary and electron oscillations are 90 degree out of phase with the applied field *E*.

The static conductivity or dc conductivity of copper is $\sigma_0 = 5.76 \times 10^7 \, \Omega^{-1} \mathrm{m}^{-1}$

Propagation of electromagnetic waves in metal

We get wave equation from the Maxwell's equation as $\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} + \frac{1}{\varepsilon_0 c^2}\frac{\partial J}{\partial t}; \quad \frac{1}{c^2} = \mu_0 \varepsilon_0$

Here $P = 0$ and $J \neq 0$; but $\tilde{J}(\omega) = \left[\frac{\sigma_0}{1 - i(\omega/\Gamma)}\right]\tilde{E}(\omega)$

Use Fourier relations as $\partial_x \equiv -ik \qquad \partial_t \equiv -i\omega$ to get transformed wave equation as

$$-k^2\tilde{E} = \frac{1}{c^2}(-\omega^2\tilde{E}) + \frac{1}{c^2\varepsilon_0}(-i\omega\tilde{J}) = -\frac{\omega^2}{c^2}\tilde{E} - \mu_0\frac{i\omega\sigma_0}{1 - i(\omega/\Gamma)}\tilde{E}$$

The wave dispersion equation is thus: $k^2 = \frac{\omega^2}{c^2} + i\left[\frac{\sigma_0\omega\mu_0}{1 - i(\omega/\Gamma)}\right]$

The wave solution is $E = E_0 e^{i(k\bullet r - \omega t)}$

For small frequency $\omega/\Gamma \ll 1$, we have $k^2 \approx i\sigma\omega\mu_0$

Skin depth

$$k^2 = i\sigma_0\omega\mu_0 = e^{i(\pi/2)}\sigma_0\omega\mu_0$$

$$k = \sqrt{e^{i(\pi/2)}\sigma_0\omega\mu_0} = e^{i(\pi/4)}\sqrt{\sigma_0\omega\mu_0} = \left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]\sqrt{\sigma_0\omega\mu_0}$$

$$k = (1+i)\sqrt{\frac{\sigma_0\omega\mu_0}{2}} \qquad \mathrm{Re}\,k = k' = \mathrm{Im}\,k = k'' = \sqrt{\frac{\sigma_0\omega\mu_0}{2}}$$

$$\mathrm{Re}\,n = n' = \left(\frac{c}{\omega}\right)k' = \sqrt{\frac{\sigma_0 c^2\mu_0}{2\omega}} = \sqrt{\frac{\sigma_0}{2\omega\varepsilon_0}} = n'' = \mathrm{Im}\,n$$

In metals wave travelling in +z direction is

$$E = E_0 e^{i(kz-\omega t)} = E_0 e^{i(k'z+ik''z-\omega t)} = E_0 e^{-k''z} e^{i(k'z-\omega t)}$$

$$= E_0 \exp(-k''z)\exp\left[i(k'z-\omega t)\right] = E_0 \exp\left(-\frac{z}{\delta}\right)e^{i(k'z-\omega t)}$$

$$\delta \triangleq \frac{1}{k''} = \frac{1}{\mathrm{Im}\,k} = \sqrt{\frac{2}{\sigma_0\omega\mu_0}} = \sqrt{\frac{2\varepsilon_0 c^2}{\sigma_0\omega}}$$

Refractive index plasma frequency of metal

Again consider the dispersion expression got for metal as

$$k^2 = \frac{\omega^2}{c^2} + i\left[\frac{\sigma_0 \omega \mu_0}{1 - i(\omega/\Gamma)}\right]$$

$$n^2 = \frac{c^2 k^2}{\omega^2} = 1 + i\left\{\frac{\sigma_0 c^2 \mu_0}{\omega\left[1 - (i\omega/\Gamma)\right]}\right\}$$

$$= 1 + i\frac{i\Gamma}{i\Gamma}\left\{\frac{\sigma_0 c^2 \mu_0}{\omega\left[1 - (i\omega/\Gamma)\right]}\right\}$$

$$n^2 = 1 - \frac{\Gamma \sigma_0 c^2 \mu_0}{\omega^2 + i\omega\Gamma} \qquad c^2 = \frac{1}{\mu_0 \varepsilon_0} \qquad \sigma_0 = \left(\frac{Nq^2}{m\Gamma}\right)$$

$$\omega_p^2 = \Gamma \sigma_0 c^2 \mu_0 = \Gamma\left(\frac{Nq^2}{m\Gamma}\right)\left(\frac{1}{\varepsilon_0 \mu_0}\right)\mu_0 = \frac{Nq^2}{m\varepsilon_0}$$

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}$$

$$\varepsilon(\omega) = n^2 = (n' + in'')^2 = (n' - n'') + i2n'n'' = \left(1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2}\right) + i\frac{\omega_p^2 \Gamma}{\omega(\omega^2 + \Gamma^2)}$$

$$\text{Metals} \quad \Gamma \approx 0 \qquad \varepsilon' = 1 - \frac{\omega_p^2}{\omega^2 + \Gamma^2} \approx 1 - \frac{\omega_p^2}{\omega^2} \qquad \varepsilon'' = \frac{\omega_p^2 \Gamma}{\omega(\omega^2 + \Gamma^2)} \approx \frac{\omega_p^2 \Gamma}{\omega^3}$$

So we obtained plasma frequency in terms of obtained static conductivity and is same as we got from discussions of plasma oscillations

Dispersion relation for em wave for bulk plasmon

The pure em waves in a media (whatever be, metal or dielectric) polarizes it and thus does not remain pure-rather becomes polariton; or plasmon (plasmon polariton). The dielectric function of the media plays important role in determining dispersion relation. The wave equation (with out current density) is

$$\nabla^2 E = \frac{\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}E \qquad \text{for air} \qquad \varepsilon = 1 \qquad \text{for metal} \qquad \varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

Fourier transforming the above we get $k^2 = \frac{\varepsilon}{c^2}\omega^2$ for air we get $\omega = ck$ is light-line

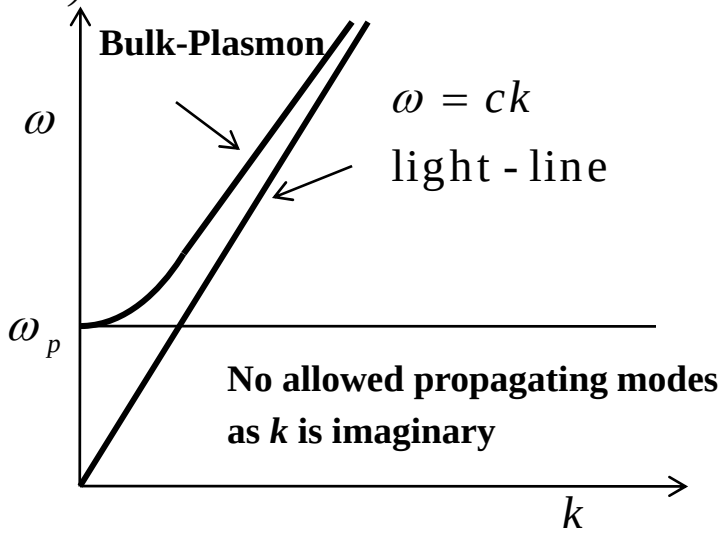
For polarized material we have bulk plasmon dispersion as: $\omega^2\left(1 - \frac{\omega_p^2}{\omega^2}\right) = c^2k^2 \qquad \omega = \sqrt{\omega_p^2 + c^2k^2}$

The Bulk plasmon line lie above light line.

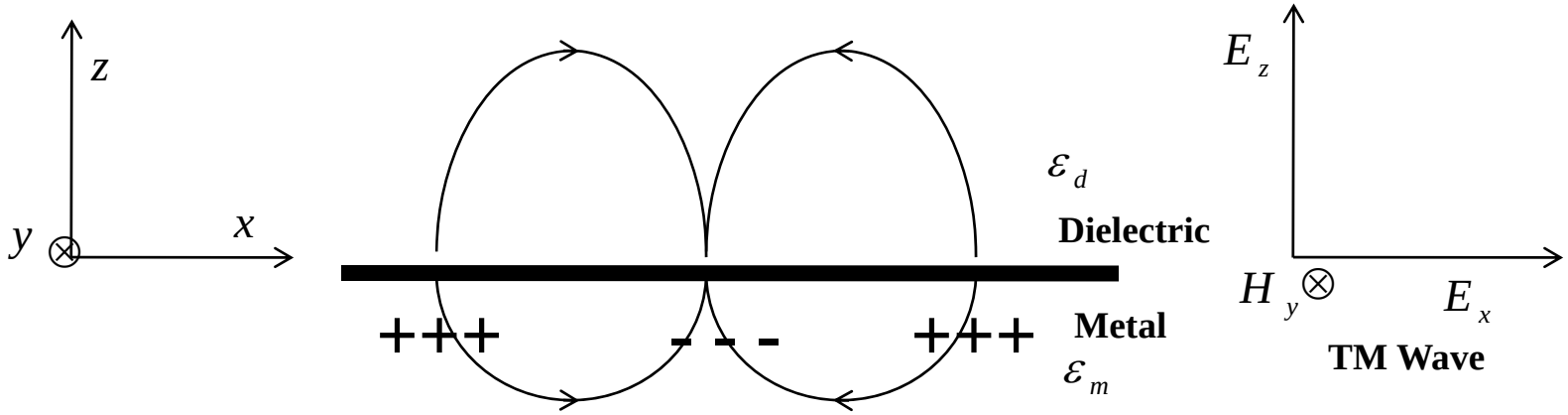
For $\omega > \omega_p$, k is real and bulk plasmon propagates

For $\omega < \omega_p$, k is imaginary and bulk plasmon evanescent waves (no propagating wave)

$$E(r,t) = \mathrm{Re}\left\{E_0 e^{i(k.r-\omega t)}\right\}$$



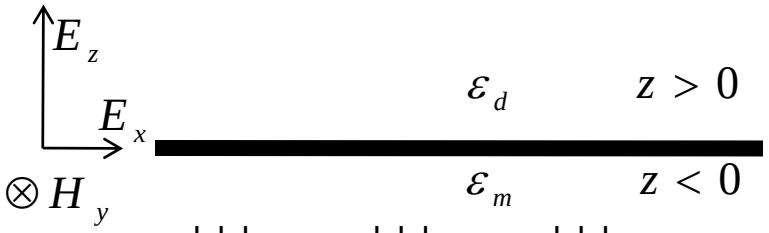
Dispersion relation of surface plasmon polariton for metal-dielectric boundary



Refer the lecture notes on LHM especially lecture-3 , for SPP as following link (LHM lectures 1-8)

http://pdfcast.org/pdf/surface-plasmon-polaritons-spp-for-left-handed-maxwell-systems-lecture-3-by-shantanu-das

Dispersion relation of surface plasmon polariton for metal-dielectric boundary



Mathematically the fields are

$$z > 0;\begin{cases} H_d = (0, H_{yd}, 0)\exp[i(k_{xd}x + k_{zd}z - \omega t)] \\ E_d = (E_{xd}, 0, E_{zd})\exp[i(k_{xd}x + k_{zd}z - \omega t)] \end{cases}$$

$$z < 0;\begin{cases} H_m = (0, H_{ym}, 0)\exp[i(k_{xm}x + k_{zm}z - \omega t)] \\ E_m = (E_{xm}, 0, E_{zm})\exp[i(k_{xm}x + k_{zm}z - \omega t)] \end{cases}$$

Maxwell equation in medium i are ($i = d$, dielectric, $i = m$, metal)

$$\nabla \bullet \varepsilon_i E = 0 \qquad \nabla \bullet H = 0$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \qquad \nabla \times H = \varepsilon_i \frac{\partial E}{\partial t}$$

At boundary we have continuity of tangential components of E_x; H_y and normal component of D_z

$$E_{xm} = E_{xd}; \quad H_{ym} = H_{yd}; \quad \varepsilon_m E_{zm} = \varepsilon_d E_{zd}$$

Dispersion relation of surface plasmon polariton for metal-dielectric boundary

Start with curl of H

$$\nabla \times H_i = \varepsilon_i \frac{\partial E_i}{\partial t}; \quad H_i = (0, H_{yi}, 0) e^{i(k_{xi}x + k_{zi}z - \omega t)}; \quad E_i = (E_{xi}, 0, E_{zi}) e^{i(k_{xi}x + k_{zi}z - \omega t)}$$

$$\left(\frac{\partial H_{zi}}{\partial y} - \frac{\partial H_{yi}}{\partial z}, \frac{\partial H_{xi}}{\partial z} - \frac{\partial H_{zi}}{\partial x}, \frac{\partial H_{yi}}{\partial x} - \frac{\partial H_{xi}}{\partial y} \right) = \left(-i\omega\varepsilon_i E_{xi}, 0, -i\omega\varepsilon_i E_{zi} \right)$$

$$\left(-ik_{zi} H_{yi}, 0, ik_{xi} H_{yi} \right) = \left(-i\omega\varepsilon_i E_{xi}, 0, -i\omega\varepsilon_i E_{zi} \right)$$

From $k_{zi} H_{yi} = \omega \varepsilon_i E_{xi}$ we get $\begin{cases} k_{zm} H_{ym} = \omega \varepsilon_m E_{xm} \\ k_{zd} H_{yd} = \omega \varepsilon_d E_{xd} \end{cases}$

$E_{\parallel}$ i.e. tangential across the boundary is continuous gives $E_{xm} = E_{xd}$ yields

$$\frac{k_{zm}}{\varepsilon_m} H_{ym} = \frac{k_{zd}}{\varepsilon_d} H_{yd}$$

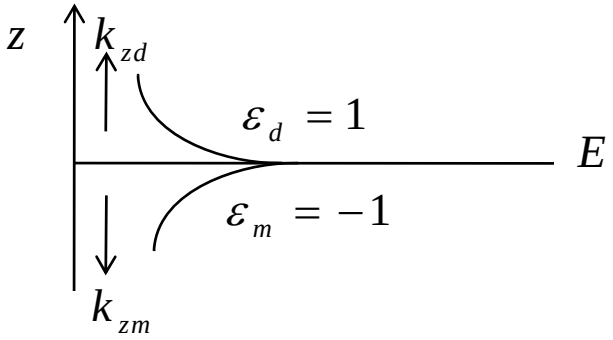
$H_{\parallel}$ i.e. tangential across the boundary is continuous gives $H_{ym} = H_{yd}$ yields

$$\frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d}$$

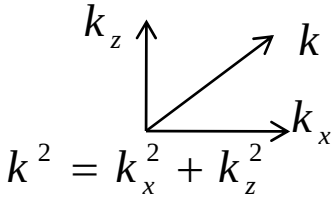
Relation between *k* vectors

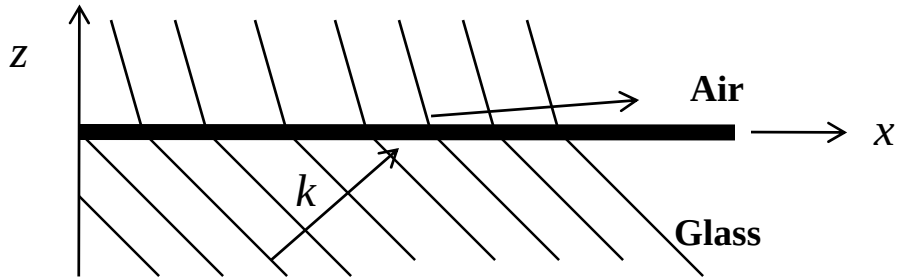
Condition for SPP to exist

$$\frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d} \qquad \text{For} \quad k_{zd} = -k_{zm} = k_z \qquad \varepsilon_m = -1 = -\varepsilon_d$$



Relation for k_x (continuity of $E_{\parallel}$; $H_{\parallel}$) gives: $k_{xm} = k_{xd}$ is true at any boundary





For any em wave $k^2 = \varepsilon_i \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_{zi}^2 \qquad k_x \equiv k_{xm} = k_{xd}$

Both in metal and dielectric we have $k_{SPP} = k_x = \sqrt{\varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_{zi}^2} \qquad \frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d}$

Dispersion relation of surface plasmon polariton

Both in metal and dielectric we have, for SPP condition

$$k_{SPP} = k_x = \sqrt{\varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_{zi}^2} \qquad \frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d} \qquad k_x = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}}$$

x:Direction

$$k_x = k_x' + ik_x'' = \frac{\omega}{c}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2} \qquad \varepsilon_m = \varepsilon_m' + i\varepsilon_m''$$

z:Direction

$$k_{zi}^2 = \varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_x^2 \qquad k_{zi} = k_{zi}' + ik_{zi}'' = \pm \frac{\omega}{c}\left(\frac{\varepsilon_i^2}{\varepsilon_m + \varepsilon_d}\right)^{1/2}$$

For bound SPP mode in *z*-direction, the k_{zi} must be imaginary, possible with $\varepsilon_m + \varepsilon_d < 0$

$$k_{zi} = \pm\sqrt{\varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_x^2} = \pm i\sqrt{k_x^2 - \varepsilon_i \left(\frac{\omega}{c}\right)^2}$$

gives condition as $\left|k_x\right| > \sqrt{\varepsilon_i}\left(\frac{\omega}{c}\right)$

\+ for z < 0, - for z >0

k_x-must be real that is $\varepsilon_m' < 0 \quad \varepsilon_m' < -\varepsilon_d$

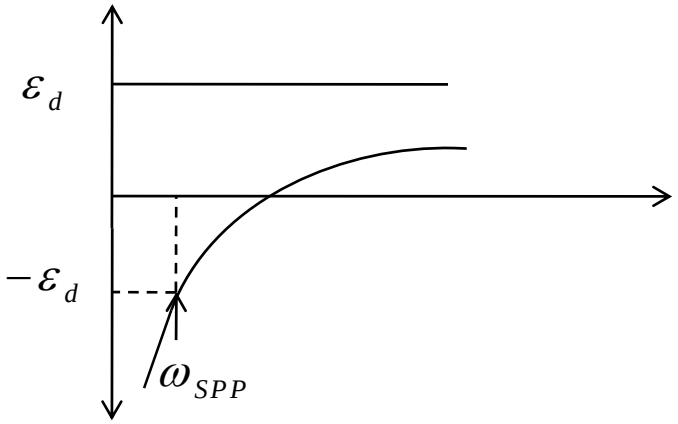
Plot of SPP dispersion

Plot of dielectric constants

$$\varepsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\varepsilon_m = -\varepsilon_d \qquad -\varepsilon_d = 1 - \frac{\omega_p^2}{\omega_{SPP}^2}$$

$$\omega_{SPP} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}$$



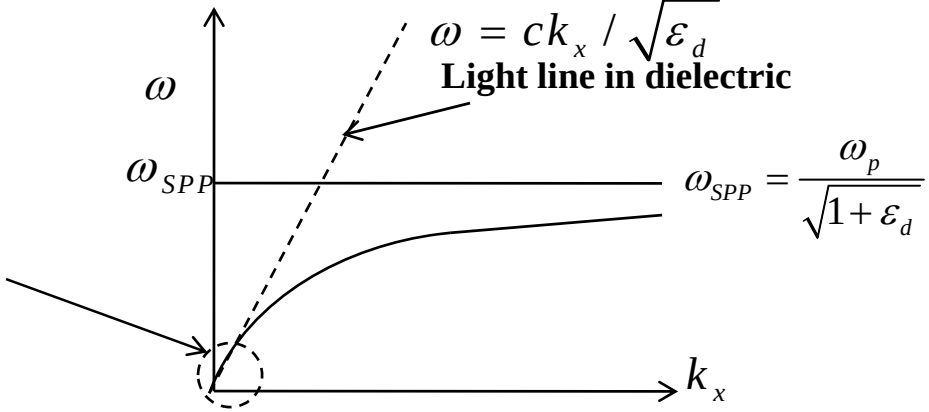
SPP Dispersion

$$k_x = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}}$$ put metals dielectric function $$k_x = k_{SPP} = \frac{\omega}{c}\sqrt{\frac{(\omega^2 - \omega_p^2)\varepsilon_d}{(1+\varepsilon_d)\omega^2 - \omega_p^2}}$$

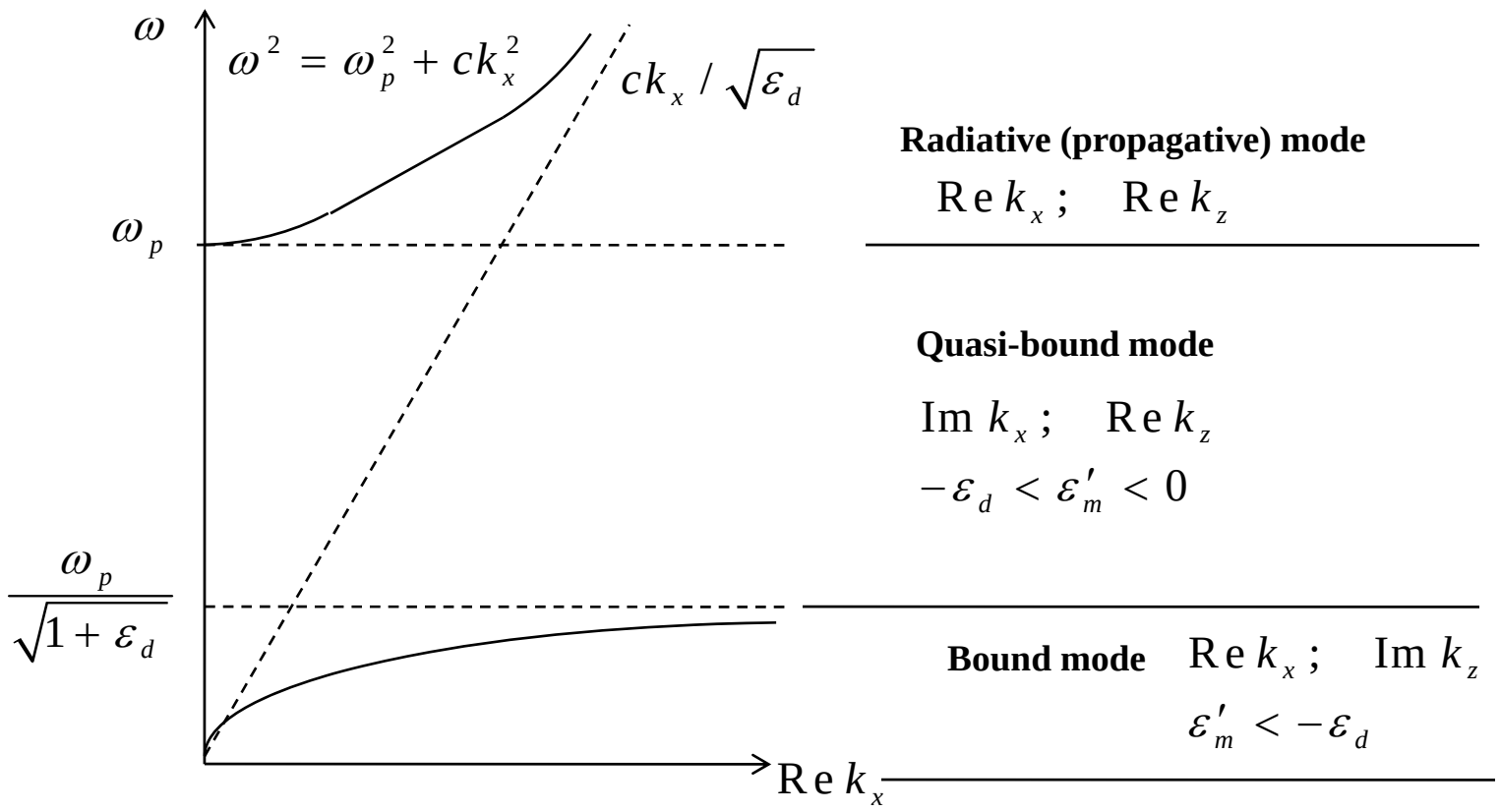
When $\varepsilon_m \to -\varepsilon_d$; $k_x \to \infty$; $\omega \equiv \omega_{SPP} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}$

For low ω

$$k_x = \frac{\omega}{c}\lim_{\varepsilon_m \to -\infty}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2} = \frac{\omega}{c}\sqrt{\varepsilon_d}$$



Modes of polariton (surface/bulk, bounded ,propagating, radiative)



Electric susceptibility χ_e in presence of static magnetic field-Faraday effect from Drude

In presence of static magnetic induction *B* and with dynamic electric field *E* the charged particle has

$m(\ddot{x} + \Gamma\dot{x} + \omega_0^2 x) = q(E + \dot{x} \times B)$ Fourier transforming this we get

$$m(-\omega^2 - i\omega\Gamma + \omega_0^2)\tilde{x} = q(\tilde{E} - i\omega\tilde{x} \times B)$$

Assume $B = B\hat{e}_z$ and also assume 'circularly polarized light' with $\tilde{E} = \tilde{E}_\pm \hat{e}_\pm$

where $\hat{e}_\pm = (\hat{e}_x + i\hat{e}_y)/\sqrt{2}$

Try $\tilde{x} = \tilde{x}_\pm \hat{e}_\pm$ also note that $\hat{e}_\pm \times \hat{e}_z = \mp i\hat{e}_\pm$; therefore we get

$$m(-\omega^2 - i\omega\Gamma + \omega_0^2)\tilde{x}_\pm = q(\tilde{E}_\pm \mp \omega B\tilde{x}_\pm)$$

$$\tilde{x}_\pm = \frac{q\tilde{E}_\pm}{m(\omega_0^2 - i\omega\Gamma - \omega^2) \pm q\omega B}$$

Recall polarization $\tilde{P} = Nq\tilde{x} = \varepsilon_0 \chi_e \tilde{E}$. Thus effect of 'quasi' static induction is

$$\chi_{e,\pm}(\omega) = \frac{Nq^2}{\varepsilon_0 m} \frac{1}{\omega_0^2 - i\omega\Gamma - \omega^2 \pm (q/m)\omega B}$$

Left & right handed polarized light sees different electric susceptibilities in magnetic field

Comments

This optical plasmonics is rapidly growing research field, which controls the manipulation of light waves at the sub wave length scale using metallic particles. At optical frequencies the free electron in noble metals can sustain volume and surface charge density oscillations as we stated called plasmons depending on the geometry of the metal structures, and their surroundings, plasmons can take different forms ranging from surface plasmon polariton SPP propagating along the metal-dielectric boundaries to localized plasmons with electrons oscillating inside metal nano particle.

Plasmonics allows optical signals to be squeezed into deep sub wavelength scale.

This application can thus be unimaginable –in meta-material parlance and optical devices and circuits!

End of part-2