

# **Left Handed Maxwell Systems In Optical Regime**

**PART-1**

**Dielectric Material in Optical Region its Electrodynamics**

**Shantanu Das**

**RR&PS**

**Reactor Control Division, B.A.R.C. Mumbai-400085**

**[shantanu@barc.gov.in](mailto:shantanu@barc.gov.in)**

## **Few salient points**

“Some men see things as they are and say ‘why’? I dream things that never were and say, why not?”-G.B.Shaw

These series of lectures is continuation of LHM series what was done for meta-material at microwave frequencies (Lecture series on LHM 1-8)

A meta-material is an artificially structured material which attains its properties from the unit structure rather than the constituent material.

A meta-material has an in-homogeneity scale that is much smaller than the wavelength of interest and its e.m. response is expressed in terms of homogenized material parameters.

Units are called ‘meta-atoms/molecules’. Even for optical regime the size of meta-atom is larger than actual atoms/molecules .

The unit cells what we constructed for microwave frequencies cannot be directly scaled to optical regime

We need to study the electrodynamics of dielectric and metals and its composites at optical frequencies

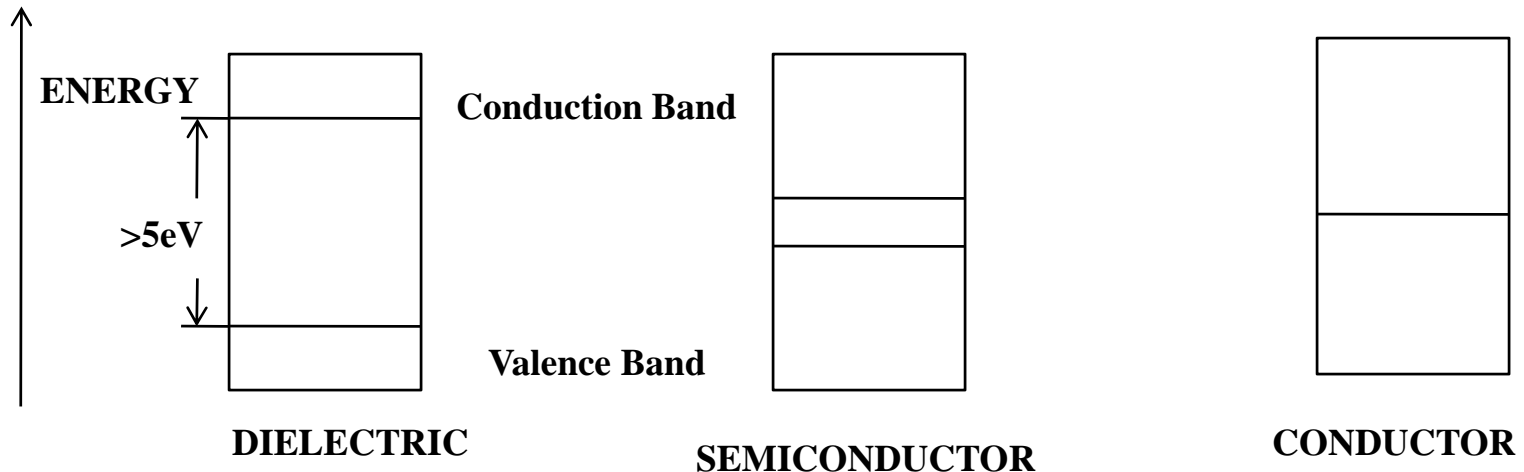
## Electronic structure of material dielectric, semiconductor, insulator

Dielectric also called insulator, and allowed energy bands are either completely empty or completely full. Consequently no electrons can move in an electric field, and thus flow of electric current is prohibited.

Conductors contain free electrons (charges) that will flow when electric field is applied.

Semiconductors that have all their energy bands entirely filled except for one or two bands, that are slightly filled or slightly empty.

Band structure distinguishes these material crystals-via band gap  $E_g$



## **Insulator conductor and semiconductor overlapping definitions!!**

When it comes to Optics one needs be careful when applying the concepts from electronics.

Many dielectrics indeed have optical properties that are closely correlated with their electronic structure. For example the photon energy of visible light which ranges between 1.5 eV and 3 eV is not sufficient to bridge the gap in common insulators. Thus those materials are typically transparent in the visible region.

As an alternate scheme it would be quite plausible to classify material based on their electronic properties. Doing so can cause further confusion. Say Indium Tin Oxide (ITO) a solid that is extremely used in making transparent electrodes in the display and lighting industries, owing to its spectacular feature of combining both electrical conductivity and optical transparency. Electronically thus ITO is meta like conductor, but optically it is dielectric. Another example as tungsten is clearly a metal because it conducts current in filament of bulbs, in the visible light spectrum however tungsten looks more like a lossy dielectric and features positive dielectric constant.

In meta-material topic, the semiconductors are treated as dielectric material, which can be lossless or absorptive. Electronic structure of insulator and semiconductor are different merely in width of the forbidden gap-thus there is not a clear dividing line that separates dielectric and semiconductor.

## Critical wavelength

The critical consideration becomes the comparison of photon energy and the band gap of a crystal. Which specifies the shortest wavelength (the critical wavelength) at which the dielectric remains transparent

$$\lambda_c = \frac{hc}{E_g} = \frac{1240 \text{ nm}}{E_g (\text{eV})}$$

When used as transparent dielectrics, many semiconductors feature a high dielectric constant, which are useful in design of many meta-material based functions

Dielectric material	Band Gap (eV)	Critical Wavelength
Diamond	5.50	0.23 $\mu\text{m}$
Si	1.12	1.10 $\mu\text{m}$
SiC	2.99	0.41 $\mu\text{m}$
GaS	1.42	0.87 $\mu\text{m}$
Alumina	9	0.14 $\mu\text{m}$

# Optical Properties of Dielectric Material

Dielectrics are by far the dominant material used for optical components.

In conventional optical systems almost all functional parts except for some reflection surfaces (mirrors) are made of crystalline glassy material.

The reason is simple the light is effectively manipulated only when it can effectively pass through a medium-meaning that the medium is, at least to some degree dielectric.

The underlying physical background of light interacting with the dielectric can be analyzed by use of Maxwell's equation plus the following two constitutive equations

$$D = \epsilon_0 E + P = \epsilon_0 (1 + \chi_e) E = \epsilon_0 \epsilon_r E$$

$$B = \mu_0 (H + M) = \mu_0 (1 + \chi_m) H = \mu_0 \mu_r H$$

**The constitutive equation for electric polarization & magnetic magnetization are:**

$$\begin{aligned}\tilde{D}(\omega) &= \varepsilon_0 \tilde{E}(\omega) + \tilde{P}(\omega) = \varepsilon_0 \{1 + \chi_e(\omega)\} \tilde{E}(\omega) = \varepsilon_0 \varepsilon_r(\omega) \tilde{E}(\omega) \\ \tilde{B}(\omega) &= \mu_0 [\tilde{H}(\omega) + \tilde{M}(\omega)] = \mu_0 \{1 + \chi_m(\omega)\} \tilde{H}(\omega) = \mu_0 \mu_r(\omega) \tilde{H}(\omega)\end{aligned}$$

The above equations are frequency domain equations, with parameters as functions of frequency  $\omega$

The electric equation specifies the relationship of electric displacement  $D$  and the electric field  $E$  and the polarization density  $P$  with electrical susceptibility  $\chi_e$  in the frequency domain (or Fourier domain)

The second equation is magnetic counterpart

Constants in SI units in vacuum are  $\varepsilon_0 = 8.85 \times 10^{-12} \text{ F / m}$        $\mu_0 = 4\pi \times 10^{-7} \text{ H / m}$

The subscripted  $r$  denotes relative permittivity and relative permeability; in further discussions we shall be dropping the subscript  $r$ , and we will be talking about these relative parameters

The time domain counterparts of the constitutive equation for electric polarization is

$$D(t) = \varepsilon_0 E(t) + \varepsilon_0 \int_{-\infty}^t \chi_e(t - \tau) E(\tau) d\tau$$

The e.m. response not only depends on instantaneous field  $E(t)$ , but values of  $E$  at all past times

## At the optical frequencies

At the optical frequencies, the magnetic susceptibility  $\chi_m$  in any conventional material diminishes and thus the relative permeability  $\mu_r$  is taken as unity (as that of vacuum).

This condition thus simplifies our description of optical material the transparent one, by assigning the refractive index as  $n = \sqrt{\epsilon}$  (we have dropped the subscript  $r$ )

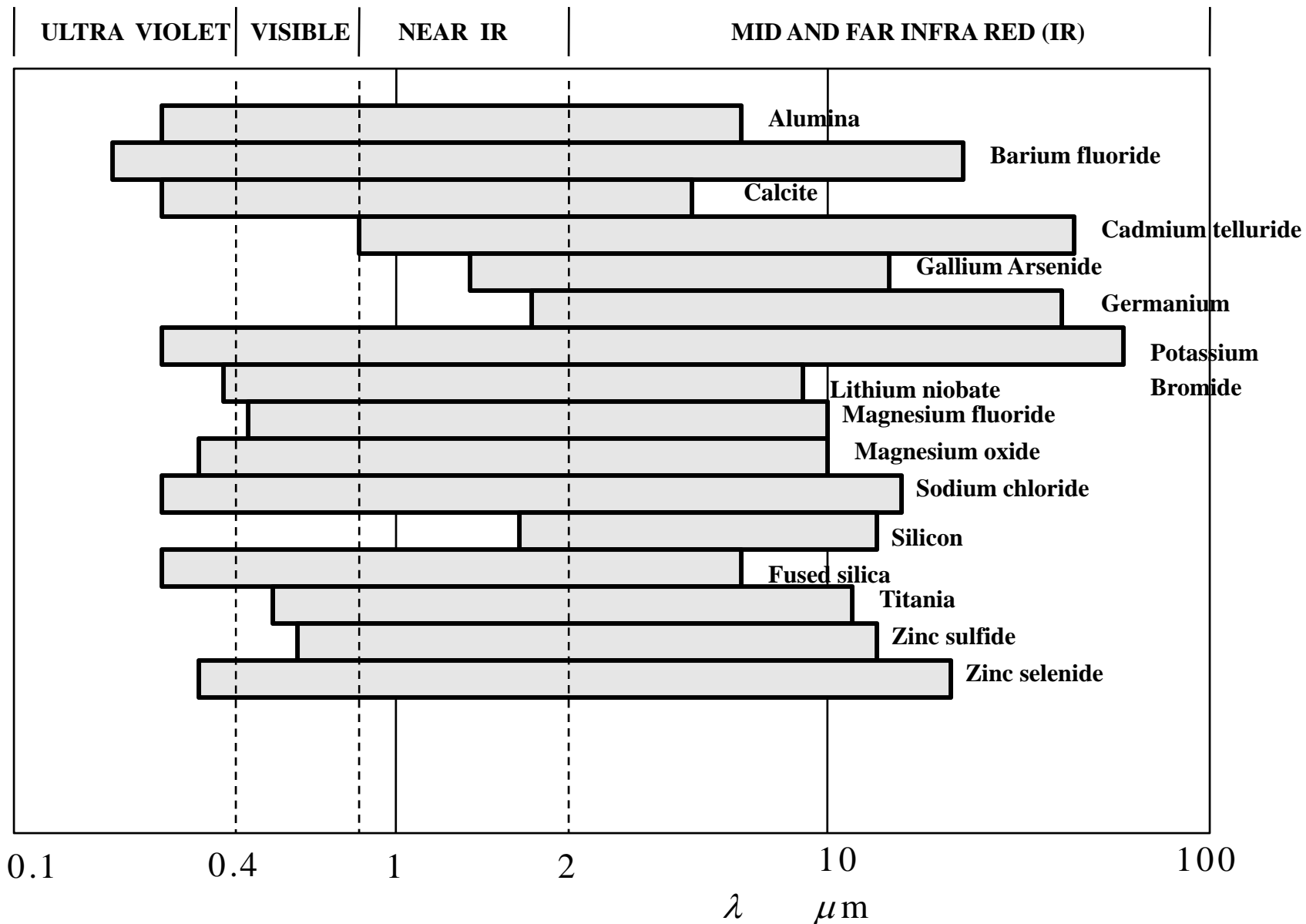
In a metal-dielectric composites (most meta-materials are), in the visible or near IR region, the origin of dispersion  $\epsilon(\omega)$  is mostly in the metallic parts, nevertheless it is important to study the frequency sensitivity of the dielectric function  $\epsilon(\omega)$  of the dielectric media; in particular when we extend our interest to the whole optical spectra from UV (200-400nm) to IR (tens of micron); the dispersion of best transparent material is no longer a negligible feature.

The transparency window of the most dielectric medium is bounded by long wavelength side by the IR absorption mode of Phonons due to lattice vibration, while at the high frequency (low wavelength) side bounded by inter-band electron-hole transition.

Thus while designing optical meta-material it is important to make sure that the selected dielectric constituent is transparent within the wavelength range of interest; else substantial loss may take place from electron or photon resonances, which are detrimental in meta-material performance.



# Spectral Range of transparency for various important dielectric material



## Drude model for dielectric function for dielectrics

The Drude model links electric properties of material with behavior of electrons (or holes)

Consider a typical electron denoted by  $x = x(t)$  the deviation for equilibrium position

External electric field  $E = E(t)$ . The motion of bound electron with mass  $m$ , charge  $q$ , friction or damping as  $m\gamma$  and spring constant as  $m\omega_0^2$  is

$$m (\ddot{x} + \gamma \dot{x} + \omega_0^2 x) = q E$$

Fourier transforming the above we get

$$m (-\omega^2 - i\omega\gamma + \omega_0^2) \tilde{x} = q \tilde{E}$$

Gives position function in frequency as  $\tilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}}{(\omega_0^2 - \omega^2 - i\omega\gamma)}$

Dipole moment of typical electron  $p(\omega) = \tilde{p} = q\tilde{x}$

## Polarization and the dielectric function from Drude model

There are  $N$  typical electrons per unit volume, then polarization  $P$  is

$$\tilde{P} = Nq\tilde{x} = \varepsilon_0\chi_e\tilde{E} \quad \text{also we have} \quad \tilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}}{(\omega_0^2 - \omega^2 - i\omega\gamma)} \quad \text{gives}$$

$$\text{Dielectric susceptibility as} \quad \chi_e(\omega) = \frac{Nq^2}{\varepsilon_0 m} \frac{1}{(\omega_0^2 - \omega^2 - i\omega\gamma)}$$

$$\chi_e(0) = \frac{Nq^2}{\varepsilon_0 m \omega_0^2} > 0$$

$$\chi_e(\omega) = \frac{\chi_e(0)\omega_0^2}{(\omega_0^2 - \omega^2 - i\omega\gamma)}$$

Recall the constitutive equation as

$$D(\omega) = \varepsilon_0 E(\omega) + P(\omega) = \varepsilon_0 \{1 + \chi_e(\omega)\} E(\omega) = \varepsilon_0 \varepsilon(\omega) E(\omega)$$

$$\varepsilon - 1 = \chi_e$$

Therefore the dielectric function is

$$\varepsilon(\omega) = 1 + \frac{\chi_e(0)\omega_0^2}{(\omega_0^2 - \omega^2 - i\omega\gamma)}$$

## Discussions

$$\chi_e(\omega) = \frac{\chi_e(0)\omega_0^2}{(\omega_0^2 - \omega^2 - i\omega\gamma)} = \chi_e'(\omega) + i\chi_e''(\omega)$$

The real part  $\chi_e'$  is called refractive and imaginary part  $\chi_e''$  is called absorptive

Introduce the normalized quantity as  $R(\omega) = \frac{\chi_e(\omega)}{\chi_e(0)}$ ,  $s = \frac{\omega}{\omega_0}$ ,  $\Gamma = \frac{\gamma}{\omega_0}$

$$R(\omega) = \frac{\chi_e(\omega)}{\chi_e(0)} = \frac{\omega_0^2}{(\omega_0^2 - \omega^2 - i\omega\gamma)}$$

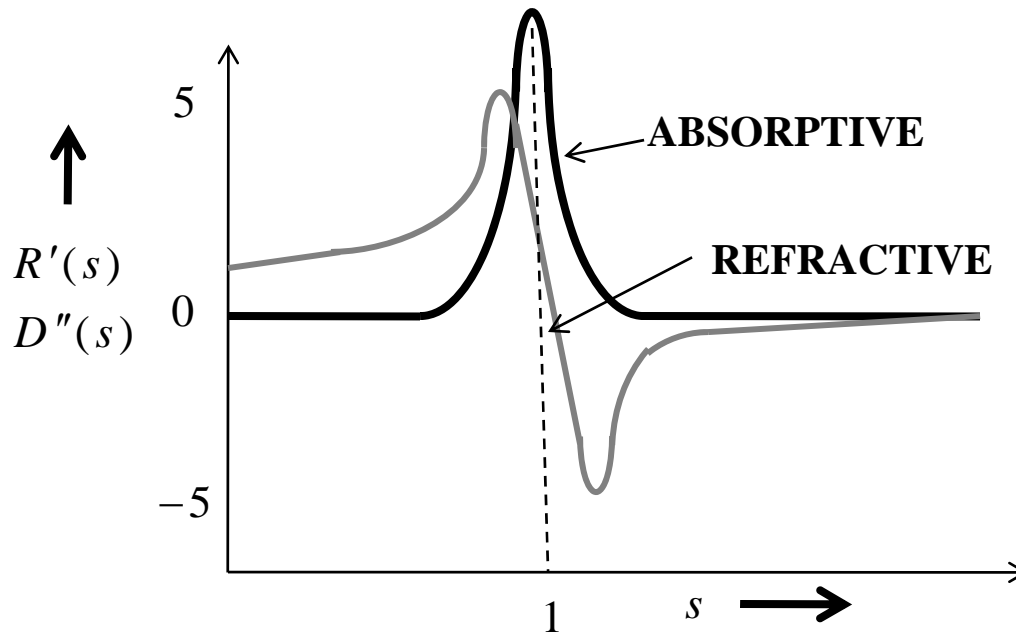
$$R(s) = \frac{1}{1 - s^2 - is\Gamma} = R'(s) + iD''(s)$$

We thus get normalized refractive and normalized absorptive parts as

$$R'(s) = \frac{1 - s^2}{(1 - s^2)^2 + \Gamma^2 s^2} \quad D''(s) = \frac{\Gamma s}{(1 - s^2)^2 + \Gamma^2 s^2}$$

Limiting cases are  $s = 0$   $s = 1$   $s \rightarrow \infty$  for small  $\Gamma$

## The plot of refractive and absorptive parts of electric susceptibility and inference



For small frequency  $\omega \ll \omega_0$  the electric susceptibility  $\chi_e$  is practically 'real'  
This is the realm of 'classical optics.'

The slope  $(\partial \chi_e' / \partial \omega) > 0$  is positive dispersion which is normal

In the vicinity of  $\omega = \omega_0$  absorption is large; negative dispersion  $(\partial \chi_e' / \partial \omega) < 0$  is accompanied by strong absorption.

For a high frequency  $\omega \gg \omega_0$  again absorption is negligible and  $\chi_e'$  is negative with normal positive dispersion  $(\partial \chi_e' / \partial \omega) > 0$  (for X-rays). Also  $\chi_e(\infty) = 0$  by first principles (causality)

## Causality

The  $\chi_e(\omega)$  must be Fourier Transform of causal time response  $\chi_e = \chi_e(\tau)$  as in  $P(t)$  below

We derived the following in our discussion

$$\tilde{D}(\omega) = \varepsilon_0 \tilde{E}(\omega) + \tilde{P}(\omega) = \varepsilon_0 \{1 + \chi_e(\omega)\} \tilde{E}(\omega) = \varepsilon_0 \varepsilon_r(\omega) \tilde{E}(\omega)$$

$$\tilde{P}(\omega) = \varepsilon_0 \chi_e(\omega) \tilde{E}(\omega) \quad \chi_e(\omega) = \frac{a}{\omega_0^2 - \omega^2 - i\omega\gamma} \quad a = \chi(0)\omega_0^2$$

In time domain we thus can write the polarization as convolution integral

$$P(t) = \varepsilon_0 \int_{-\infty}^t \chi_e(\tau) E(t - \tau) d\tau \quad \text{with Fourier - invert} \quad \chi_e(\tau) = \mathfrak{F}^{-1} \{ \chi_e(\omega) \}$$

From definition of Fourier invert we have thus

$$\chi_e(\tau) = a \int \frac{d\omega}{2\pi} \frac{e^{-i\omega\tau}}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

$$\chi_e(\omega) \text{ has poles at } \omega_{1,2} = \pm \varpi - i \frac{\gamma}{2} \quad \varpi = + \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$$

Indeed  $\chi_e(\tau) = 0$  for  $\tau < 0$

$$\chi_e(\tau) = a \frac{\sin \varpi \tau}{\varpi} e^{-\gamma\tau/2} \quad \text{for } \tau > 0$$

## The dielectric function

$$\varepsilon(\omega) = 1 + \frac{\chi_e(0)\omega_0^2}{(\omega_0^2 - \omega^2 - i\omega\gamma)}$$

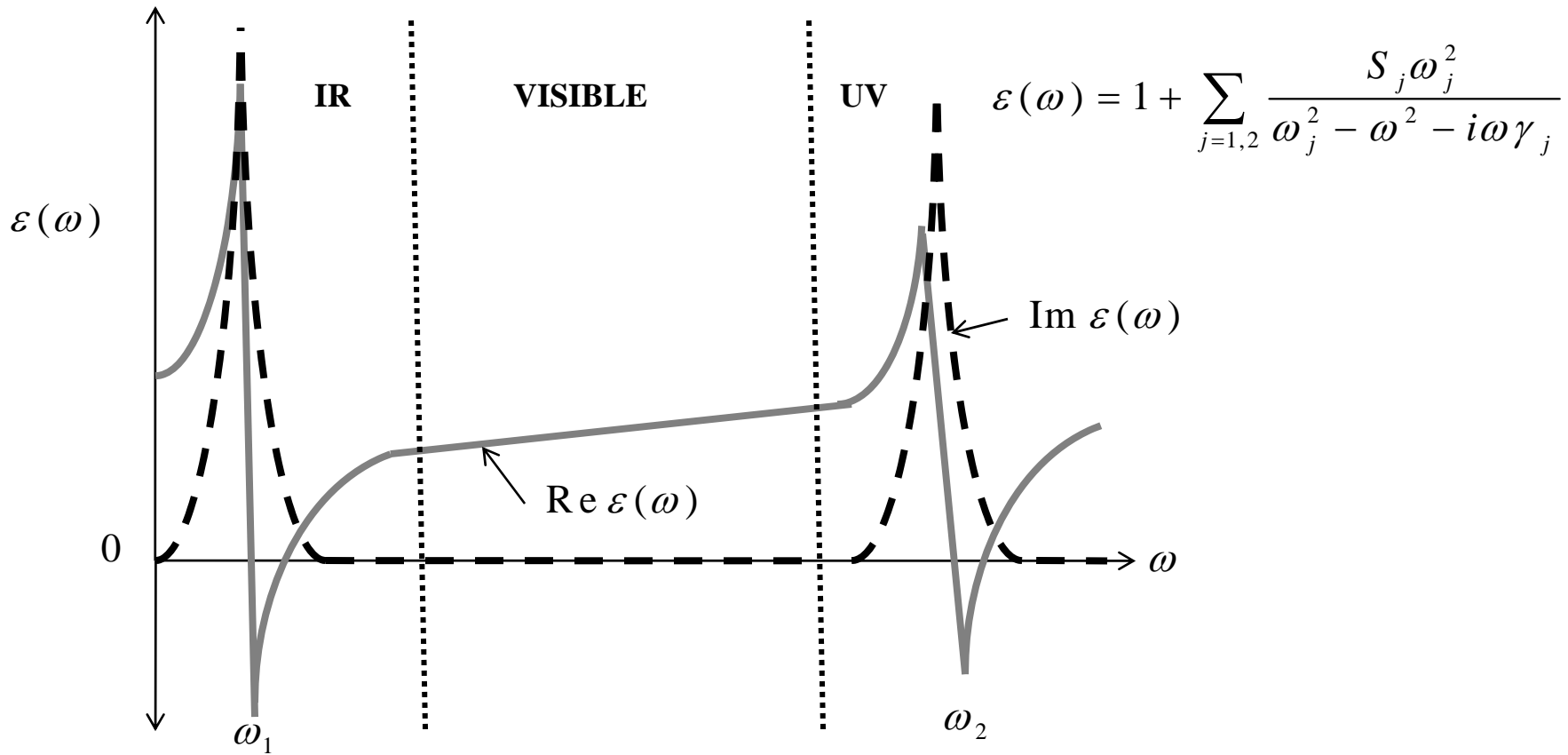
We obtained for one harmonic oscillator. In reality electrons are bound in atom via several harmonic such oscillators with several  $\omega_0$ 's and  $\gamma$ 's ; each representing a different oscillator. Thus we write the dielectric function as

$$\varepsilon(\omega) = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

Where  $\omega_j$  represents the resonance frequency of the  $j$ -th mode and  $S_j$  and  $\gamma_j$  representing strength and damping constant of the  $j$ -th mode.

The transparency of the dielectrics are bound in optical region in the IR side by phonon resonance and at the UV side by electron transition due to crystal band gap. The se two resonance frequencies we call  $\omega_1$  and  $\omega_2$  respectively.

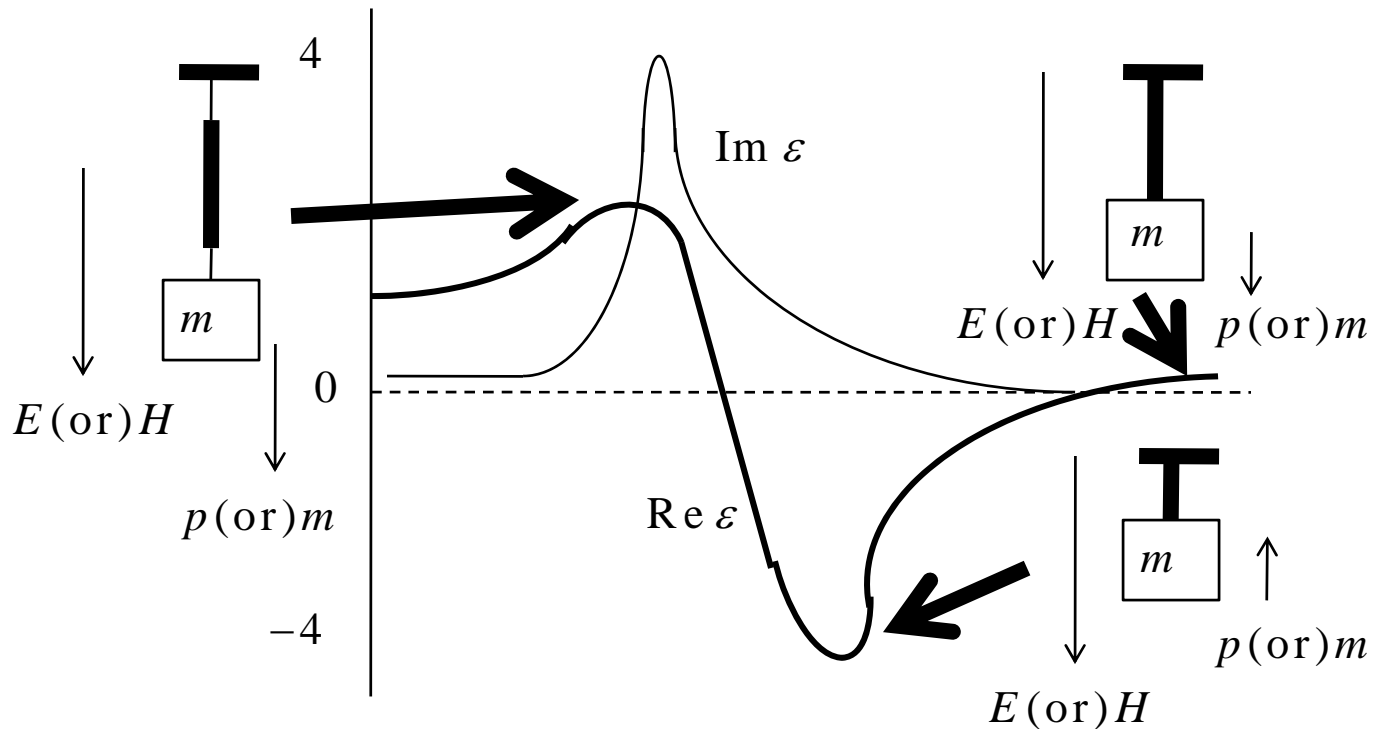
# The dielectric function real and imaginary parts & discussion



Between the two resonance frequencies the permittivity curve is rather flat with almost negligible imaginary part. This explains why common dielectrics like quartz alumina is transparent to the visible light. The real part is large positive at the low-frequency side, of the resonance, and it has a negative value when frequency is slightly higher than that of resonance. A negative real part implies that polarization density  $P$  or the electric displacement  $D$  is directed opposite to the electric field  $E$ .



## The negative value of real part of dielectric function



A negative dielectric constant is somewhat counterintuitive, because transparent materials have positive dielectric constants in the visible range. A negative value of the real part of  $\epsilon$  implies that the response, which is the polarization  $P$  or the electric displacement  $D$ , is directed opposite to the electric field  $E$ . The electromagnetic response is indicated by a mass-spring system. Away from resonance, the response is able to follow the driving force with no delay. When the frequency is slightly lower than the resonance frequency  $\omega_0$ , the function  $\epsilon$ ,  $\mu$  is increasingly positive.  $P$  or  $M$  is enhanced. On the other side of the resonance peak, the function can take a negative value, thus the response is opposite to the driving force.

## Remark about refractive index

The simple square root relationship between refractive index and the permittivity still holds in the frequency domain. Therefore the frequency dependent refractive index  $n(\omega)$  being complex value in general is related to dielectric function as:

$$n(\omega) = n'(\omega) + in''(\omega) = \sqrt{\varepsilon(\omega)}$$

The real part  $n'(\omega)$  is called refractive index and imaginary part  $n''(\omega)$  is absorption index. From above, we obtain some useful relations as:

$$\varepsilon' = n'^2 - n''^2; \quad \varepsilon'' = 2n'n''$$
$$n'^2 = \frac{\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}}{2}; \quad n''^2 = \frac{-\varepsilon' + \sqrt{\varepsilon'^2 + \varepsilon''^2}}{2}$$

The frequency dependence of  $n(\omega)$  is very similar to that of  $\varepsilon(\omega)$ , plot, except that the real part  $n'(\omega)$  is raised or offsetted along vertical axis to avoid negative value in real part  $n'(\omega)$ .

## Dispersion of refractive index for transparent dielectrics Sellmier dispersion formula

Since the imaginary part  $n''(\omega)$  is negligibly small usually less than  $10^{-5}$  for common transparent dielectrics in visible range, the refractive index of such materials can be modeled similar to that obtained from Drude model for dielectric function  $\varepsilon(\omega)$  with out involving imaginary parts or the damping constants

$$\varepsilon(\omega) = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2 - i\omega\gamma_j} = 1 + \sum_j \frac{S_j \omega_j^2}{\omega_j^2 - \omega^2}$$
$$n(\lambda)^2 = 1 + \sum_j \frac{C_j \lambda^2}{\lambda^2 - \lambda_j^2}$$

Some modified forms of the Sellmier formula also exist in which an empirical approximation to the  $n(\lambda)$  instead of  $n(\lambda)^2$  is done . Power series approximations to the Sellmier formula is provided where  $n(\lambda)$  or  $n(\lambda)^2$  is expressed as sum of power of wave-length as  $\sum_j A_j \lambda^j$  the index  $j$  can be both positive or negative.

## Absorption coefficient for intensity in transparent dielectrics

For weakly absorptive media the absorption coefficient  $\alpha$  is routinely used to characterize the attenuation of light propagating in the material. The absorption coefficient is exponential index appearing in Beer's law, which says that the intensity  $I_0$  of light will decrease as light travels  $z$ .

$$I(z) = I_0 e^{-\alpha z}$$

The electric field  $E$  changes along the propagation direction  $z$  follows the following

$$E(z) = E_0 e^{i2\pi(n'+in'')z/\lambda_0} \quad I(z) \propto E(z)^2 \quad E(z)^2 \sim e^{-4\pi n''/\lambda_0} e^{i(4\pi n'/\lambda_0)}$$

It is easy to see that absorption coefficient is  $\alpha = 4\pi n'' / \lambda_0$

$$[\alpha] \equiv \text{cm}^{-1}$$

## Comments about dielectric in optical regime

We have covered some fundamental information on the optical properties of dielectric, that may be useful in the meta-material studies. The properties of bulk crystal are not covered, including the optical anisotropy, nonlinear responses. Briefly we comment on them here. Optical anisotropy and birefringence are intrinsic property are intrinsic property of crystalline dielectrics. In an anisotropic media such as biaxial crystals the optical response to external field is directionally dependent. Moreover the polarization field  $P$  is not necessarily aligned with the electric field so tensors of rank 2 instead of scalar values of must be used for electric susceptibility  $\chi_e$  and permittivity  $\epsilon$ .

In optical meta-material however the length scale of the continuous dielectric portion of the unit cell is so small-being much smaller than the light wavelength-that all the dielectric component tend to have “amorphous” state without preferred direction. Thus in design of optical meta-material a scalar value is usually sufficient for permittivity or refractive index of the dielectric constituent.

As far as the nonlinear effect of the dielectric they are normally neglected as most meta-material are in linear operation. While having nonlinear meta-material, the metallic parts in the dielectric will be important, but the non linearity of dielectric will not play part.

**End of part-1**