Indian Science Congress
Physical Sciences

Advances in Photonics and Meta-materials
3-7 January, 2012 Bhubaneswar

Electromagnetic Momentum and Energy inside Negative Refractive Indexed Material: A new look at concept of photon

Dedicating this new thought to my beloved mother Purabi Das

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“We do not have time to blink at the night sky; the twinkling stars as they are, have travelled several and several years to reach us”- Rabindra Nath Tagore.

In this, the “99th Indian Science Congress” and the 150th anniversary of Gurudev Rabindra Nath Tagore’s “birth anniversary”
and “birth centenary” of Prof. S. Chandrashekhar we take a look at the photon...
...and its momentum...
especially when index of refraction is negative, and a few problems we are still having with it!

It is 103 years since controversy began about the photon’s momentum inside a medium with refractive index other than unity…and still continuing; so let me lead (or mislead) you through yet another ‘controversy’ if we may admit this as a ‘controversy’, regarding this rather conceptual concept….
‘the photon an electromagnetic pulse inside negative refractive indexed material!’

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Key words

Negative Indexed Material (NRM), Meta-Material, Group Refractive Index Phase refractive index, Phase Momentum, Wave momentum, Pseudo-Momentum, Hidden Momentum, Left Handed Cross Product, Group and Phase velocity, Mechanical Momentum, Reactive energy, Polariton, Phonon, nihility.

Abstract

Electromagnetic (EM) radiation in dispersion less free space vacuum is represented by a photon, with corpuscular and wave nature. The discussions, for the past century aimed at the nature of photon (rather polariton) inside a media having dispersion in the refraction property, other than free space. Still the debate is continuing regarding Minkowski, Abraham momentum in dispersive media; and to add to this debate the introduction of Veselago's negative refractive indexed media has opened new thoughts to these classical momentum-energy concepts. We call mechanical momentum, wave-momentum (pseudo-momentum), and try to match our 'thought experiments' with intriguing property of this 'photon' (polariton) or pulse carrying EM energy packet, and more so we try to find its property energy, momentum inside a media a positive refractive media. Well if the media show a negative refractive index behavior, then these queries are profound, and suitable explanations to these classical concepts of corpuscular-wave nature of photon (polariton) inside these media are quest for the scientists dealing with these meta-materials. Here in this deliberation, some of this counterintuitive phenomena of corpuscular-wave nature of photon inside negative indexed material is brought out, with possible 'new definition' of its 'pseudo or wave-momentum', the concept of 'reactive energy' inside negative indexed material, along with possible 'new wave equation'. Also we deliberate and point out several counterintuitive observations regarding propagation of electromagnetic radiation inside the negative refractive indexed material. We will also show that century old Minkowski and Abraham momentums manifests as mechanical momentum and gets transferred to the dispersive media and the transfer of these two type of momentum are same-thus we distinguish these mechanical and 'new-defined possible' wave (pseudo) momentum separately and also try to give Electrical Engineering's Power system's analogy of active and reactive power to the transport mechanism of Energy and Momentum of the EM radiation to Negative Refractive Media.

1. Introduction

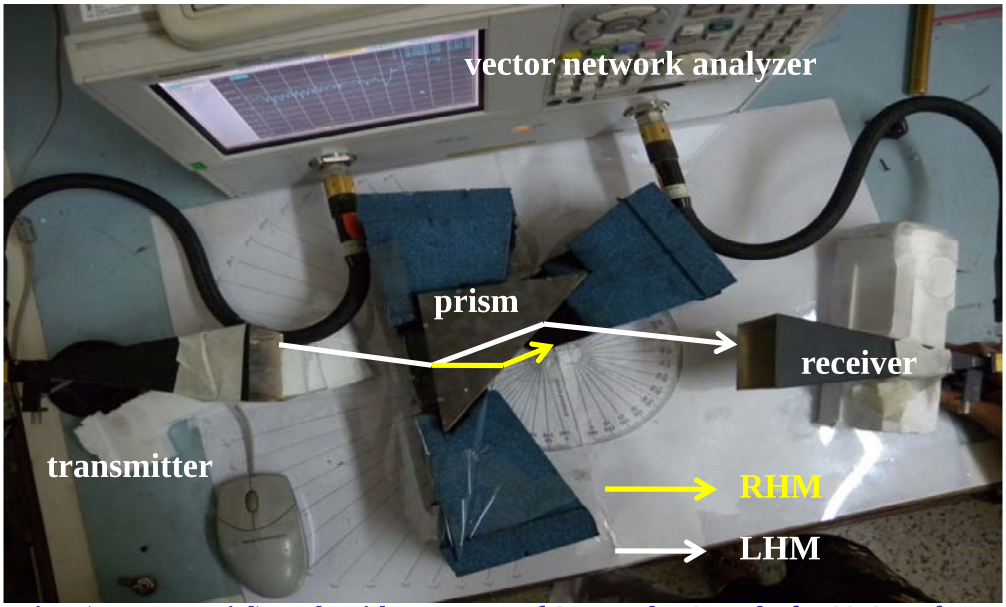
This detailed write-up, I have not kept as 'paper' format. Here I present a detailed note based on published journal papers those are very recent and they are "A new look at the nature of linear momentum and energy inside Negative Refractive Media", Physica Scripta 84 (2011), "A new mechanics of corpuscular wave transport of momentum and energy Inside negative refractive media; Fundamental Journal of Modern Physics (2011)" and "Quantized Energy Momentum & Wave for an Electromagnetic Pulse- A single photon inside Negative Refractive Media"; Journal of Modern Physics (2011). This is a detail write up, dealing with detailed derivations what I could not deal in detail in those papers due to size limitations of size-along with what I am presenting in this session in limited time slot of this Indian Science Congress, the topic namely, "Electromagnetic Momentum and Energy inside Negative Refractive Indexed Material: A new look at concept of photon".

I am doing a project as a team (with SAMEER Kolkata Scientists guided by Professors from University of Calcutta and other institutions) called "Left Handed Maxwell Systems", have demonstrated negative refractive index 'meta-material' plasmonic structures in Ka-band (33 GHz). In our experimental investigation, we have made these plasmonic meta-material prisms of 45, 30 and 15 degrees to get enhanced transmittance of more than 15 dB from background; at negative angles indicating a refractive index of about -1.8. Further experiments at this point of time are progressing with novel structures to demonstrate the same at X-Band about 10 GHz a lower frequency; and higher frequencies structures for about 100 GHz. The experimental set up with meta-material prism is depicted in figure-1, and the difference between Right Handed Material (RHM) normal prism, and Left Handed Material (LHM) for reversal of refraction is compared in the same figure-as produced by ray diagram.

Why I have called the project as "Left Handed Maxwell Systems" is due to counterintuitive nature of the cross product I need to take in the Maxwell equations, to satisfy that wave vector is opposite to the Poynting vector when the dielectric permittivity and magnetic permeability are both negatives; (that is giving me a media of refractive index negative). The figure-2 will elucidate the gist that I make here. The detailed note on this is too included in subsequent sections, with mathematical and possible physical explanations-via thought experiment.

The meta-material is though new to all of us; but was first conceived by Sir Jagadeesh Chandra Basu (Sir J C Bose) in 1898; I salute him and that I depict in figure-3.

This discussion is not aimed here for this experimental design, where the meta-material realized by my team is based on simple wire-array and Labyrinth resonators, but to focus on possible (new) theory of the wave mechanics coupled to particle nature of the EM radiation, energy and momentum transport anomalies, a possible new momentum energy description. The meta material 'theory' is really counterintuitive-several interesting explanations are given in reference section; I suggest that my class room lectures as available in Google Search and listed in reference be made use of by interested readers, namely "Lectures: Parts 1-8 on Left Handed Maxwell Systems" (Google search), these are class room lectures for the "reversed electrodynamics”. Several students in this country as well as abroad have used these lecture notes via Google Search, and did comment on its usability for their research work and orientation to these counter-intuitive phenomena.



Prism (meta-material) made with LR a type of SRR and WA stacks for 33 G Hz frequency, the excitation is TE10 mode of EM from 26GHz to 40GHz, with *E* parallel with the wire structure (that is the negative EPSILON sheet interlaced SRR), *H* perpendicular to SRR

Figure-1 Experimental set up to demonstrate reversal of Snell’s law

Though several approaches to explain these counterintuitive phenomena have been evolving, yet it is interesting if in the meta-material parlance particle-wave theory be founded! Here I give possible classical explanations to these counterintuitive phenomena and also a new explanations regarding energy momentum, wave equation if applied to this negative indexed material: how shall they look, vis-à-vis positive indexed systems. This problem is a topic subject of investigation in modern optics also. Here, I propose the concept of reactive energy and expression for new wave-momentum for pulse of electromagnetic energy inside a medium (negative refractive indexed), with suitable derivation along with possible new wave equation.

The contents list gives the idea of organization and presentation of this write-up. Sometimes, I may take detour to explain some basics-which otherwise we all take as granted as though the concept are very simple, for instance, I have spent considerable effort here to bring out dispersion concept of refractive index via classical electrodynamics principles. Also I tried to give clear simple concept of radiation pressure via kinetic theory of gases. I tried to elaborate via simple thought experiments the meaning of energy momentum of a single radiation pulse a photon. I have tried to give visual picture of a single photon and then its mathematical representation. I also have included simple thought experiments to elucidate the principle of the mechanical, and wave momentum; along with an interpretation of (constant multiplier that is) c^2, as equivalent (and not always equal to) as product of group and phase velocities. Also I have elaborated that this relation is exact in the wave-guides and the wave guide equations I showed they are equivalent to quantum mechanical counterparts. I tried to explain the evolution of 'backward wave' in meta-material resonating elements; and tried to explain why shall I call my project a "Left Handed Maxwell" System; by deriving via simple reasoning that the cross product of wave vector with electric or magnetic field vectors needs be taken via 'left hand'. Here I also elaborate via thought experiment, the physics of 'concept of hidden momentum' arising out of no movement of the resonating element of the meta-material structure, that too in reverse direction to the 'energy flux flow'. In this detailed note, I have tried to derive via simple technique a 'single photon's' quantized momentums; and given explanations to momentum transfers schemes to different momentums (mechanical, and wave momentum). The explanations of real and imaginary energy concepts as appears here, I have tried to give simile to active and reactive power of electrical engineer's; and explained the phenomena as both components are required in electromagnetic as compared to sound waves. It gives picture a clearer one to the concept of corpuscular and wave momentum of a single electromagnetic pulse. I have tried to generalize the classical wave equation and proposed a new one so that wave propagation of negative indexed materials is covered. Before concluding (which I must confess is difficult) I have tried to evolve the basic discussion on why shall a negative root be taken, for product of two negative numbers (negative epsilon and negative mu)!

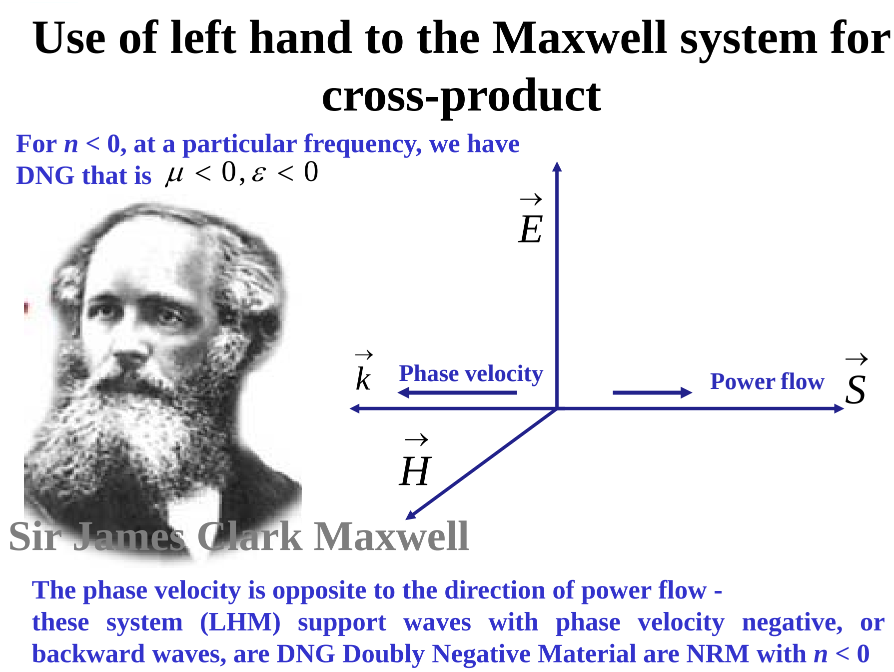


Figure-2: The Left Handed Maxwell Systems

Lastly I must confess I have several light years to travel before I know what exactly a photon is when the media is negative or even positively refracting with dispersion.

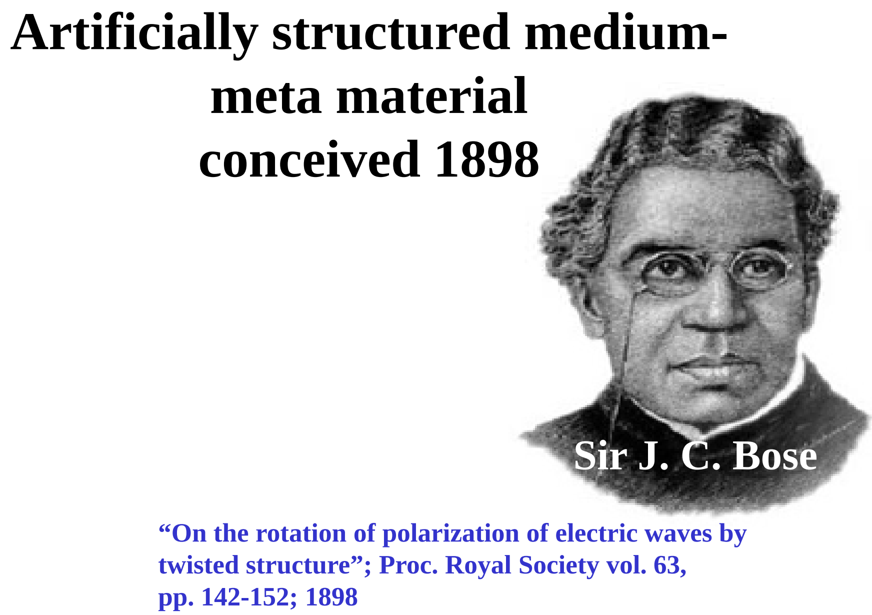


Figure-3 First meta-material conceived by Sir J. C. Bose in 1898

2. The Origin of Refractive Index

Is it so simple that after electromagnetic wave (or light) from a source enters a media with refractive index n the propagation speed from free space, that is c gets reduced to c/n, will explain me dispersion? Let me in this section make a re-visit to electro dynamical explanation of these phenomena. The source electric field (assume in x-direction) with EM wave travelling in z direction at a particular distance z, is oscillating source field as: $E_s = E_0 \cos(\omega t - \omega z/c) = E_0 e^{i\omega(t-z/c)}$. This source field when strikes a medium (placed perpendicular to direction of plane wave travel that is in xy-plane having a thickness of Δz), interacts with electrons of the medium plate. Therefore there are extra forces on the electrons; and these extra forced oscillations of the media due to all the electrons give an extra field, call it E_a. Thus I should observe at the other end (refraction) a vector summation of $E_s + E_a$. If say Δz thick media were absent, the EM waves would travel the distance Δz in time $\Delta z/c$. But if the EM waves appear to travel at speed of c/n; then it should take $n\Delta z/c$; or the additional time is thus (due to presence of media) is amounting to $\Delta t = (n-1)\Delta z/c$. After the travel of extra thickness of media, the waves continue to travel towards receiver with speed of c . Replacing t with $(t-\Delta t)$ or $(t-(n-1)\Delta z/c)$, in the expression for E_s I can get the waves as

$$E_{\text{after-media}} = E_0 e^{i\omega\left(t-(n-1)\Delta z/c - z/c\right)} = e^{-i\omega(n-1)\Delta z/c} E_0 e^{i\omega(t-z/c)} = e^{-i\omega(n-1)\Delta z/c} E_s$$

Origin of refractive index

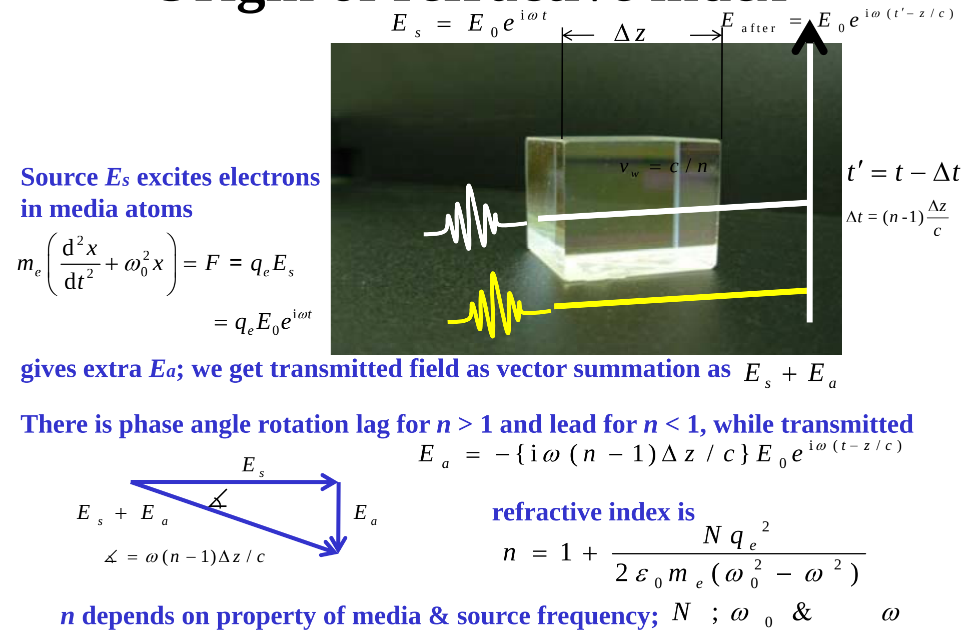


Figure-2 The Origin of refractive index

What the media has done to the source wave? That is after interaction with the media's atomic electrons changes the phase of the source got 'retarded' (due to negative sign) by an amount equaling $\omega(n-1)\Delta z/c$. In terms of complex numbers, I may say if the E_s is a 'real 'quantity, (say zero degree as phase angle) the field due to media has rotated by negative angle $\theta = -\omega(n-1)\Delta z/c$. The above explanation show phase angle rotation due to media and these phenomena I have summarized in figure-2.

But I have earlier stated that extra E_a gets added to E_s, due to forced oscillations of charges? The addition comes from approximating the exponential (rotational term) as $e^{-i\omega(n-1)\Delta z/c} \cong 1 - i\omega(n-1)\Delta z/c$. Putting this approximation I will get

$$E_{\text{after-media}} = E_0 e^{i\omega(t-z/c)} - i[\omega(n-1)\Delta z/c]E_0 e^{i\omega(t-z/c)}$$

Thus I now have E_a, the extra field due to atomic interactions produced by oscillating charges on the slab, being added (as vector), as quadrature to the original source field. The, term $-i$, indicates the extra field E_a is phase lagging field being added as vector to E_s, thereby generating refracted composite field as $E_s + E_a$. So the extra wave is what I should get is

$$E_a = -\frac{i\omega(n-1)\Delta z}{c}E_0 e^{i\omega(t-z/c)}$$

Can I relate E_a, to the motion of charges (electrons) in the media? Let me place the media slab of thickness Δz at coordinate $z = 0$. Therefore, the source field is at this place is $E_s = E_0 e^{i\omega t}$. Each of the electrons (or charges) in the atoms would feel the source electric field and would

climb up and down in x direction (transverse to travel direction of the source plane wave), by a force $F = q_e E = q_e E_0 e^{\mathrm{i}\omega t}$, with frequency ω. I have assumed positive charges.
The forced harmonic motion of the charges (consider positive charges) is thus:

$$m_e\left(\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \omega_0^2 x\right) = F = q_e E_0 e^{\mathrm{i}\omega t}$$

The solution to this is $x = x_0 e^{\mathrm{i}\omega t}$, substituting this in above I obtain the displacement equation as

$$x_0 = \frac{q_e E_0}{m_e(\omega_0^2 - \omega^2)} \qquad\qquad x = \frac{q_e E_0}{m_e(\omega_0^2 - \omega^2)} e^{\mathrm{i}\omega t}$$

This above expression is motion of electrons in the slab plate, and is same for every charge, except that the mean position (the zero point of the motion) is of course different for each charge!

Well if I have a plate at say at $z = 0$, having a 'charge density per unit area' η, on that plate, and the charges are moving up-down (say in x-direction) with velocity in $x - y$ plane, then due to their velocity (retarded at z) I will get electric field as

$$E_a = -\frac{\eta q_e}{2\varepsilon_0 c}\left[\text{velocity of charges}\right]_{\text{at}(t-z/c)} = -\frac{\eta q_e}{2\varepsilon_0 c}\mathrm{i}\omega x_0 e^{\mathrm{i}\omega(t-z/c)}$$

Let me derive the above. Well, the radiating field is proportional to acceleration of charges ($\ddot{x}$) that is $-\omega^2 x_0 e^{\mathrm{i}\omega t}$ (at $z = 0$; where the plate of oscillating charges is kept). The geometry of the system is like following. The plate (symmetrical) is at xy plane the centre of the plate is at the origin. From centre I want to have perpendicular i.e. z at distance where I need the field. Say I put an oscillating charge, away from origin on the plate at radius ρ, at coordinate $(x, y, 0)$. Then from the point of observation $(0,0,z)$ this point is called at distance, r so that I have $\rho^2 + z^2 = r^2$.

Due to this oscillating charge at $(x, y, 0)$ the field (with retarded time) is proportional to $-\omega^2 x_0 e^{\mathrm{i}\omega(t-r/c)}$. From the electric field expression only taking the $(1/r)$ term and neglecting the $(1/r^2)$ terms of Coulomb's law; I write the (far) radiating field as

$$E_Z \cong \frac{q_e}{4\pi\varepsilon_0 c^2}\frac{\omega^2 x_0 e^{\mathrm{i}\omega(t-r/c)}}{r}$$

I have put approximation sign, as the above expression is actual, only if the line or plane of movement of charges is perpendicular to line of sight; only true if the charge were at origin. The approximate is due to the line of sight r is not perpendicular to charge movement at $(x, y, 0)$. Had I taken the projection of the movement of charges perpendicular to r, the equality sign would have appeared in above expression. This is field (radiating field) due to one charge. The other charges too contribute to the field. To that I have to integrate the effect of each charge on the plate.

I thus consider a circle of ring width $\mathrm{d}\rho$ at a radius ρ in the plate. Number of charges in the ring is $(2\pi\rho\mathrm{d}\rho)(\eta)$. Thus the integration of these types of rings from radius zero to infinity will give me the total field as

$$E_{\text{Total}-Z} = \int_0^\infty \frac{q_e}{4\pi\varepsilon_0 c^2} \frac{\omega^2 x_0 e^{\mathrm{i}\omega(t-r/c)}}{r} \eta.2\pi\rho\mathrm{d}\rho$$

In the above integral expression all other terms are independent of ρ and r are put aside, to leave the (naked) integral as I , that is

$$I = \int_0^\infty \frac{e^{-\mathrm{i}\omega r/c}}{r} \rho\mathrm{d}\rho$$

With $r^2 = \rho^2 + z^2$, I have $2r\mathrm{d}r = 2\rho\mathrm{d}\rho$ (z is independent of ρ and r), the integration is thus

$$I = \int_{r=z}^\infty e^{-\mathrm{i}\omega r/c}\mathrm{d}r = -\frac{c}{\mathrm{i}\omega}\left[e^{-\mathrm{i}(\infty)} - e^{-\mathrm{i}\omega z/c}\right]$$

Looking at the integral's value the term $e^{-\mathrm{i}\infty}$ is mysterious to me, the real part of this is; $\cos(-\infty)$, which has no stable value-it can be between $(-1,+1)$; fortunately bounded. I apply two physical senses that is let me include η, the surface charge density of the plate. The modified integral reads as I', that is

$$I' = \int_{r=z}^\infty \eta e^{-\mathrm{i}\omega r/c}\mathrm{d}r$$

To have uniformly extended constant surface charge density at infinity is not practical though. The surface charge density tapering to zero at infinity will make $(\eta_{\text{at}-\infty})(e^{\mathrm{i}(-\infty)})$ zero. Another physical picture which I consider is; as r goes to infinity, the radius ρ of the 'charged' circle ring also is infinity. The charges oscillating at that large radius will not have effective projection perpendicular to the line of sight! This also enables me to make contribution of upper limit of the integral as zero. With these two arguments, I may write the contribution of integral (I) as $I = (c/\mathrm{i}\omega)e^{-\mathrm{i}\omega z/c}$, taking contribution of, $e^{-\mathrm{i}\infty} \cong 0$. Mathematicians won't like this though!

I bring all other terms (inside naked integral) and with this evaluated integral I get

$$E_{\text{Total}-Z} = E_a = -\frac{\eta q_e}{2\varepsilon_0 c}\mathrm{i}\omega x_0 e^{\mathrm{i}\omega(t-z/c)} = -\frac{\eta q_e}{2\varepsilon_0 c}\left[\text{velocity of charges}\right]_{\text{at}(t-z/c)}$$

Putting the value of x_0, or differentiating x obtained in forced oscillation solution, and then putting the same in the above expression of E_a , using retarded time I have

$$E_a = -\frac{\eta q_e}{2\varepsilon_0 c}\left[\mathrm{i}\omega \frac{q_e E_0}{m_e(\omega_0^2 - \omega^2)} e^{\mathrm{i}\omega(t-z/c)}\right]$$

Above is driven motion of charges which produce 'extra wave' which travels to the right of the plate ($+z$). Equating the above E_a, to the earlier obtained E_a via phase lag method, I get the following

$$(n-1)\Delta z = \frac{\eta q_e^2}{2\varepsilon_0 m_e(\omega_0^2 - \omega^2)}$$

Assuming the N as volume charge density (number of charges per unit volume) and Δz being the thickness of the medium plate, I have therefore, the surface charge density as $\eta = N\Delta z$. When I substituted this in above expression, the Δz gets eliminated from both sides and I get index of refraction as function of frequency, $n(\omega)$ as:

$$n = 1 + \frac{N{q_e}^2}{2\varepsilon_0 m_e(\omega_0^2 - \omega^2)}$$

The above description of refractive index origination and the dispersion expression is very simple approach. I have neglected that any source field striking the media gets reflected and backward waves travel to the source. Though reflection appears from surface, the backward waves do generate within the media. This complication I have neglected. This transmission case too I have simplified, as to striking field in media makes forced oscillation for a single charge-and simply I have multiplied the effect by charge number density! I have not considered effect of all other charges to our test charge and its effect on other charges! A very complicated one, though I can say near about $n \cong 1$, that is media having very less charge density (gases, air, even Bose-Einstein Condensate BEC etc) this approximate explanation is valid.

In my explanation of mechanical oscillator of mass m_e of charge q_e, bound by restoring force $m_e\omega_0^2 x$, I have not considered damping, the 'frictional' loss factor is γ. If the oscillator is damped oscillator I shall replace $\omega_0^2 - \omega^2$ by $\omega_0^2 - \omega^2 + \mathrm{i}\gamma\omega$. In above I have considered only one type of oscillator. But a system of charges in atom can have several types of oscillators with various ω_0's and γ's. Assume N_k electrons per unit volume where ω_0 is ω_k with damping γ_k, gives me expression for refractive index as

$$n = 1 + \frac{{q_e}^2}{2\varepsilon_0 m_e}\sum_k \frac{N_k}{\omega_k^2 - \omega^2 + \mathrm{i}\gamma_k\omega} = n' - \mathrm{i}n'' = \mathrm{Re}\, n + \mathrm{i}\,\mathrm{Im}\, n$$

I represent this expression in figure-3. Due to damping I get 'complex' refractive index.

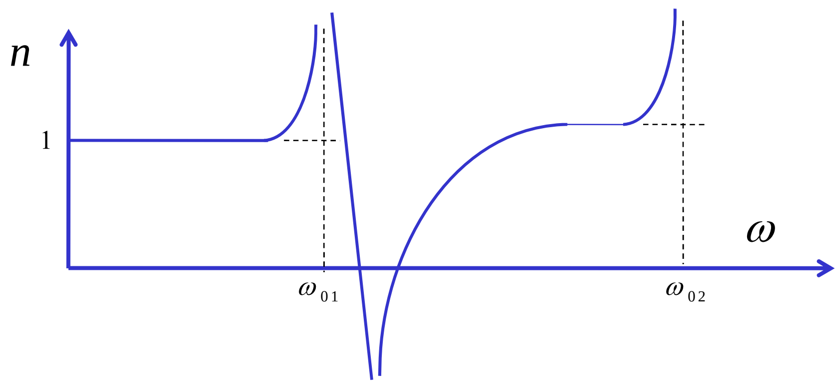
3. Dispersion in refractive index

When EM radiation moves from vacuum into an isotropic medium it is generally refracted. Its frequency $f = 2\pi\omega$ remains same, but the wavelength, that is λ is reduced by the refractive index $n > 1$. The corresponding phase velocity $f\lambda$ is thus reduced to $1/n$. Here I set $c = 1$. But I must point out here that this is an effective description valid on large scales where discrete atoms in the media are replaced by continuous media! On atomic scale (what I described in previous section) the EM radiation is moving with vacuum velocity c even between interactions with charges and atoms. These will scatter the EM radiation in such a way that in forward direction the scattered waves add up to the incoming plane wave; however the scattered waves are delayed (phase shifted) by $\pi/2$ relative to the source wave. The interference between these two waves will then effectively slow down the propagating wave. This is normal dispersion.

The relation what I obtained for refractive index in previous section, by this method of 'wave-mechanics' gives me the idea of dispersion else I could never understand the dispersion in refraction, by the concept of slowing down of travel of EM radiation in the media with index of n . In the expression of n obtained the quantities N and ω_0 pose problem, as they are different for different composites and materials. The figure-3 shows the same. The figure-3 and the formula of refractive index as obtained suggest to me that there can be case near and higher to ω_0 that we may get negative refractive index! The phenomena in this region is anomalous (the region in the figure with negative slope, and the region where refractive index increasing with frequency just near and lesser than ω_0 , is normal dispersion.

Dispersion of *n*

$$n = 1 + \frac{q_e^2}{2\varepsilon_0 m_e} \sum_k \frac{N_k}{\omega_k^2 - \omega^2 + \mathrm{i}\gamma_k \omega}$$



The ω_{0k} s' are the natural rotational frequency of bound charges for the unbound charges as in case of free electrons of 'plasma' $\omega_{0k} = 0$

The dispersion expression is 'phase refractive index'

Figure-3 Dispersion of refractive index

For gases air etc ω_0 , that is natural frequency of charge harmonic oscillator, is close to Ultra Violet (UV) frequency. So if the incident EM wave of source E_s has a frequency as $\omega \ll \omega_0$, I can disregard the ω in the formula of n . The media thus show a constant index of refraction for wide range of frequencies (up to UV).

In this case when frequency of source ω of E_s is close to visible region, then I cannot neglect ω in the formula of n . In this zone as I keep on increasing the source frequency ω , the n also increases. This I call normal dispersion-the, n for blue light is more than n for red one-in normal dispersion. This normal dispersion all we know.

When $\omega = \omega_0$ (say measuring n for a glass slab near UV); then n , becomes infinitely high. The interesting case while $\omega > \omega_0$, (the case when glass is illuminated by X-rays) the formula of n suggest that the index becomes less than one. Here the speed is more than c , in cases of $n<1$. (Do not worry 'phases' the crest and trough can be having speed more than c, also inside a wave-guide the phase velocity of phases of crest and troughs are greater than c).

I can set $\omega_0 = 0$ then the formula is $n = 1 - (Nq_e^{\ 2} / 2\varepsilon_0 m_e \omega^2)$, is the dispersion relation of index of refraction. Well I can set the resonating frequency of harmonic oscillator for charges as zero, for free charges-as these charges are not bound the case in plasma. Utilizing the expression for plasma frequency as; $\omega_{ep}^2 = Nq_e^2 / \varepsilon_0 m_e$, (to be specific 'electric plasma frequency'), I therefore, have dispersion relation as $n = 1 - (\omega_{ep}^2 / 2\omega^2) = 1 - (\omega_R^2 / \omega^2)$. Well this can give me $n = -1$, when $\omega = \omega_R / \sqrt{2} = \omega_{ep} / 2$. At $\omega = \omega_R$ I get $n = 0$ a case for 'nihility'.

This is dispersion, of refractive index near this Surface Plasmon Polariton resonance where $\varepsilon_r = -1$; at $\omega = \omega_{ep} / \sqrt{2}$ in system of free charges (with designed electric and magnetic plasma frequency) is key to make 'meta-materials' exhibiting negative refraction. Well negative refraction is a 'resonance' phenomenon-as the EM interaction with charges of media gives an index of refraction which is dispersive and anomalous dispersion. In meta-material design I approximate the dielectric dispersion as $\varepsilon_r = 1 - (\omega_{ep}^2 / \omega^2)$, and $\mu_r = 1 - (\omega_{mp}^2 / \omega^2)$. Both values as negative unity, gives me a media with refractive index minus one! The nihility in these approximate cases for permittivity and permeability appears at plasma frequencies and not half the plasma frequencies as I got in the formula of refractive index! I carry on with this discrepancy of approximation.

The above explanation what I gave for several cases, suggests also the phase shift to the source field by the system generated field can give me a positive or negative phase shift. The negative angle (lagging case) I have depicted in figure-2. When phase shift is leading it means the displacement of charges in the equation of harmonic oscillator, that is x is opposite and reverse of the source field E_s. Therefore, I do have in this case (especially for $n<1$), whence the response of the charges motion opposite to the source field, that is sign has gotten reversed. When electric field is pulling in one direction the charge (in my explanation I have taken positive charge q_e magnitude equaling electronic charge), is moving in other direction. So the phase of the transmitted (refracted) field, that is $E_s + E_a$ can appear to be ahead with respect to the source wave! It is this advance phase which implies phases are travelling more than speed of EM radiation in vacuum that is $v_p > c$.

4. How the negative mu and negative epsilon are are realized, and makes "Left Handed Maxwell System"

As I have pointed out that our refractive index needs to be anomalous for negative indexed systems of meta-material, and most importantly that the interaction of EM waves to the system media should be with 'bound-free' charges. I have metals where I have free electrons-so metals can give me negative index of refraction! At UV frequency the metal's free electrons behave as ‘plasma’. As I stated that free electrons does not have ω_0, so I can write the EM interaction with inclusion of damped motion of free electrons as

$$m_e\ddot{x} + m_e\gamma\dot{x} = -eE_0 e^{\mathrm{i}\omega_c t}$$
$$m_e x(-\omega^2 + \mathrm{i}\omega\gamma) = -eE_0 e^{\mathrm{i}\omega_c t}$$
$$x = \frac{-eE_0 e^{\mathrm{i}\omega_c t}}{m_e(-\omega^2 + \mathrm{i}\omega\gamma)}$$

I have used $\mathrm{d}/\mathrm{d}t$ as $\mathrm{i}\omega$ in above to get displacement’s solution. The incident field has frequency ω_c, and γ the damping coefficient and e the free electronic charge. Here I assume that wave length of the EM shining the metals is substantially larger than path length of electron, so that effectively the electron sees a spatially constant field, and velocities are low so that I can forgo magnetic fields and its effect. This gives me ‘polarization’ per unit volume and I write that as

$$P = \varepsilon\varepsilon_0 E_0 e^{\mathrm{i}\omega_c t} - \varepsilon_0 E_0 e^{\mathrm{i}\omega_c t} = (\varepsilon - 1)\varepsilon_0 E_0 e^{\mathrm{i}\omega_c t} = -Nex$$

Where N is the number density of free electrons and all of them contributing to polarization. On this polarization expression substituting the displacement expression for x, with algebraic manipulations and also I use the expression for plasma frequency as ${\omega_{ep}}^2 = (Ne^2)/m_e\varepsilon_0$; I obtain ‘dispersion’ expression for the ‘effective dielectric’ constant of the metals as

$$\varepsilon = 1 - \frac{\omega_{ep}^2}{\omega^2 - \mathrm{i}\gamma\omega}$$

So I have a negative dielectric permittivity below plasma ω_{ep} frequency for metals. This plasma frequency in case of metal is in UV ranges. So naturally metals behave as epsilon negative materials (ENG) below plasma frequency. I can bring down this plasma frequency to G Hz range of X, Ka band by making the number density very low-by using thin metal wires embedded sparsely in a dielectric (or in air). The ENG behavior I owe it to ‘electric polarization’. Well with ENG I can realize negative index as $n^2 = -\varepsilon_r$ for $\omega_c < \omega_{ep}$; and waves will be of ‘evanescent’ bounded waves in ENG (here the wave vector is imaginary in direction of propagation). I can also comment in the ENG region the response of the material up to plasma frequency is ‘out-of-phase’ to the driving field. I can approximate this epsilon’s dispersion as $\varepsilon_r = 1 - ({\omega_{ep}}^2/\omega^2)$.

I will briefly now state the artificial realizations of the ‘negative’ magnetic permeability. These are resonating elements. The classical structure to realize the negative mu is split ring resonator SRR. The SRR are concentric rings split at one at 0^0 and the inner ring separated by gap d, split at 180^0 relative to the first one. The SRR works on principle of the magnetic field of EM radiation, which drives a resonant LC element (circuit) through inductance, and resulting in dispersive magnetic permeability as

$$\mu = 1 - \frac{\pi r^2/a^2}{1 - (3d/\mu_0\varepsilon_0\varepsilon\pi^2\omega^2 r^3) + \mathrm{i}(2\rho/\mu_0\omega r)}$$
$$= 1 - \frac{F\omega^2}{\omega^2 - {\omega_R}^2 - \mathrm{i}\Gamma\omega}$$

With circuit resonance frequency ω_R , filling factor is $F = \pi r^2 / a^2$, the 'magnetic plasma' $\omega_{mp}{}^2 = (3d)/\left[\mu_0 \varepsilon_0 \varepsilon \pi^2 r^3\right]$ frequency is $\omega_R{}^2 = (3d)/\left[(1-F)\mu_0 \varepsilon_0 \varepsilon \pi^2 r^3\right]$. Say I design the SRR with $d = 0.2\text{mm}$, radius of outer ring as $r = 1.5\text{mm}$, lattice spacing for SRR repeated structure as $a = 5\text{mm}$, the copper resistivity $\rho \cong 0$, with ε the dielectric permittivity of gap (capacitance) I get magnetic plasma frequency as 7.56GHz and the SRR circuit resonance frequency as 6.41GHz. The dispersion in mu I can approximate as, $\mu_r = 1 - F\omega^2 / (\omega^2 - \omega_R^2)$, with magnetic plasma frequency as $\omega_{mp} = \omega_R / \sqrt{1-F}$; at this mu is zero and below this the value is negative.

The 'artificially' structured magnetic activity I obtain a negative value from the circuit resonance frequency ω_R to 'magnetic' plasma frequency ω_{mp}. This negative realization is 'resonance' where very high resonating EM fields are obtained. Therefore response of SRR, near the resonance ω_R, and up to ω_{mp} I expect 'out of phase' response to the driving EM field! This way I can get negative mu material (MNG).

This ENG and MNG together if I tune I get a region where both mu and epsilon are negatives; and that I get a region as DNG (Double Negative), a region where the refractive index is negative, and media DNG supports propagation of EM waves.

Does it help me to make 'Left Handed Maxwell' system having realized artificially a media with ENG $\varepsilon < 0$ and MNG $\mu < 0$? Let me take a plane wave travelling in $+z$ direction, with electric field E_R pointing towards $+x$ direction, the magnetic field H_R pointing in $+y$ direction, a travelling wave($E_R = E_0 e^{(\mathrm{i}k_R z - \mathrm{i}\omega t)}$) in normal dispersive media, with $\varepsilon_R > 0$ and $\mu_R > 0$ the media properties. A wave travelling in RHM right handed media! Well the Poynting flux with right hand cross product is (the power flow direction), that is $S_R = E_R \times H_R$, in $+z$ direction. The Maxwell's equations in RHM are

$$\nabla \times E_R = -\mathrm{i}\omega\mu_R H_R$$
$$\nabla \times H_R = +\mathrm{i}\omega\varepsilon_R E_R$$

In this RHM the wave vector k_R is in direction of propagation that is in $+z$ direction, and let me re write the RHM Maxwell's equations in terms of wave vector as (comes from above curls)

$$k_R \times E_R = +\omega\mu_R H_R$$
$$k_R \times H_R = -\omega\varepsilon_R E_R$$

Above cross product I obtain via right hand.

Now as I pointed out that the media what I made is a resonating structure, especially the realization of the MNG via SRR. This resonance gives me a response of very high electric and magnetic fields at near about resonance. As I pointed out the response at the electric and magnetic resonance in the region of ENG and MNG will give 'phase opposition' to the excitation EM signal; so I should get a strong E_L in $-x$ direction (opposite to E_R) and strong H_L in

$-y$ direction (opposite to H_R). The near resonance response of the DNG media with ENG and MNG is very strong, so resultant response is with E_L and H_L only (the incident is negligible compared to these giant fields, and I obtain 'out of phase' response!). Now if I check with the right hand the cross product $E_L \times H_L$, gives me the power flow S_L in the original direction of $+z$ directed!

Now if I re-write the Maxwell's equation by putting the media properties as negatives, that is by putting the values as, $\mu_L < 0$, and $\varepsilon_L < 0$ I get the curl expressions for NRM DNG media as

$$\nabla \times E_L = +\mathrm{i}\omega\mu_L H_L$$
$$\nabla \times H_L = -\mathrm{i}\omega\varepsilon_L E_L$$

I cannot use k_R to make the equations with MNG and ENG so I write inequality and equality as

$$k_R \times E_L \neq -\omega\mu_L H_L = +\omega\mu_L H_L$$
$$k_R \times H_L \neq +\omega\varepsilon_L E_L = -\omega\varepsilon_L E_L$$

The, vector k_R is in $+z$ direction, so if I use right hand and take cross with E_L my thumb points towards $-y$ or in H_L's direction and not in desired $-H_L$'s way! Similarly I do right handed cross of k_R with H_L, I find my thumb towards $+x$ that is towards $-E_L$ as against desired direction E_L. From here I infer that k_R is not the direction of 'wave-vector'!

Let me reverse the wave vector and point it towards $-z$ direction (opposite to k_R), and I call it k_L and write the equations as desired (off course the following will come from the revised curls)

$$k_L \times E_L = -\omega\mu_L H_L$$
$$k_L \times H_L = +\omega\varepsilon_L E_L$$

I find that k_L with E_L and H_L is following the right handed cross product rule to satisfy above. The fact is $+x$, $+y$ and $+z$ follows right handed cross product rule, but $-x$, $-y$ and $-z$ will follow the 'left handed cross product' rule. But the above cross product equations for NRM DNG media satisfy right handed cross product rule since I have made 'phase inversion' of E and H too. So where is left handed cross product? But what is my reference coordinate, that is E in $+x$, H in $+y$ and let me write the following.

Now if I write a general for DNG or NRM media that k is opposite (that is in $-z$ direction) to $S = E \times H$ ($+z$ direction) with, E in $+x$ and H in $+y$ direction the Maxwell's equations for $\varepsilon < 0$ and $\mu < 0$ are

$$\nabla \times E = +\mathrm{i}\omega\mu H \quad \text{and} \quad k \times E = -\omega\mu H$$
$$\nabla \times H = -\mathrm{i}\omega\varepsilon E \qquad\qquad k \times H = +\omega\varepsilon E$$

The reference remains E as pointing towards $+x$, H pointing towards $+y$, and travelling in $+z$ as for source incident field of RHM. The Poynting flux follows the right handed cross product while the k E H triad follows a cross product with left hand! This is why called this system with ENG and MNG (DNG) an NRM media a 'Left Handed Maxwell's system'.

The NRM media having k opposite to the Poynting vector means that its phase velocity is opposite to a media of RHM say vacuum, the phases travel in opposite direction as respect to energy flow. This media supports backward waves. This left handed cross product is depicted in figure-2, a definition of Left Handed Maxwell's system. I will try and explain the 'backward waves' generation physically in subsequent section.

5. The problem of scaling the Meta-Material Structures to T Hz and beyond and its differences with photonic structures

Well, I have in short described in previous sections, how we are making artificial structures and probably will go up to 350 THz of electromagnetic artificial structures (requirements of nano-scaled structures). Although most materials exhibit good electric response, can be found at almost all the frequencies from RF to UV; magnetic response of most materials is limited to low microwave (GHz) level. Magnetic polarization usually results from either unpaired electron spin or orbital electron currents, and collective excitations of these usually tend to occur at low microwave frequencies. Some materials exhibit some magnetic activity at even 100GHz, but are rare and BW is too narrow. But now possibility of artificially structuring materials at micro and nano scales enable us to generate a variety of meta-materials with magnetic activity at almost up to IR including RF the microwave frequency and THz frequency in between. Thus even magnetic activity let alone negative permeability are special at High Frequency.

Maxwell's equations appear to suggest that I can scale the phenomena by simple scaling of length-scales. However, main problem to scale to IR and optical frequency is that metals no longer behave as Perfect Electrical Conductor PEC, and EM fields penetrate considerably into metals. This means that dissipative nature of the metals must be taken into account for scaling to HF. Also technological ability on nm scales is to be overcome. I can thus, see that limitation to scale up to IR comes due to inertia of electrons. The limit of magnetic resonance is about 350 T Hz.

A normal *L-R* circuit when excited by a step voltage has following relations

$$V(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + Ri(t) \text{ and } i(t) = \frac{V_0}{R}[1-\exp(-t/\tau)], \text{ with } \tau = L/R \qquad (1)$$

The current seems to be delayed due to 'time constant'. I may ask a question, is anything else in electrical circuit that cause delay? Clearly in order to have current, the charge carriers must be accelerated and it takes time to accelerate a particle of mass m_e. Therefore current will necessarily lag behind voltage causing it to rise. Consider a conductor of length l cross section area $dS = \pi r^2$. If V volts are applied across the conductor the current is $i = JS = Ne\mathrm{v}S$. Where J is current density, N is number of charges with e as electronic charge in coulombs, and v the velocity of charges. This charge e with electronic mass m_e movement, in electric field E I can write as force balance relation for this charged particle in electric field E as in (2)

$$eE = m_e\left(\frac{\mathrm{dv}}{\mathrm{d}t} + \frac{\mathrm{v}}{\tau_{\mathrm{coll}}}\right) \qquad V = \frac{lm_e}{Ne^2S}\left(\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{i}{\tau_{\mathrm{coll}}}\right) \qquad (2)$$

Comparing (2) with the *L-R* circuit (1) equation I get kinetic or inertial inductance and kinetic resistance as follows:

$$L_i = \frac{lm_e}{Ne^2 S}, \qquad R_i = \frac{lm_e}{Ne^2 S \tau_{\text{coll}}} \tag{3}$$

Looking at expression of kinetic/inertial resistance, I find, it is nothing but 'ordinary resistance'. On the other hand the expression for kinetic inductance is new. As the area of conductor becomes smaller and smaller, the kinetic inductance becomes comparable to magnetic inductance. It can even become dominant inductance as in example in circuit containing nano-rods. Since $\omega_{ep}^2 = Ne^2 / \varepsilon_0 m_e$, that is 'electric plasma frequency'; then I can write and have, kinetic inductance as $L_i = l / (\pi r^2 \varepsilon_0 \omega_{ep}^2)$.

The presence of this additional inductance can be explained by noting that at high frequencies the currents are hardly diffusssive, and almost ballistic; because the distance through which electrons moves with in a period of wave becomes comparable to the mean free path in metals. This means that if the frequencies are too high, the electrons can hardly be accelerated and the response falls.

The mass of electron contribute additionally to the inductance. Current density is $J = Ne\text{v} \cong Ne(-\text{i}\omega eE / m_e)$ then the potential drop is $V \cong \{m_e l / Ne^2\}(\partial J / \partial t)$, implying an inductance that is proportional to electron mass. The effective damping factor $\Gamma = L_i \gamma / (L_i + L_g)$ also increases becomes much larger as the size of the ring is reduced (where $L_g = \mu_0 \pi r^2$ is geometrical inductance).

This is due to the fact that the proportional energy in ballistic motion of the electron increases as size gets reduced and resistive losses are then very large indeed. Thus even if the size of the ring were negligible the inertial/kinetic inductances would still be present preventing scaling to higher frequencies. Well, this effect of inertial inductance is also there even if super conducting Split Ring Resonators (SRR) is employed. The large increase in damping as the sizes are scaled down broadens the resonance and permeability does not rapidly disperse; and the regions of MNG (Mu-Negative) vanish altogether. This increase in damping is matter of great concern for optical frequencies. SRR with these two splits tends to tail off at wave length of 5 micro—meters, (IR region). By adding more capacitive gaps to lower the net capacitance and adjusting the dielectric constant of substrate MNG with this scaled down is achievable at 1.5 micro-meters. Parallel metal sticks, say pair of wire of 100nm length periodically embedded in dielectric behaves as MNG at IR. The figure-4 will give indication of scaling up to T Hz of frequency, the structure for having ENG and MNG together.

Well whatever is today's limitation to scale up the frequency, let me believe that I can have a material which has a refractive index as negative value. So what is the nature of classical photon if it were to enter the NRM material?

Here I briefly distinguish meta-materials from photonic crystals as follows. Meta-materials, in some sense, can be strictly be distinguished from other structured photonic material (Photonic Crystal or Photonic Band Gaps). In photonic crystal or the band-gap materials the stop-band arise as a result of multiple Bragg's scattering in a periodic array of dielectric scatterers. In fact the periodicity of the structure here is of the order of wavelengths, and hence homogenization in the sense cannot be carried out. In meta-materials the periodicity is by comparison far less important and all the properties mainly depend on single scatter resonances. So I shall discuss as though I have got a homogeneous meta-material exhibiting negative index of refraction. What shall be then photon (polariton) momentum inside this NRM?

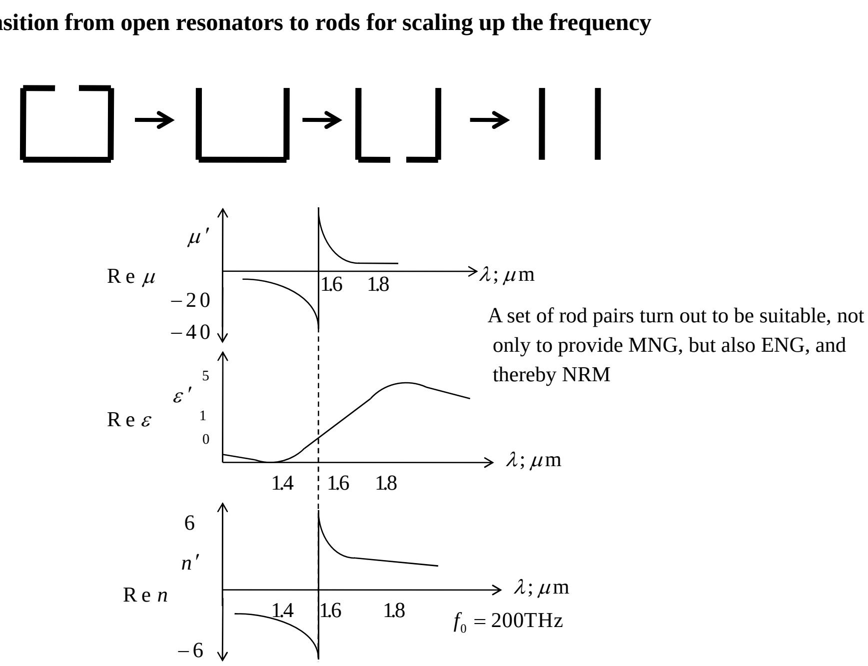


Figure-4: Scaling up to T Hz the NRM region one possibility with nano-wires

6. The Controversy regarding Photon-Polariton momentum in a media

EM radiation is refracted from the material having refractive index $n \neq 1$. Classically it is frequency f remains fixed while the wavelength of radiation λ of free space is changed to λ / n. The corresponding phase velocity $v_p = f\lambda$ is lowered to c / n. But I should not forget that this is effective description valid on large scales. On micro scales the EM radiation is still moving with phase velocity as c, between the interactions with electrons and atoms. These interactions will scatter the EM radiation in such a way that in forward direction the scattered 'waves' add up as plane waves. However, it is delayed by a phase shift of $\pi / 2$, relative to incoming waves. The interference between these waves will then effectively slow down the propagating wave. As a result of this microscopic process the resulting wave is highly complex object. This I have dealt in detail earlier.

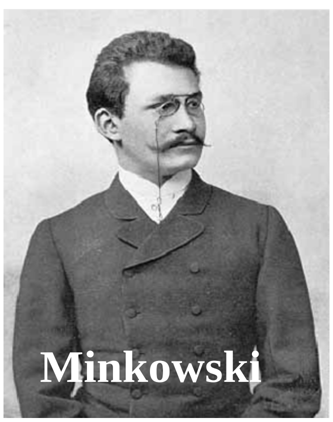
The dielectric constant and magnetic permeability characterizes 'macroscopic' response of a homogeneous medium, to the applied E-M fields. These are macroscopic parameters because one usually seeks time-averaged and spatially-averaged responses averaged over sufficiently long times and sufficiently large volumes. All that survives the averaging in macroscopic measurement are the frequency components of the individual (atomic or molecular) oscillators driven by external field.

This idea I can now extend to a higher class of in-homogeneous materials where in-homogeneity are on the length-scales much smaller than the wavelength of EM radiation, but can be large compared with atomic or molecular length scales. The radiation then does not resolve these individual meso-structures, but responds to the (atomically) macroscopic resonances of the structure. These are meta-materials, and can be characterized by macroscopic parameters permittivity and permeability, that define their responses to the EM field much like homogeneous medium.

So, which is it?

$$p_M = \hbar k = \frac{nh}{\lambda} = \frac{nhf}{c} \quad \textbf{or} \quad p_A = m\mathrm{v} = \frac{\mathrm{E}}{c^2}\frac{c}{n} = \frac{hf}{nc} \quad \textbf{?}$$

Minkowski 1908 **or** **Abraham 1909 ?**



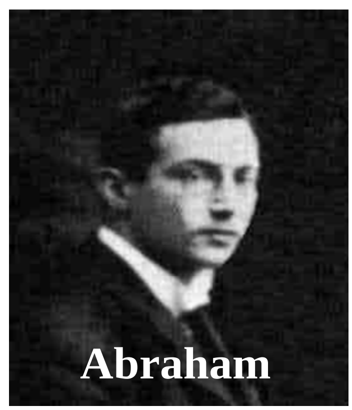


Figure-5 Controversy about momentums

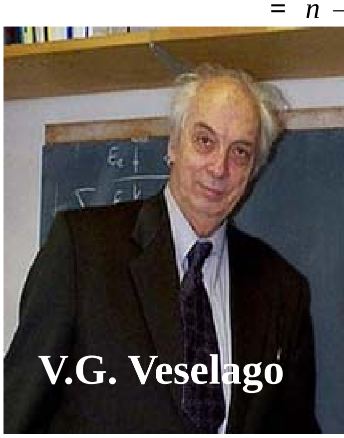
Well the question is about nature of photon while it enters the material having refractive index n. The controversy about this is over more than a century (starting from Minkowski's definition of photon's momentum in 1908). Moreover the controversy stems from the 'dual' nature of the energy packet carried by photon. I have naïve approach number-one, and I say the free space wave length of photon is $\lambda_0 = c/f$, with c as speed of electro-magnetic (EM) radiation in free space, and f is the frequency of oscillations (which remains constant) of the EM radiation. While this packet of energy enters the medium the wavelength is now $\lambda_{\text{medium}} = \lambda_0 / n$. The free space (vacuum) momentum is thus $p_0 = \hbar k = h/\lambda_0 = hf/c$; while the momentum of photon in media is $p_{\text{medium}} = \hbar k = h/\lambda_{\text{medium}} = nh/\lambda_0 = nhf/c$. If for medium $n > 0$, the momentum of photon increases in the media. This is Minkowski's definition.

Let me have second naïve approach. In vacuum the photon momentum is $p_0 = m\text{v}$, where $m = \text{E}/c^2$. Thus I write expression that is, $p_0 = mc = (\text{E}/c^2)c = hf/c$. Same as Minkowski's free space definition. While here, when photon enters the medium, I get $p_{\text{medium}} = m\text{v}$, $\text{v} = c/n$ thus $p_{\text{medium}} = (\text{E}/c^2)(c/n) = \text{E}/(cn) = hf/(nc)$. I have used $\text{E} = \hbar\omega_0 = hf$ .This is Abraham's answer; (1909). Well Abraham's formulation gives a momentum of photon inside a medium (with $n > 0$) a lesser momentum than in free space.

These two definitions are compared in figure-5. Comparing these two definitions I can say at the outset that if a photon pulse strikes and enters a block of material with refractive index $n > 0$ and $n \neq 1$, then with Abraham's formulation, the block (assuming on frictionless surface; and showing zero reflection) will move in the direction of travel of photon. While the same block would move opposite if we take Minkowski's momentum. Several experiments with Bose Einstein Condensate (BEC), as block and with Electrically Induced Transmission (EIT) method of slow-light have been performed- all are not without any un-controversy though.

So, which is it when media is with negative refraction $n < 0$?

$$p_M = \hbar k = \frac{nh}{\lambda} = \frac{nhf}{c} = n\frac{\hbar\omega}{c} \quad \textbf{or} \quad p_A = m\text{v} = \frac{\text{E}}{c^2}\frac{c}{n} = \frac{hf}{nc} = \frac{1}{n}\frac{\hbar\omega}{c} \quad \textbf{?}$$



V. G. Veselago

1967 proposed that media with $n < 0$ has: Poynting Vector S opposite to wave vector k

both momentums negative for $n < 0$?

shall radiation exert negative pressure when $n < 0$?

Figure-5 Veselago's NRM will make both the momentums negative?

In 1967, Victor Vaselago, proposed that a medium can have negative refractive index, with permittivity and permeability both negatives, giving rise to a wave-vector opposite to the Poynting's vector. In 1999 John Pendry suggested to use SRR as resonating element to get realize magnetic permeability as negative in the magnetic plasma frequency range (we have used LR a variant of SRR in our experiment of NRM prism figure-1); with wire array structure for realizing negative electric permittivity, at electric plasma frequency. Refer figure 5 and 6. However, let me put $n < 0$ for the block (NRM), then momentum of the photon is negative in both the formulations! Meaning if I have NRM, the photon would be taking a negative direction that is: 'will it then come out of the block from the same side?' Or in block of NRM, with photons 'negative' electromagnetic radiation pressure is exerted? The problem seems to be not in the definitions of Minkowski's or Abrahams momentums, but about n the refractive index!

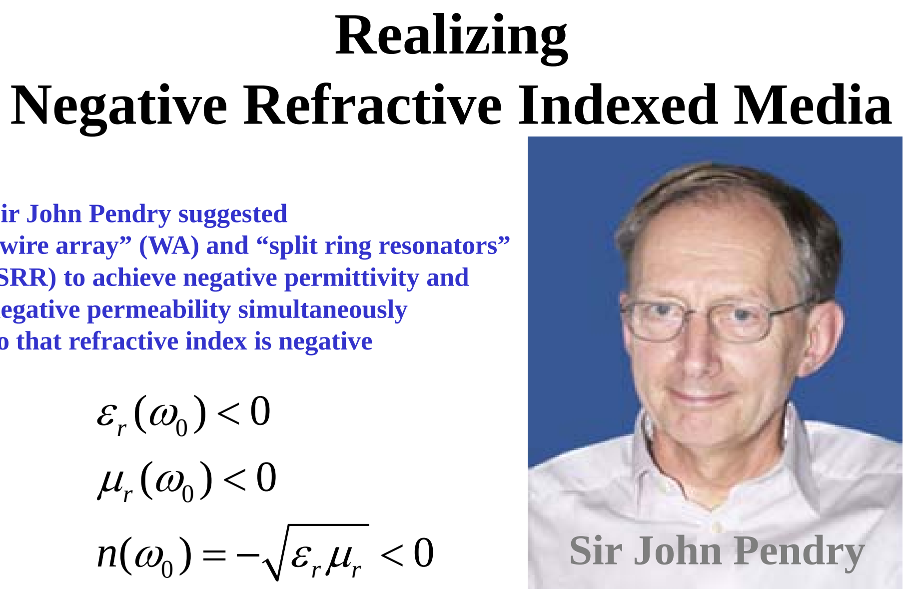


Figure-6 Realizing Negative Refractive Indexed Material by Pendry

7. Demarcating Phase and group refractive index as n_p and n_g

In the section of origin of refractive index, I have made use of interaction of incident EM radiation to atomic electrons of the refracting media thereby obtained a formula for dispersion in refractive index as:

$$n = 1 + \frac{N q_e^{\,2}}{2\varepsilon_0 m_e(\omega_0^2 - \omega^2)} \qquad n = 1 + \frac{q_e^{\,2}}{2\varepsilon_0 m_e}\sum_k \frac{N_k}{\omega_k^2 - \omega^2 + \mathrm{i}\gamma_k \omega}$$

The introduction of damping in the 'charge equation of motion' and thus in the above formula gives me a refractive index as complex number, with real and imaginary parts, as

$$n = n' - \mathrm{i}n''; \quad n'' > 0$$

I shall point out the fact that the imaginary part as indicated above is responsible for dissipation, as the propagating source signals travel the dispersive media. The real part is responsible for phase speed modification in the media, and also in meta-material theory the resonance near the electric, magnetic plasma frequency. With the complex index of refraction the Electric Field rotated after passing through the media (as obtained in the earlier section) I re-write as follows

$$E_{\text{after-plate}} = e^{-\omega n'' \Delta z/c} e^{-\mathrm{i}\omega (n'-1)\Delta z/c} E_0 e^{\mathrm{i}\omega(t-z/c)}$$

Near the electronic oscillator's natural frequency ω_0, the function is analytic, and I can Taylor expand the same. That I demonstrate in this section. The discussions in earlier section for the above derivation suggest that, the n used in the discussion is pertaining to 'phase' manipulations by the media. Thus I shall call this as, phase refractive index. Let me demarcate the two refractive indices, and this demarcation is essential in explaining the NRM theory and the photon's momentum. Take the refractive index dispersive that is a function of frequency call it $n_p(\omega)$ call it phase refractive index. This is basic refractive indices by which the velocity of phases of traveling gets modulated inside a dispersive media. This I call phase index n_p. Similarly velocity of a group of frequency travelling wave gets modulated in the media that gives group refractive index n_g. In case of NRM the phase refractive index if it were $n_p(\omega_c) = -1$ at a particular frequency, that is ω_c , it would imply that in that media the phases would be travelling with speed of light but in opposite direction (as it would have been in vacuum). There is a backward wave inside NRM. Refer figure-7 C; where it is demonstrated that phase gets reversed while inside NRM compared to the free space propagation. Now if there is no change in the refractive index for phases with respect to frequency, meaning that $\{\mathrm{d}n_p(\omega)/\mathrm{d}\omega\} = 0$, I can call it dispersion less medium. In that case the phase velocity $v_p(\omega)$ of the wave and group velocity $v_g(\omega)$ of the wave are same. In the free space (refer figure-7 A) both group of frequencies and the crests and troughs of phases are travelling with $v_p(\omega_c) = v_g(\omega_c) = c$. In the free space I have same modulation for the phases of the signal and group of frequency at a particular frequency and thus I say phase and group index are same $n_p(\omega_c) = n_g(\omega_c) = 1$.

If the media were dispersive I take phase refractive index as an 'analytic' function of the frequency that is; $n_p = f_{\text{analytic}}(\omega)$ at a particular frequency ω_0, natural frequency of harmonic electron oscillator. Expansion of Taylor series, for the dispersive 'phase index' (4) for this dispersive phase refractive index; taking the origin at $\omega = \omega_0$ that is frequency of NRM behavior; can happen only if 'free electrons exists' and for free electrons $\omega_0 = 0$; since they are not bound to any potential (only to its first derivative term at the frequency ω_0 near electric plasma and magnetic plasma resonance where, $\varepsilon_r < 0$ and $\mu_r < 0$ for NRM), is defined as group refractive index, which needs be positive. Meaning that

$$\begin{aligned} n_p(\omega) &\cong n_p(\omega_0) + (\omega - \omega_0)\left(\frac{\mathrm{d}n_p(\omega)}{\mathrm{d}\omega}\right)_{\omega=\omega_0} + \ldots \\ n_g(\omega) &\cong n_p(\omega) + \omega\left(\frac{\mathrm{d}n_p(\omega)}{\mathrm{d}\omega}\right) = \frac{\mathrm{d}}{\mathrm{d}\omega}(\omega n_p) \end{aligned} \tag{4}$$

In the (4) I have expanded analytic function of 'phase-index', at $\omega = \omega_0$ and, then defined 'group index', by putting $\omega_0 = 0$; truncating the Taylor series; and also replacing value $n_p(\omega_0)$, by dispersive function $n_p(\omega)$, making $\omega_0 = 0$ for this function . Note that $n_g(\omega_0) > 0$. From here, for free electron system (as for meta-materials) I can write the group velocity as $v_g = c/n_g = c/\left[n_p + \omega(\mathrm{d}n_p/\mathrm{d}\omega)\right]$ and phase velocity as $v_p = c/n_p$.

This demarcation of phase and group refractive index is very important in understating the behavior of NRM especially the nature of photon inside NRM. NRM have unusual properties and in particular Snell's law predicts that the refracted ray of EM signal on entering such a medium would be refracted on the same side of normal to the surface of the incident beam. The wave number that is $k = n_p\omega/c$ has the opposite sign to its value in positive indexed media. Also inside NRM It is shown that however that Poynting vector and flow of energy points in opposite direction to the wave vector, hence in the expected direction of the propagation of the EM wave-pulse packet (a photon) travels in the direction of energy, while phases move the opposite. (See figure-7 C). The existence of negative values of ε_r and μ_r tends to suggest 'negative energy density'; but that is not the case when dispersion is taken into consideration. Indeed NRM can only exist if the media is dispersive. Moreover causality (in form of Kramer-Kronigs KK-rule) requires that group refractive index defined in (4) $n_g(\omega_s) > 0$ and group velocity; $v_g(\omega_s) > 0$ are always positive. I will not detail this KK-rule and its causality here.

In the introduction I have made a statement of our prism experiment showing a negative value of refractive index of – 1.8. I clarify here, that the, observed negative refraction is for 'phase-refractive-index' as; $n_p(\omega_s) \cong -1.8$, at $\omega_s/2\pi \cong 33\mathrm{GHz}$, with region of NRM as $\Delta\omega/2\pi \cong 0.85\mathrm{GHz}$ where as the group refractive index $n_g(\omega_s) > 0$, as this gives positive group velocity. I thus can say that I can observe a negative phase refractive index but the group refractive index is always shall be positive. Equation (4) should be read at a particular frequency ω_s of interest, where I shall be observing a negative refractive index, in our experimental case it were around 33 GHz.

I can emulate and model by a simplest expression as in (5); an NRM (phase refractive index), by a function such that ω_s is a frequency below which the phase refractive index is negative and above which the phase refractive index is positive.

$$n_p(\omega) = 1 - \frac{\omega_s^2}{\omega^2} \tag{5}$$

This (5) is simplest form of model where I gets ENG (Epsilon Negative) and MNG (Mu Negative) material representation as $\varepsilon_r(\omega) = 1 - (\omega_{ep}{}^2/\omega^2)$ and $\mu_r(\omega) = 1 - (\omega_{mp}{}^2/\omega^2)$. Where ω_{ep} and ω_{mp} are respectively electric and magnetic frequencies below which the permittivity and permeability are respectively negative-designed to overlap. In (5) ω_s is chosen in the region where $\varepsilon_r(\omega_s)$ and $\mu_r(\omega_s)$ both are negative so that $n_p(\omega_s) < 0$. This is design issue, I am not dealing here.

From (5) when I do the differentiation with respect to ω gives me $\mathrm{d}n_p(\omega)/\mathrm{d}\omega = 2(\omega_s^2)/\omega^3$, when I put this and (5) in (4) I get

$$n_g(\omega) = \left(1 - \frac{\omega_s^2}{\omega^2}\right) + \omega\left(2\frac{\omega_s^2}{\omega^3}\right) = 1 + \frac{\omega_s^2}{\omega^2} \quad (6)$$

I call ε_{r-} and μ_{r-} explicitly to distinguish NRM, for ENG and MNG with negative permittivity and negative permeability, respectively. For plasmonic system to achieve NRM I need $\varepsilon_{r-} < 0$ and $\mu_{r-} < 0$, and for ideal case for $n_p = -1$, I need $\varepsilon_{r-} = -1$ and $\mu_{r-} = -1$. Well I can have electric plasma and magnetic plasma frequency overlapped, as $\omega_{ep} = \omega_{mp}$ below which ε_{r-} and μ_{r-} are negatives, so I get NRM as (5). At the Surface Plasmon Polariton resonance frequency $\omega_s = \omega_{ep}/\sqrt{2} = 0.7\omega_{ep}$, the value of $\varepsilon_{r-} = -1$; thereby, giving the value of phase refractive index as, $n_p(\omega) = -1$. Also from (5) I find that $n_p(\omega) = -1$, when $\omega_s^2/\omega^2 = 2$. Putting this value of frequency, I obtain that $n_g = 3$ when $n_p = -1$ at the frequency of operation Surface Mode Resonances. Thus I state that the phase refractive index is negative for NRM and the group refractive index in positive for NRM.

Well if I can write the Minkowski and Abraham's momentum modified (via quantization rule) as $p_{\text{Minkowski}} = n\hbar\omega/c = (n_p^2\hbar\omega)/n_g c$ and $p_{\text{Abraham}} = (\hbar\omega)/(nc) = (\hbar\omega)/(n_g c)$; then both the momentums remain positive even if the media is NRM. I will quantize and derive the radiation momentums in subsequent sections and see a single photon's momentum is same as described above. I will show that with this modified definition the both the momentums are mechanical in nature and shall deliver same momentum to the media of interaction!

I shall now from here call these mechanical momentums as p_{m1} and p_{m2}. The modified momentums are 'single photon' realization, when the two definitions are quantized. They are 'narrow band-width' representation of wave 'packet' travelling with $v_g = c/n_g$ - a single-photon (polariton). Also I assume EM radiation is quasi monochromatic meaning $\Delta\omega \ll \omega_0$; with $\Delta n = \Delta\omega\left|(\partial n/\partial\omega)_{\omega=\omega_0}\right| \ll \left|n(\omega_0)\right|$. Note that ω_0 here, is not the electron oscillator's natural frequency, instead frequency of the 'monochromatic' incident EM signal. From here onwards ω_0 will be indicating monochromatic EM signal's frequency; or carrier frequency for a single photon (EM pulse of radiation); not to be confused with 'natural frequency of charge oscillator' as I used in earlier while deriving the formula for index of refraction.

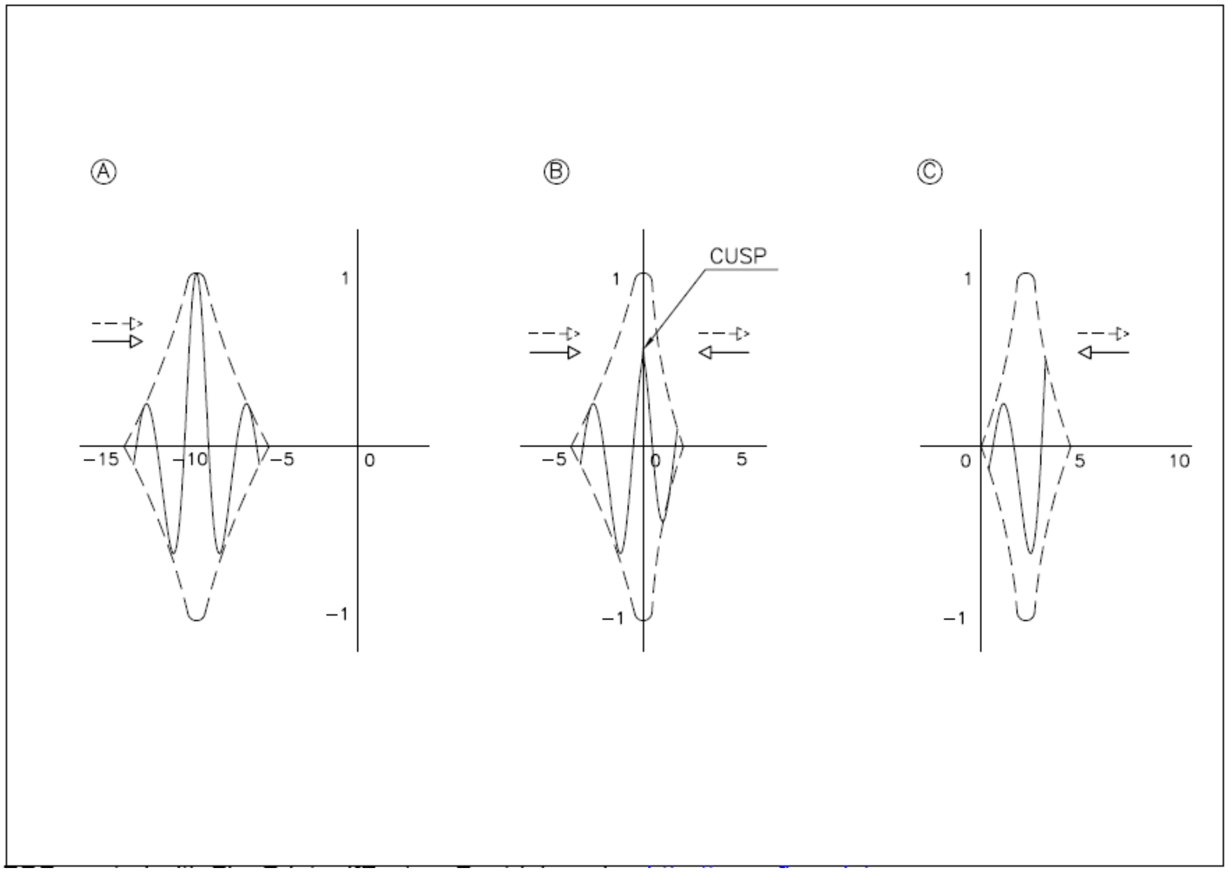
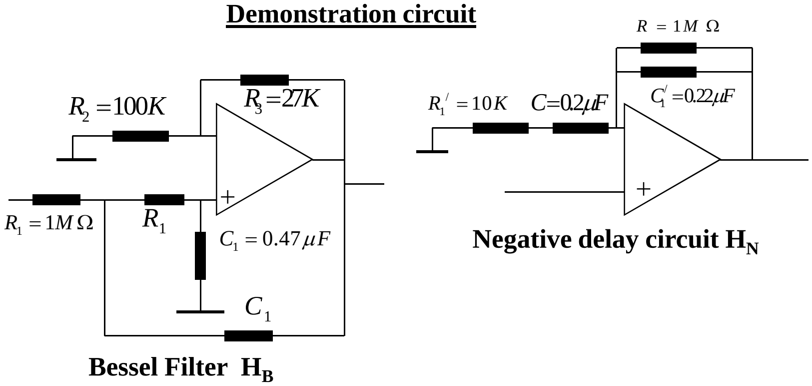


Figure-7: Propagation of electromagnetic pulse. A. Pulse propagating towards right in free space, having envelope (dashed) and phases (solid) traveling with velocity *c* in same direction. B. The same pulse touches the media with NRM with phase index as -1, and group index as +3; shows that at the boundary there is 'cusp' formation and envelope retards. Here the phases travel in opposite direction and the group (envelope) travels in same direction. This cusp oscillates at the surface of the NRM boundary. C. The pulse is traveling as envelope with squeezed envelope inside NRM towards the right direction with velocity $+c/3$ whereas the phases are traveling opposite to envelope, with velocity $-c$. The pulse is sharpened and squeezed. This is ideal case of loss-less NRM while lossy structures will have attenuated pulse as it travels.

Demonstration circuit



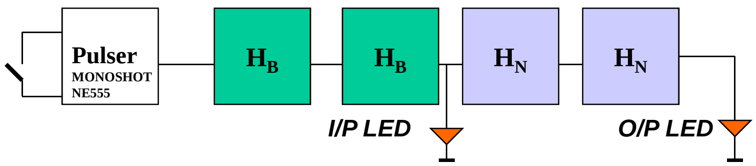
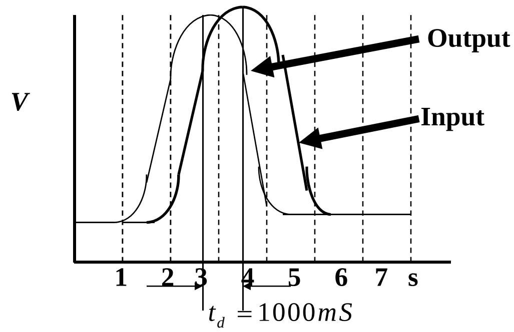


Figure-8 Demonstration circuit of backward wave in LHM idea of faster than light propagation?

Demonstration Circuit pulses observations and comments



Output LED glows before input LED.
CRO gives the measure of <u>negative delay</u> about one second.

Figure-9 CRO output showing peak of output appears before 1 second as compared to input peak

8. The Backward Wave Realization with its Physical Generation and concept of 'Hidden Momentum'

I can demonstrate backward wave, by a circuit presented in figure-8. Here I have emulated LHM via circuit techniques, where the output LED glows before the input LED; giving idea of faster than light propagation! The CRO record of about one second pulse peak advancement is depicted for this circuit in figure-9. The same or rather similar effect is obtained when I make Periodically Loaded Transmission Line (PLTL) depicted in figure-10. The values of the Transmission line are depicted in Table-1, and the 'faster than light' effect is shown in figure-11. The figure-12 depicts its dispersion characteristics ($\omega-\beta$) dispersion diagram, the region where I get effect of Negative Group Velocity ($\mathrm{d}\omega/\mathrm{d}\beta<0$), though $\omega/\beta>0$, the region where phase and group velocity are of opposite signs, is NRM region. Is it so? I will justify this anomaly in the last section as to actually I should have NPV and positive group velocity; after all figure-12 is in first quadrant! Wait for last section, for explanation of this anomaly.

Realizing NGV/ NGD in strip-line transmission line

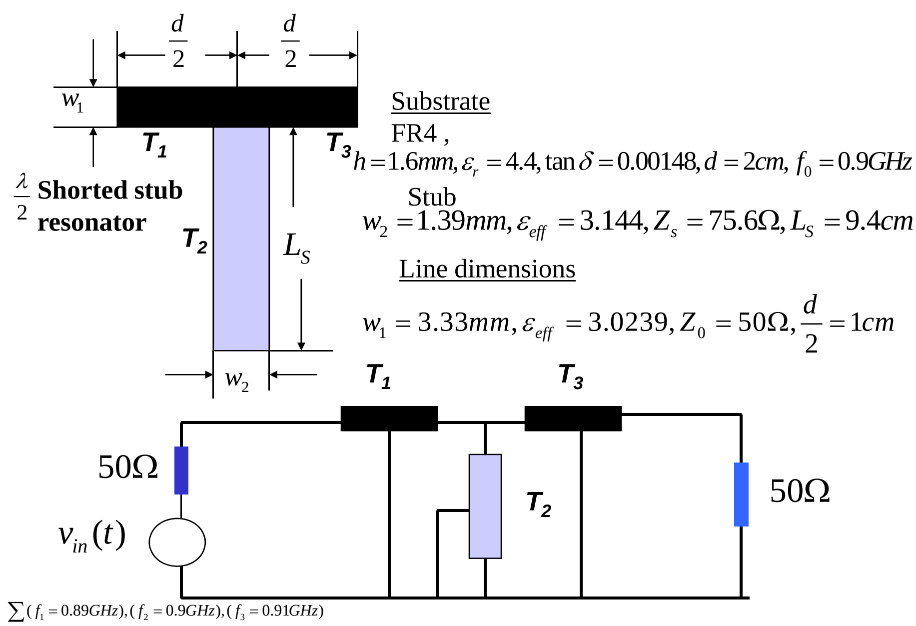


Figure-10 Periodically Loaded Transmission Line PLTL to make LHM

Negative Group Velocity/ Negative Group Delay in time domain

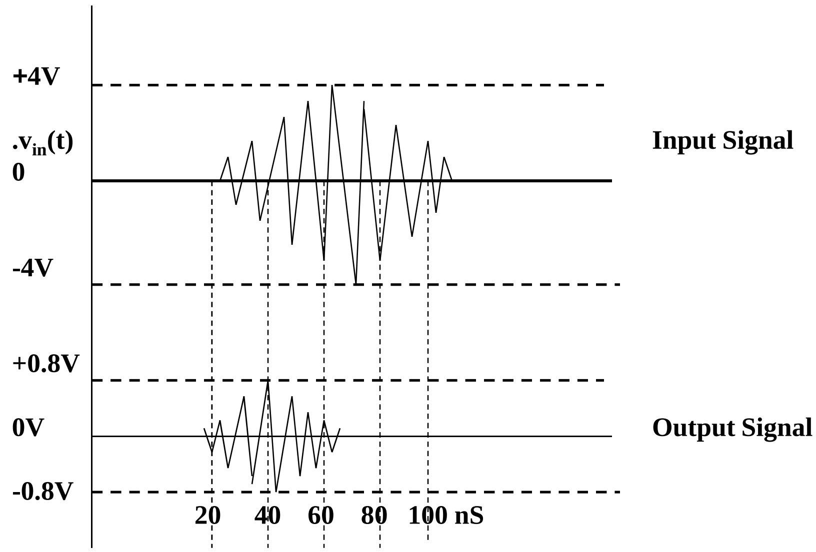


Figure-11 Input output response to have feel of 'faster than light' propagation through LHM

NGV /NGD from dispersion diagram of TL-RLC Shunt structure

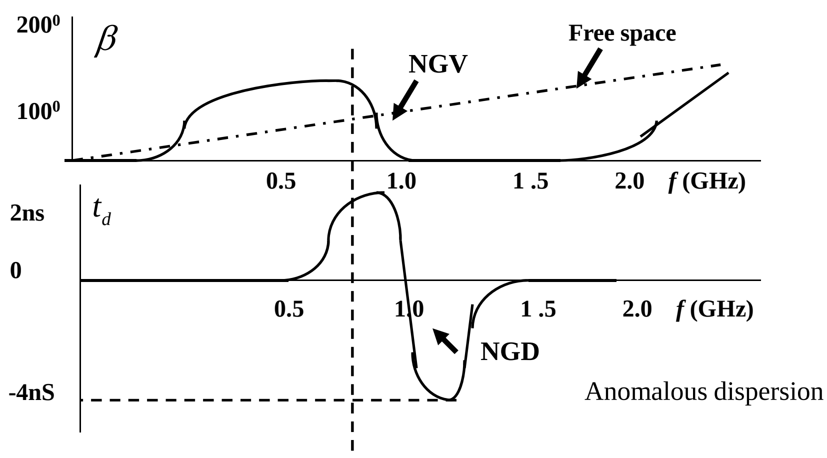


Figure-12 Dispersion diagram of PLTL showing region of Negative Group Delay and Negative Group Velocity

Values of electrical parameters of the strip-line sections

Line	R Ω / m	L nH / m	G $\mu S / m$	C pF / m	length (cm)
T_1	5.0928	304.15	10200	121.66	1.000
T_2	11.079	446.84	6500	78.18	9.4
T_3	5.09 28	304.15	10200	121.66	1.000

Table-1 Showing Parameters Values of PLTL

In the previous section I stated about 'backward waves' in the meta-material. Here I shall try and explain the physical behavior of resonating element. Refer figure 13, depicting lattice of meta-material comprising of the wire-array structure and (square) split ring resonator. The excitation or driving plane wave has magnetic field directed towards the $+y$ axis, the driving plane waves travel in $+z$ direction, with electric field in the $+x$ direction.

In the SRR via Lenz's law there will be induced currents direction shown in the figure, to oppose the driving field, and at resonance there is very strong 'phase opposition' to the driving field. Thus a resultant field will be in $-y$ direction. Similarly the ENG realized by the wire-array will give a strong phase opposed electric field response, giving resultant electric field in the $-x$ direction. The same I had explanted in justifying the term 'Left Handed Maxwell's' Systems; that these meta-materials are. The figure-13 depicts the response fields as of opposite phase, and very large-call them giant fields.

Thus here a magnetic dipole is formed, what it does under action of E_L is our 'thought experiment' in this section. As I may guess this action of electric field to the dipole moment can have associated energy flux and a momentum even if the dipoles are not moving! A hidden momentum then!

Backward Wave & Hidden Momentum

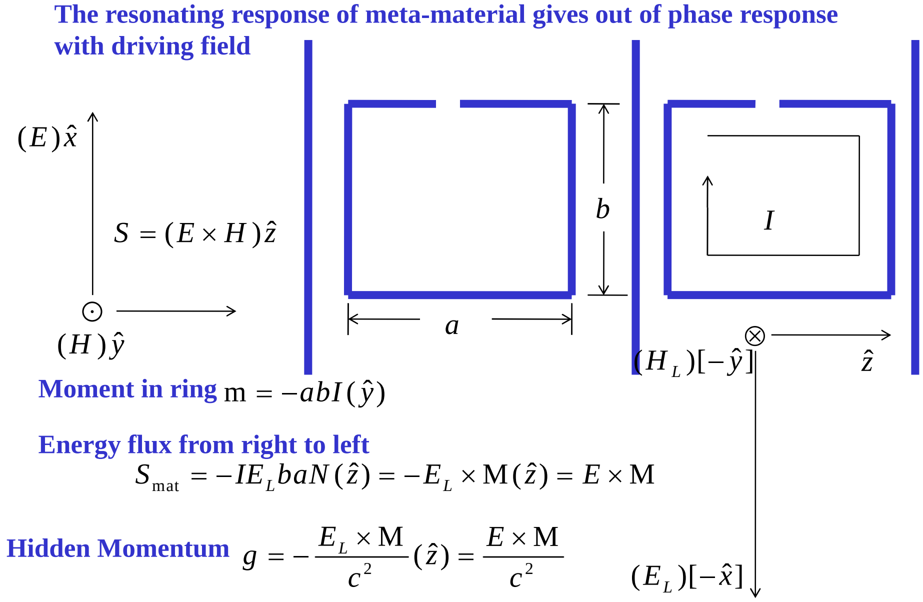


Figure-13 Physics of Backward Waves and Hidden Momentum.

I can write the magnetic moment as $\mathrm{m}=-abI(\hat{y})$, directed towards $-y$ direction. Now carefully look at the figure, where I have shown the current I in the resonating ring, the direction of movement of positive charges. The electric field (the resultant one) that is E_L pointed in $-x$, does 'positive' work on the positive charges that goes down at the right arm of the resonating square ring; and does a negative work at the left arm, while the positive charges are climbing up the left side arm. So a positive charge (say q_e) moving from right side of the square ring to the left side has extra energy, which I can quantify as q_eE_Lb (with respect to situation when the charge moves from left to right of the ring).

This argument generates an 'energy flux' at a point (and pointing towards $-z$ direction). In each magnetic dipole this energy flux is there, and says there are N such rings, where dipoles are formed. I can state that in each ring the energy flux at a point per unit time is IE_Lb. This is my flow quantity of energy crossing dz. So the energy flux per ring (integrate dz from 0 to a) I get, as $-IE_Lba(\hat{z})$ directed towards $-z$. Thus total energy flux from N magnetic dipoles I can write as $S_{\text{mat}}=-IE_LbaN(\hat{z})=-E_L\times\mathrm{M}=E\times\mathrm{M}$. This M is magnetization of the medium. Therefore if I have a magnetization M as in the ring resonator, with source electric field E, there is a 'hidden momentum' density $g=(E\times\mathrm{M})/c^2$, directed towards $-z$ direction.

The momentum of charges that move towards the left of the ring is higher because they pose higher energy. The ratio between momentums to energy of relativistic particle is v/c^2. The hidden momentum concept is therefore a purely relativistic effect as I have not considered $m\mathrm{v}$ type of momentum! This 'hidden momentum' is phase reversal, thus in meta-materials with

NRM there is a concept of 'backward wave' the phases travel in backward direction to the power flow direction!

I have explained just above the hidden momentum that is due to interaction of electric field to the magnetization of resonating split-rings. Is there a converse that is any other momentum of the EM radiation, as effect of 'magnetic field'? I will now see this interaction. The electric field acts on charges (free charges) of wire-array, as they drive them up down with them. When I consider say positive free charges, and the resonance effect of NRM then the displacement as well as velocity will be directed opposite so I can take velocity of the free positive charge in $-x$ direction. The induced B_L is resultant due to phase opposition in $-y$ direction of H_L (as explained in the figure-13). Thus there force on the charge as $F = q_e \mathrm{v} \times B_L$ and is directed in $+z$ direction. If I take $B_L = E_L / c$, I get $F = q_e(\mathrm{v}E_L)/c$, directed in $+z$ direction. But $q_e E_L$ is electric force on the charges, times v the velocity is work done per unit time. Thus I can state that the force gives a 'pushing momentum' directed in $+z$ direction-this is also related to a mechanical pressure of radiation; which I must state is positive by this argument in NRM. I will detail the radiation pressure later.

Thus I see a hidden momentum, and a pushing momentum for EM radiation which are directed opposite in NRM media.

9. Electromagnetic Pulse Sharpening inside NRM

A wave with crest and trough moving and carrying a Gaussian pulse a 'packet' of energy, in free space travelling with speed of light c, (refer figure-7 A) when entering the NRM with $n_p = -1$, will retard the wave-packets speed to c / n_g in this case $c/3$, (refer figure-7 B and C) though the direction of travel of wave-packet, energy will be in same direction as was in free space; but the phases crests and troughs will here start travelling in opposite to free-space with velocity $-c$. This is implication of the phase and group refractive index in NRM. The implication at NRM boundary of these opposite phases meeting will form a 'cusp' which be oscillating at the junction of NRM to the free space (refer figure-7 B). This phenomenon of retardation of the wave-packet envelope and change of direction of travel of crest & trough the phase, inside NRM gives the 'pulse-sharpening' effect, and flattening of wave-front effect, what I have been observing in our experiments also, (refer figure-7 C). The pulse sharpening effect is too observed in figure-11 (the output input relation of PLTL of figure-10). The cusps at the NRM boundary is due to counter propagation of the 'phases' of the waves inside and outside the NRM, they are surface charges, and at the boundary Electric Field at this cusp oscillates; as two sets of impinging wave fronts meet at the interface with ENG (Epsilon Negative Material $\varepsilon_{r-} < 0$). The same cusp will be obtained for the MNG, (Mu Negative Material $\mu_{r-} < 0$) and it may be argued that 'surface' currents in that case for TE polarized incidence, will be at the boundary and magnetic field at the cusp then will oscillate. However, these points are valid when the wave hits a slab with ENG and MNG i.e. NRM here however there will be propagating modes inside NRM-from evanescent. In the case of Double Negative slab (NRM) there will be cusp formation at the boundary too. The formation of surface states or excitation of surface Plasmon poalriton is altogether different field in modern optics, where matching of wave vectors and phase velocities are mandatory, I will not deal with this subject here; however this is important.

10. Electromagnetic Pulse of Energy travelling in free space and inside medium its transmission and reflection at the interface boundary

The figure-11, 14 gives me an idea what is an electromagnetic pulse. I shall relate corpuscular and wave nature to the same. The probability amplitude is what is of interest to say where the particle ought to be at space-time. Amplitude to find a particle (photon) at a place can in some circumstances, vary in space and time in a manner say $\psi = Ae^{i(\omega t - kz)}$, where ω is the frequency, which is related classically to energy by $\mathrm{E} = \hbar\omega$, and k-the wave number, is related to momentum through $p = \hbar k$. In the figure-11 the carrier wave frequency is 0.9 G Hz, thus $\omega = 0.9\mathrm{GHz}/(2\pi)$. I would say the particle (photon) had a definite momentum, p if wave number k were 'exactly' that particular wave number (without any spread or uncertainty); that is a perfect wave, which goes on with same amplitude everywhere. The amplitude equation as described just above, then gives me amplitude and probability (square of amplitude) for finding particle (photon) as function of space-time. Thus for a perfect wave the probability is constant which means probability to find particle (photon) is the same anywhere!

This is not the situation with single particle (photon) travelling as in figure-11 and 14. The amplitude modulated pulse as shown has maxima and dies out at both the sides. It was possible for me to get this by adding waves of nearly same ω and k, in figure-11 signals of 0.89 G Hz and 0.91 G Hz I added to get the electromagnetic pulse a single photon. Thus the particle (photon) is more likely to be near the maxima (lump) of figure-11, 14. After a few moments this wave with lump will be elsewhere, as it is traveling with group velocity v_g, should be related to particle (photon) velocity.

I have classical Energy momentum relativistic expressions as $\mathrm{E} = (mc^2)/\sqrt{1-(\mathrm{v}^2/c^2)}$, and $p = (m\mathrm{v})/\sqrt{1-(\mathrm{v}^2/c^2)}$. Eliminating the v, from these two expressions leads me to, expression a well known one as $\mathrm{E}^2 - p^2c^2 = m^2c^4$. This relation is depicted in figure-17, and I will use in subsequent sections. The E is related to ω and p is related to k, using them in the total energy expression of above I get $(\hbar^2\omega^2/c^2) - \hbar^2k^2 = m^2c^2$, a quantum-mechanical relation between frequency and wave number, for a quantum-mechanical amplitude wave representing a particle of mass m. From this derived expression I get $\omega = c\sqrt{k^2 + (m^2c^2/\hbar^2)}$, which gives me phase velocity $v_p = \omega/k$, as $v_p = c/[k\sqrt{k^2 + (m^2c^2/\hbar^2)}]$. Differentiating, and substituting several related quantities I obtain after algebraic simple manipulations expression of the group velocity as $v_g = \mathrm{d}\omega/\mathrm{d}k = kc/\sqrt{k^2 + (m^2c^2/\hbar^2)} = c^2k/\omega = c^2p/\mathrm{E}$. Recognizing that $c^2p/\mathrm{E} = \mathrm{v}$, I can say that group velocity of the wave packet is particle velocity v. Here I have a special case as $v_p v_g = c^2$, which in general need not be equal, but are equivalent. These I shall deal in separate section and use this concept further to arrive at momentum and energy transfer to medium, by a single photon.

When I consider a narrow range of frequencies about ω, with in which absorption is negligible; within this range of frequencies I can write electric field as following

$$E(r,t) = \mathrm{Re}\left[E_0(r,t)e^{-\mathrm{i}\omega t}\right] = \mathrm{Re}\left[\int \mathrm{d}\alpha \tilde{E}_0(r,\alpha)e^{-\mathrm{i}\alpha t}e^{-\mathrm{i}\omega t}\right]$$

Where $E_0(r,t)$ is slowly varying compared to $e^{-\mathrm{i}\omega t}$ that is $\alpha \ll \omega$ for frequencies α for which $E_0(r,t)$ is non-negligible. This is in short I say a high frequency carrier is modulated by low frequency signal, a case of amplitude modulation! This fundamental I will use to construct and visualize a photon (an EM pulse).

The discussion on this section is from classical electrodynamics principles. Let me take following example a pulse of EM energy travelling in free-space at a particular frequency ω_0, thus carrying an energy packet of $\hbar\omega_0$. This packet of EM radiation may be represented as a Gaussian pulse; that will strike a medium (other than free-space) located at $z = 0$, by (7), this is derived in (8).

$$E = E_0\sigma\sqrt{\pi}\left\{e^{+\mathrm{i}z\omega_0/c}e^{-\mathrm{i}\omega_0 t}\right\}\exp\left[-\frac{\sigma^2}{4}(t - z/c)^2\right] \tag{7}$$

The field incident at $z = 0$ is adequately represented by complex Electric field as:

$$\begin{aligned} E^{\mathrm{in}} &= E_0\int \mathrm{d}\omega \exp\left[-\left(\omega-\omega_0\right)^2/\sigma^2\right]\exp\left[\mathrm{i}(kz-\omega t)\right] \\ &= E_0\sqrt{\pi}\sigma\exp\left[-\mathrm{i}\omega_0(t-z/c)\right]\exp\left[-\frac{\sigma^2}{4}(t-z/c)^2\right] \end{aligned} \tag{8}$$

The (7) expression is for travelling Electric field that has two parts. The phase part given inside the $\{\ \}$ brackets, and multiplied by Gaussian travelling envelope in free space as $\exp[-\sigma^2(t-z/c)^2/4]$, having variance σ^2 i.e. the width of the packet (Full Width Half Maxima FWHM). The packet is travelling from left to right thus phases (crest and trough are translating in $+z$-direction) with a phase velocity $v_p = c$, and the group i.e. the envelope carrying the information / energy is travelling with group velocity $v_g = c$ in the same direction of $+z$ in free space having $n_p = n_g = +1$. Refer figure-7 A the (7) is depicted there traveling towards right with envelope as dashed and phases as solid lines. The (7) is a single photon an EM pulse depicted as in figure-14 that is how single photon is depicted perhaps!

Visualizing single photon

A photon is an electromagnetic pulse in vacuum- a packet; this packet has a spread in space enveloping a carrier with frequency *f* ; the energy carried by this photon is *E* = *h f*

Figure-14 An EM Pulse visualizing a single 'monochromatic' photon!

Now I will investigate what happens when this (7) (8) incident Gaussian Electromagnetic pulse enters a medium. This Gaussian pulse is centered at angular frequency ω_0 and I shall assume that this energy beam is weakly focused so I can take spatial spread in only one dimension. The reflection and refraction of Electromagnetic waves at an interface are described by Fresnel law. For normal incident I have reflection coefficient $\rho(\omega)$ and transmission coefficient $\tau(\omega)$ described as (9); both being function of frequency since impedance of media is dispersive.

$$\rho(\omega) = \frac{Z_0 - Z}{Z_0 + Z} \qquad \tau(\omega) = \frac{2Z}{Z_0 + Z} \tag{9}$$

Where, $Z = \sqrt{\mu / \varepsilon}$ is impedance of medium and Z_0 is free space impedance. Note for a NRM with $\mu_r = \varepsilon_r = -1$ the $Z = Z_0$, the incident beam suffers no reflection and is 100% transmitted. The forms of reflected and transmitted waves follow from the spectrum of the incidence pulse (7) as (8) and (9).

$$E^{\mathrm{ref}} = E_0 \int \mathrm{d}\omega \exp\left[-\left(\omega - \omega_0\right)^2 / \sigma^2\right] \rho(\omega) \exp\left[\mathrm{i}\omega(t + z / c)\right] \tag{10}$$

$$E^{\mathrm{trans}} = E_0 \int \mathrm{d}\omega \exp\left[-\left(\omega - \omega_0\right)^2 / \sigma^2\right] \tau(\omega) \exp\left[-\mathrm{i}\omega(t - n_p\{\omega\} z / c)\right] \tag{11}$$

It suffices for my purpose to assume that spectrum is narrow so that I can approximate $\rho(\omega)$ and $\tau(\omega)$ by their values at ω_0 and $n_p(\omega)$ by first two terms of Taylor series expansion (4). This leads to simple Gaussian forms for (10) and (11) as (12) and (13)

$$E^{\mathrm{ref}} = \rho(\omega_0) E_0 \sqrt{\pi} \sigma \exp\left[-\mathrm{i}\omega_0 (t + z / c)\right] \exp\left[-\frac{\sigma^2}{4}(t + z / c)^2\right] \tag{12}$$

$$E^{\text{trans}} = \tau(\omega_0)E_0\sqrt{\pi}\sigma \exp\left[-\mathrm{i}\omega_0(t-n_p z/c)\right]\exp\left[-\frac{\sigma^2}{4}(t-n_g z/c)^2\right] \tag{13}$$

For 100% transmission when $Z = Z_0$ say for NRM when $\varepsilon_r = \mu_r = -1$, with $n_p(\omega_0) = -1$ and $n_g(\omega_0) = 3$ I will get $E^{\text{ref}} = 0$ since $\rho(\omega_0) = 0$, $\tau(\omega_0) = 1$ and transmitted field inside NRM is thus given below (14).

$$E^{\text{trans}} = E_0\sqrt{\pi}\sigma \exp\left[-\mathrm{i}\omega_0(t+n_p z/c)\right]\exp\left[-\frac{\sigma^2}{4}(t-3z/c)^2\right] \tag{14}$$

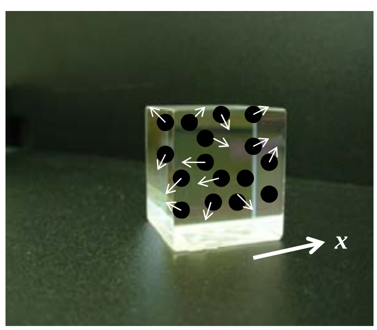
11.Pressure due to photons and 'radiation compressibility'

Here I take a detour from electrodynamics (for a while) and see the 'kinetic theory of gases' and put that theory to a conglomerate of photons. Well, if I can confine radiation inside a volume region there will be a gas like 'photon' pressure P, in a volume V; like a gas law will have a radiation compressibility law as $PV^{\gamma} = \mathrm{C}$. The gas constant γ indeed will not be same for radiation! The C, is constant. The radiation pressure and compressibility of radiation comes from kinetic theory of gases. Consider large number N photons (at very very high temperature) packed in volume V. The photons are confined and thus they bombard the boundary of this 'photon' star, and give a pressure $P = 2(N/V)\,p_x v_x$. While considering the volume is of NRM then I become uncomfortable with this basic relation, where classically speaking $p_x = np_0$, is negative for NRM with $n < 0$.The scenario is of a very very hot star-a hot star even hotter than sun! The photons are travelling in random direction (similar case as for confined gas depicted in figure-15).

Photon's pressure inside *V* with *n* < 0

***N* photons are confined in volume *V* gives compressibility of radiation**

$$P = 2\left(\frac{N}{V}\right) p_x v_x$$



$$P V^{4/3} = \mathrm{C}$$

$$p_x = n p_0$$

or

$$p_x = p_0 / n$$

with

$p_0 = \hbar\omega / c$ **while *n* < 0 makes us uncomfortable with *P*, the pressure exerted by photon's to *V* !**

Figure-15 photons confined in a volume

In figure-15, let area of the face of the volume V be of A , on which the x-component of momentum that gives kick, and twice this x component of momentum, p_x which is given in the kick. If v_x is the velocity of x component, I get kick amount as $2mv_x$ i.e. $2p_x$, the momentum transferred to the wall of area A, by a single photon. Now I need the number of collisions made by the photons in a second, or in a certain time (t). The number of photons per unit volume is N/V . So in this case only the photons which are at distance $v_x t$ from the wall are going to hit the wall in time t. Far away photons won't take part in collision, or kicks. Thus the number of collisions in time t is equal to the number of photons which are in the region within a distance $v_x t$ and since area of the wall is A; the volume occupied by the photons which are going to hit the wall is $v_x tA$. But the number of photons that are going to hit the wall is that volume ($v_x tA$) multiplied by photon density (N/V), that is $(N/V)v_x tA$.

The pressure to the wall is $F/A=P$. The differential work in compressing the volume by $\mathrm{d}x$, or by $\mathrm{d}V$ is $\mathrm{d}W = F(-\mathrm{d}x) = -PA\mathrm{d}x = -P\mathrm{d}V$. Force on the wall is thus number of photons colliding in time t, times change in momentum divided by time t, that is

$$F = \left[\left(\frac{N}{V}\right)v_x At\right]2mv_x \div t = \left(\frac{N}{V}\right)v_x A(2mv_x)$$

The above is coming from impulse that is $(F)(t)$, is total change in momentum. The pressure $P = F/A$ is

$$P = 2\left(\frac{N}{V}\right)(mv_x)v_x = 2\left(\frac{N}{V}\right)p_x v_x$$

This above expression made me uncomfortable, when the volume is negative indexed. I can write average as

$$PV = N\langle p_x v_x\rangle$$

The number 2 is dropped since in the collision process, half of the photons will be moving the other way away from the wall, thus while averaging the pressure expression 2 gets cancelled. Now as in kinetic theory I take all the three components of randomly directed velocity v, that is v_x, v_y and v_z, and average them to obtain

$$PV = \frac{N\langle p.v\rangle}{3}$$

Recognizing energy $U = pv$ (rather in free space $U = pc$), for photons I have $PV = U/3$. (While monatomic gases in volume of V, having gas pressure, as P follows the relation $PV = (2/3)U$, with internal energy $U = (1/2)mv^2$). Compressibility of radiation can be obtained as follows, by rewriting $PV = (1/3)U$ as

$$PV = (1/3)U \quad PV = (\gamma-1)U \text{, with } \gamma = 1+\frac{1}{3} = \frac{4}{3}$$

$$U = \frac{PV}{(\gamma-1)}$$

$$\mathrm{d}U = (P\mathrm{d}V + V\mathrm{d}P)/(\gamma - 1)$$

As already pointed earlier above $\mathrm{d}W = \mathrm{d}U = -P\mathrm{d}V$ substituting this above I get

$$P\mathrm{d}V = -(P\mathrm{d}V + V\mathrm{d}P)/(\gamma - 1)$$

$$\gamma P\mathrm{d}V = -V\mathrm{d}P$$

$$\gamma \frac{\mathrm{d}V}{V} + \frac{\mathrm{d}P}{P} = 0$$

$$\gamma \ln V + \ln P = \ln \mathrm{C}$$

$$PV^{\gamma} = \mathrm{C}\text{, with } \gamma = 4/3 \text{ for photons}$$

This section I had detoured but nevertheless I showed the apprehension regarding photon pressure if the media is NRM! Therefore I need to define photon momentum in some different way; and thus I can avoid this anomaly in 'radiation' pressure.

12. Energy Momentum of Gaussian Electromagnetic pulse

Let me now turn attention to a single photon, a single EM pulse (figure-14). To this Gaussian pulse there is a packet of energy $\hbar\omega_0$; where I can associate momentum $\hbar\omega_0 / c$ with this pulse. Inside a medium I can have scenario where the momentum can have different interpretation if I say $p = n_p \hbar\omega_0 / c$ as phase 'wave' momentum inside medium, then if the media has $n_p = -1$, I get confused by this negative momentum indicating a decrease in pressure for radiation of electromagnetic wave, when it strikes a boundary. Well call this momentum $n_p \hbar\omega_0 / c$ as 'wave' momentum, to distinguish from 'mechanical' momentum (15) (16) (containing group velocity and group index) as, Minkowski or Abraham;

$$p_{m1} = n_p^2 \hbar\omega_0 / n_g c = v_g n_p^2 \hbar\omega_0 / c^2 \qquad (15)$$

$$p_{m2} = \hbar\omega_0 / n_g c = v_g \hbar\omega_0 / c^2 \qquad (16)$$

From the discussions of Minkowski and Abraham's momentum in earlier section I wrote $p_{\mathrm{Abraham}} = nhf / c$ and $p_{\mathrm{Minkowski}} = hf / nc$. These are modified and placed as p_{m1} as Abraham's and p_{m2} as Minkowski's. Which at present let these be, and subsequently I will derive in next section. These definitions of mechanical momentum ensure that they are positive, in side NRM as well. Well these mechanical momentum definitions (15) (16) give me non-confusing thought that even with $n_p < 0$ still there is positive electromagnetic pressure, as against definition of 'wave' momentum or pseudo-momentum $p = n_p \hbar\omega_0 / c$, where I let believe if the electromagnetic pressure be negative in case of NRM! Well, only for phase reversal I make use of wave-momentum (the hidden momentum), and for energy transport and electromagnetic energy pressure I shall make use of mechanical momentum. The confusion is arising because of dual nature of radiation, particle as well as wave nature.

Now when the Gaussian pulse of this Electromagnetic energy enters a slab with $n_p \neq 1$ and $n_g \neq 1$, assuming 100% transmission into that slab I have different Electric field as from (17)

$$E = E_0 \sigma \sqrt{\pi}\left\{e^{\mathrm{i} n_p \omega_0 z/c} e^{-\mathrm{i}\omega_0 t}\right\} \exp\left[-\frac{\sigma^2}{4}(t - n_g z/c)^2\right] \qquad (17)$$

Now if I state that $n_p = -1$, and $n_g = 3$, then I will observe that the Gaussian pulse envelope will compress itself and keep propagating inside NRM block in the same direction of $+z$,with group velocity $c/3$ but the phases will keep now translating in space in opposite direction but with phase velocity $-c$, refer figure-7 C. The meeting of the two opposite phases, (refer figure-7 B) at the NRM boundary gives rise to cusps-owing to surface modes, which travel and oscillate in direction perpendicular to propagation direction and along the surface of the interface.

Well I ask a query that is if (17) can be called a photon as it has now become inside NRM of my choice as-better be called as ‘negative-photon’ (17) (18)

$$E_{\mathrm{N}}^{\mathrm{photon}} = E_0 \sigma \sqrt{\pi}\left\{e^{-\mathrm{i} z \omega_0/c} e^{-\mathrm{i}\omega_0 t}\right\} \exp\left[-\frac{\sigma^2}{4}(t - 3z/c)^2\right] \qquad (18)$$

is different from original (7), that is $E_P^{\mathrm{photon}} = E_0 \sigma \sqrt{\pi}\left\{e^{+\mathrm{i} z \omega_0/c} e^{-\mathrm{i}\omega_0 t}\right\} e^{-\frac{\sigma^2}{4}(t - z/c)^2}$ in the free space. The (18) seems to suggest that the pulse envelope and the phases travel are in opposite direction, this packet need not be thus called a photon packet rather ‘negative’ photon packet! (Refer figure-1 C). This is also depicted pictorially in figure-16.

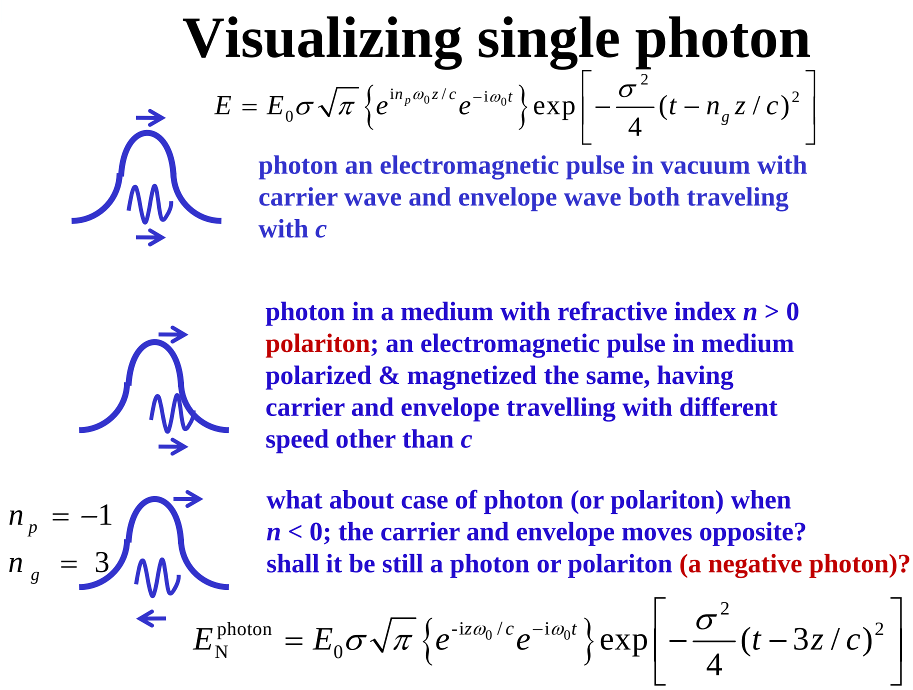


Figure-16 A positive photon and negative-photon

My argument of ‘negative’ photon stems from the fact that had there be 100% reflection to (7), $\rho(\omega_0) = 1$, then I get, a packet of original photon as in (19), but where the envelope and phases are travelling in $-z$ direction after hitting the boundary at $z=0$, thus retaining the character of original photon.

$$\begin{aligned} E^{\mathrm{ref}} &= \rho(\omega_0) E_0 \sqrt{\pi}\sigma \exp\left[-\mathrm{i}\omega_0 (t+z/c)\right] \exp\left[-\frac{\sigma^2}{4}(t+z/c)^2\right] \\ &= E_P^{\mathrm{photon}} = E_0 \sigma\sqrt{\pi}\left\{e^{-\mathrm{i}z\omega_0/c} e^{-\mathrm{i}\omega_0 t}\right\} e^{-\frac{\sigma^2}{4}(t+z/c)^2} \end{aligned} \tag{19}$$

Reflected photon is original photon as incident photon, while transmitted photon inside NRM is 'negative' photon. Well, inside media the photon polarizes the same and thus becomes polariton, so the above is 'negative' polariton!

13. Electromagnetic momentum and energy quantization for a single photon inside weakly dispersive dielectric media

Essentially the EM field itself is quantized. Of course I know that it is made up of photons (Plank's black body $\hbar\omega$). I also understand that these 'modes' are oscillations of radiation are 'simple harmonic oscillators' with energy as integral multiples of $\hbar\omega$ and just as a mass spring oscillator has fluctuations in ground state $\langle x \rangle = 0$ but $\langle x^2 \rangle \neq 0$ the x being the position; similarly for EM I have $\langle A \rangle = 0$, with $\langle A^2 \rangle \neq 0$, with A as vector potential associated with travelling plane EM wave. I shall use this vector potential and quantize the same in a dispersive media; and obtain energy and momentums quantized.

I wrote the mechanical momentums of single photon in dispersive media with inclusion of group velocity group index phase velocity phase index as in (20), in earlier section, now I shall try and derive the same

$$p_{m1} = n_p^2 \hbar\omega_0 / n_g c = v_g n_p^2 \hbar\omega_0 / c^2, \; p_{m2} = \hbar\omega_0 / n_g c = v_g \hbar\omega_0 / c^2 \tag{20}$$

The peculiar situation about momentum of electromagnetic radiation is long standing controversy, starting from Mikowski's (subscripted M) definition (1908) and followed by Abraham's (subscripted A) definition (1909). Where the former is referred to canonical one and later is referred to mechanical one classically. The traditional electromagnetic momentum density in a medium with averaging over a time period, are:

$$\begin{aligned} \bar{g}_A(\bar{r},t) &= \frac{\langle \bar{E}(\bar{r},t)\times\bar{H}(\bar{r},t)\rangle}{c^2} = \frac{\langle S(\bar{r},t)\rangle}{c^2} \\ \bar{g}_M(\bar{r},t) &= \langle \bar{D}(\bar{r},t)\times\bar{B}(\bar{r},t)\rangle = \varepsilon\mu\langle \bar{E}(\bar{r},t)\times\bar{H}(\bar{r},t)\rangle = \frac{\langle S(\bar{r},t)\rangle}{v_p{}^2} \end{aligned} \tag{21}$$

In the definition of g_M above, (21), in dispersive media I have used $(\varepsilon\mu)^{-1} = v_p{}^2$, similar to if it was free space then $(\varepsilon_0\mu_0)^{-1} = c^2$.

The quantization scheme I will use a very simple one, starts with standard classical expression for the electromagnetic energy density in a dispersive dielectric medium (non-magnetic one to keep the derivation simpler). For classical fields in such a dispersive medium the effective energy is

$$U_{em} = \frac{\mathrm{d}(\varepsilon\omega)}{\mathrm{d}\omega}\frac{1}{2}\int \mathrm{d}^3 r\left|\bar{E}_0\right|^2 + \frac{\mathrm{d}(\mu\omega)}{\mathrm{d}\omega}\frac{1}{2}\int \mathrm{d}^3 r\left|\bar{H}_0\right|^2 \tag{22}$$

For a non magnetic media, then $\mu = \mu_0$, and from above (22) I obtain, by putting $\mu_0 \bar{H} = \bar{B}$ (23)

$$U_{em} = \frac{\mathrm{d}(\varepsilon\omega)}{\mathrm{d}\omega}\frac{1}{2}\int \mathrm{d}^3 r\left|\bar{E}_0\right|^2 + \frac{1}{2\mu_0}\int \mathrm{d}^3 r\left|\bar{B}_0\right|^2 \tag{23}$$

Note that in (22) (23) I have taken average over the carrier period $T_0 = 2\pi/\omega_0$, for a monochromatic radiation. Therefore ½ is appearing in the expressions, that is average of sinusoidal square. The amplitudes $\bar{E}_0$ and $\bar{B}_0$ are the peak values of the field. For monochromatic fields of interest, the power Fourier spectrum is concentrated at a particular frequency ω_0 with spectral width $\Delta\omega \ll \omega_0$. The medium is assumed to be weakly dispersive with respect to this wave packet (a single photon), that is a 'narrow band' case too (24)

$$\Delta n_p = \Delta\omega\left|\frac{\partial n_p(\omega)}{\partial\omega}\right|_{\omega=\omega_0} \ll \left|n_p(\omega_0)\right| \tag{24}$$

The quantum theory of the electromagnetic field starts by Fourier expanding the vector potential and then substituting operators for the amplitude term. Consider the classical field described by a vector potential in Fourier series having Fourier (root mean squared) amplitude as $\bar{A}_s(\bar{k})$, that is

$$\bar{A}(\bar{r},t) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_s \bar{A}_s(\bar{k})\mathbf{e}_s(\bar{k})e^{\mathrm{i}(\bar{k}\bullet\bar{r}-\omega t)} = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_s \bar{A}_s(\bar{k})\mathbf{e}_s(\bar{k})e^{-\omega t}\bar{F}_\omega(\bar{r}) \tag{25}$$

The amplitudes in k space are in root mean squared (RMS). The term $\mathbf{e}_s$ is unit 'polarization' vector in a plane perpendicular to $\bar{k} = (\bar{k}/k)[n_p\omega/c]$. The ω is function of wave vector k in (25) and $\bar{F}_\omega(\bar{r})$ is mode function satisfying the transversality condition and Helmholtz equation that is

$$\begin{aligned} &\nabla \bullet \bar{F}_\omega(\bar{r}) = 0 \\ &\nabla^2 \bar{F}_\omega(\bar{r}) + \frac{\omega^2}{c^2}\varepsilon\mu\bar{F}_\omega(\bar{r}) = 0 \end{aligned} \tag{26}$$

From the vector potential (25) I can obtain the electric and magnetic field from $\bar{B}(r,t) = \nabla\times\bar{A}(r,t)$ and $\bar{E}(r,t) = -(\partial\bar{A}(r,t)/\partial t)$ (assuming scalar electric potential is a constant and using $\nabla = \mathrm{i}k$), as

$$\bar{E}(\bar{r},t) = \mathrm{i}\omega\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_s \bar{A}_s(\bar{k})\mathbf{e}_s(\bar{k})e^{-\omega t}\bar{F}_\omega(\bar{r}) \tag{27}$$

$$\bar{B}(\bar{r},t) = \nabla\times\bar{A}(\bar{r},t) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\mathrm{i}k\sum_s \bar{A}_s(\bar{k})e^{-\omega t}\bar{F}_\omega(\bar{r})\times\mathbf{e}_s(\bar{k}) \tag{28}$$

From (27) and (28) I get peak value of the field, from RMS expression of Fourier components as $\left|\bar{E}_0\right| = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \omega\sqrt{2}\left|\bar{A}_s(k)\right|\left|\bar{F}_\omega(\bar{r})\right|$ and. $\left|\bar{B}_0\right| = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \sqrt{2}k\left|\bar{A}_s(\bar{k})\right|\left|\bar{F}_\omega(\bar{r})\times\mathbf{e}_s\right|$ I then substitute this in (23) and write (29).

$$U_{em} = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \omega^2 \frac{\mathrm{d}(\omega\varepsilon)}{\mathrm{d}\omega}\left|\bar{A}_s(\bar{k})\right|^2 \int \mathrm{d}^3r\left|\bar{F}_\omega(\bar{r})\right|^2 + \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \frac{k^2}{\mu_0}\left|\bar{A}_s(\bar{k})\right|^2 \int \mathrm{d}^3r\left|\bar{F}_\omega(\bar{r})\times\mathbf{e}_s\right|^2 \quad (29)$$

I now employ the identity $\int \mathrm{d}^3r\left|\bar{F}_\omega(\bar{r})\times\mathbf{e}_s\right|^2 = \int \mathrm{d}^3r\left|\bar{F}_\omega(\bar{r})\right|^2$ for the mode function, assuming the mode function is normalized such that this integral is unity we simplify (29) to get

$$U_{em} = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \left\{\omega^2 \frac{\mathrm{d}(\omega\varepsilon)}{\mathrm{d}\omega} + \frac{k^2}{\mu_0}\right\}\left|\bar{A}_s(\bar{k})\right|^2 \quad (30)$$

Also with this peak values, of $\bar{E}_0$ and $\bar{H}_0 = \bar{B}_0/\mu_0$ I can write time averaged magnitude of the Poynting flux as $S(\bar{k}) = \bar{E}_0\times\bar{H}_0 = (\sum_s \sqrt{2}\omega\left|\bar{A}_s(\bar{k})\right|)(\sum_s \sqrt{2}[k/\mu_0]\left|\bar{A}_s(\bar{k})\right|) = 2\varepsilon_0\omega k c^2 \sum_s \left|\bar{A}_s(\bar{k})\right|^2$. In this expression I have manipulated by using $c^2 = (\varepsilon_0\mu_0)^{-1}$ to get the Poynting flux, and the Fourier expansion as indicated above is for plane wave expansion, thus E and H are orthogonal, we get simplified Poynting or energy flux expression. This expression I will use later for momentum quantization.

I now use the relation $n_p{}^2 = (\varepsilon/\varepsilon_0)$ and $v_g = (\mathrm{d}\omega/\mathrm{d}k) = c/[n_p + \omega(\mathrm{d}n_p/\mathrm{d}\omega)]$, $v_p = c/n_p$ to rewrite (30) after a simple algebraic manipulation as

$$U_{em} = 2\varepsilon_0 \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \frac{\omega^2 n_p c}{v_g}\left|\bar{A}_s(\bar{k})\right|^2 \quad (31)$$

This electro-magnetic energy is a harmonic oscillator can be expressed as sum of energies $\hbar\omega$ of several radiation oscillators with new amplitudes as $a_s(\bar{k})$, that is

$$U_{em} = 2\varepsilon_0 \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \frac{\omega^2 n_p c}{v_g}\left|\bar{A}_s(\bar{k})\right|^2 = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \hbar\omega_s\left|a_s(\bar{k})\right|^2 \quad (32)$$

So I get by this quantization rule, a standard, the Fourier amplitude of vector potential

$$\bar{A}_s(\bar{k}) = \sqrt{\frac{\hbar v_g}{2\varepsilon_0 n_p \omega c}}\, a_s(\bar{k}) \quad (33)$$

The Hamiltonian and the vector fields are represented as

$$\mathcal{H}_{em} = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \hbar\omega_s a_s(\bar{k}) a_s^\dagger(\bar{k}) \qquad \bar{A}(\bar{r},t) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \sqrt{\frac{\hbar v_g}{2\varepsilon_0 n_p \omega c}}\, a_s(\bar{k})\mathbf{e}_s(\bar{k}) e^{\mathrm{i}\bar{k}\bullet\bar{r}} e^{-\mathrm{i}\omega t} \quad (34)$$

The quantized vector potential for free space would be, where $n_p = 1, v_g = c$, that is photon in free space, is $\bar{A}(\bar{r},t) = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sum_s \sqrt{\frac{\hbar}{2\varepsilon_0\omega}}\, a_s(\bar{k})\mathbf{e}_s(\bar{k}) e^{\mathrm{i}\bar{k}\bullet\bar{r}} e^{-\mathrm{i}\omega t}$.

In this section of Fourier expansion has considered only the positive frequency ω, and I wrote the Fourier series representation. Applying the quantization rule to the momentum density definitions (21) I get two momentums quantized as

$$\begin{aligned}
p_A = \int (\mathrm{d}^3 r)\bar{g}_A(\bar{r},t) &= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\frac{S(\bar{k})}{c^2}\frac{\bar{k}}{k} \\
&= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\frac{2\varepsilon_0 \omega k c^2 \sum_s \left|\bar{A}_s(\bar{k})\right|^2}{c^2}\frac{\bar{k}}{k} \\
&= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_s \hbar\omega \frac{v_g}{c^2}\frac{\bar{k}}{k} a_s^\dagger(\bar{k}) a_s(\bar{k}) \\
&= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_s \frac{v_g v_p}{c^2}\hbar\bar{k} a_s^\dagger(\bar{k}) a_s(\bar{k})
\end{aligned} \tag{35}$$

$$\begin{aligned}
p_M = \int (\mathrm{d}^3 r)\bar{g}_M(\bar{r},t) &= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\frac{S(\bar{k})}{{v_p}^2}\frac{\bar{k}}{k} \\
&= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\frac{2\varepsilon_0 \omega k c^2 \sum_s \left|\bar{A}_s(\bar{k})\right|^2}{{v_p}^2}\frac{\bar{k}}{k} \\
&= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_s \hbar\omega n_p^2 \frac{v_g}{c^2}\frac{\bar{k}}{k} a_s^\dagger(\bar{k}) a_s(\bar{k}) \\
&= \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\sum_s \frac{v_g}{v_p}\hbar\bar{k} a_s^\dagger(\bar{k}) a_s(\bar{k})
\end{aligned} \tag{36}$$

The $a_s^\dagger(\bar{k}) a_s(\bar{k})$ operation with complex amplitudes represent modal-number operator for photons in the s-th mode, the expressions (35) and (36) imply that a single photon in a dispersive dielectric medium has the momentums

$$p_A = \frac{v_g v_p}{c^2}\hbar\bar{k} = \frac{v_g}{n_p c}\hbar\bar{k} = \frac{v_g \hbar n_p \omega_0}{n_p c^2} = v_g\left(\frac{\hbar\omega_0}{c^2}\right) = \frac{1}{n_g}\left(\frac{\hbar\omega_0}{c}\right) \tag{37}$$

$$p_M = \frac{v_g}{v_p}\hbar\bar{k} = \frac{n_p v_g}{c}\hbar\bar{k} = \frac{n_p v_g}{c}\frac{\hbar n_p \omega_0}{c} = n_p^2 v_g\frac{\hbar\omega_0}{c^2} = \frac{{n_p}^2}{n_g}\left(\frac{\hbar\omega_0}{c}\right) \tag{38}$$

I will use these definitions and show that these photon momentums are actually mechanical in nature. The (37) and (38) are the expressions of momentum which I wrote in the previous section, the ‘wave-momentum’ I shall re-define in the following sections-imbibing the phase and group indices and velocities as they got included in the Minkowski’s and Abraham’s definitions.

14.Momentum transfer to the medium from photon

Taking clue from the above discussion let me define 'phase momentum', or 'wave-momentum' of a (single) photon packet as (39); this choice will be clear as I proceed for proof below, in next section.

$$p_c \overset{\text{def}}{=} \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}} \frac{\hbar\omega_0}{c} = N\frac{\hbar\omega_0}{c}, \qquad n_p > 0; \quad \operatorname{sgn} n_p = +1; \qquad n_p < 0; \qquad \operatorname{sgn} n_p = -1 \quad (39)$$

Well if the photon is in free space then (39) would be $p_c = \hbar\omega_0 / c$ or if it were in my chosen NRM with $n_p = -1$ and $n_g = 3$, then inside NRM this 'negative' photon has wave-momentum as $p_c = -(1/\sqrt{3})\hbar\omega_0 / c$. Well I could have chosen (39) to be as $p_c = (n_p / n_g)\hbar\omega_0 / c$ too, but the chosen square root for $|n_p n_g|$ will be explained in the next section, by total energy balance formulation.

I start my discussion of effect of my single photon entering the medium from region of free space. If the photon is totally reflected then because of the momentum conservation it transfers $2\hbar\omega_0 / c$ momentum to the medium. If the photon passes into the medium in that case momentum will be transferred to the medium at the interface surface where there will be reflection and transmission, the momentum transferred to the medium at surface is given as:

$$p^{\text{media}} = (1+R)\frac{\hbar\omega_0}{c} - Tp \quad (40)$$

Where the reflection probability R and transmission probability T with respect to free-space impedance Z_0 and impedance of medium Z are defined as (41):

$$R = \left(\frac{Z_0 - Z}{Z_0 + Z}\right)^2 \qquad T = (1-R) = \frac{4Z_0 Z}{(Z_0 + Z)^2} \quad (41)$$

Putting (41) in (40) and using p as $p = Np_0$ (with free space momentum $p_0 = \hbar\omega / c$) I get the following algebraic manipulations

$$\begin{aligned} p^{\text{media}} &= \frac{\hbar\omega_0}{c} + \left(\frac{Z_0 - Z}{Z_0 + Z}\right)^2 \frac{\hbar\omega_0}{c} - \frac{4Z_0 Z}{(Z_0 + Z)^2} N \frac{\hbar\omega_0}{c} \\ &= \frac{2\hbar\omega_0}{c} + \left(\frac{Z_0 - Z}{Z_0 + Z}\right)^2 \frac{\hbar\omega_0}{c} - \frac{4Z_0 Z}{(Z_0 + Z)^2} N \frac{\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c} \\ &= \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left[1 + \frac{4Z_0 Z}{(Z_0 + Z)^2} N - \frac{(Z_0 - Z)^2}{(Z_0 + Z)^2}\right] \\ &= \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}(1+N)\frac{4Z_0 Z}{(Z_0 + Z)^2} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}(1+N)T \end{aligned} \quad (42)$$

In the expression, (42), the values of N, are as $N = 1/n_g = v_g / c$ for Abraham's, $N = n_p^2 / n_g = v_g n_p^2 / c$ for Minkowski's and $N = \operatorname{sgn}(n_p) / \sqrt{|n_p n_g|}$, for the new wave momentums.

The (42) is expression for momentum transferred to the media. Therefore with the definition of wave-momentum as in (39) I get momentum transferred to the media, at the surface as (43)

$$p^{\text{media}} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left(1 + \frac{\text{sgn}(n_p)}{\sqrt{|n_p n_g|}}\right)T \tag{43}$$

Using the mechanical momentum definitions of (37) and (38), and doing the same algebraic manipulations I get the mechanical momentums transferred to the medium at the surface as:

$$p_{m1}^{\text{media}} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left(1 + \frac{n_p^2 v_g}{c}\right)T \qquad p_{m2}^{\text{media}} = \frac{2\hbar\omega_0}{c} - \frac{\hbar\omega_0}{c}\left(1 + \frac{v_g}{c}\right)T \tag{44}$$

I have already started speaking about Minkowski and Abraham momentum as 'mechanical' momentum; the proof will be given shortly. Well, all these momentums transferred to medium at the surface of all types (43) and (44) reduces to $2\hbar\omega_0 / c$ for a perfectly reflecting surface when $T = 0$, corresponding to change in momentum due to reflection. It is also clear that mechanical momentum transferred to medium by definition of p_{m2} will always be positive as $v_g < c$, however the definition of p_{m1}, and p_c, when used the momentum transfer to the medium at surface can be positive or negative depending on the property of media. Here I can state that in order that Minkowski's momentum that is p_{m1}^{media} be positive, the property of media with condition $[1 + (n_p^2 v_g)/c]T < 2$.

Let me take an example of ideal case whence $R = 0$ and $T = 1$, zero reflection and 100% transmission for, NRM with $n_p = -1; \quad n_g = 3; \quad v_g = c/3$. The condition for this is $\varepsilon_{r-} = \mu_{r-} = -1$, gives $Z = Z_0$, thus $R = 0$. Otherwise for making $R = 0$ the incident angle of photon at the Boundary must be at Brewster's angle. Here the photon passes into NRM with 100% probability ($T = 1$). For this NRM condition the momentum transfer associated with mechanical momentums are identical, corresponding to $(1 - v_g / c)$, that is 2/3, of the original photon mechanical momentum transferred to the media. The mechanical momentum retained by photon is (1/3) the original photon momentum. Whereas the wave-momentum transferred (43), for these values is $(\sqrt{3}+1)/\sqrt{3} = 1.577$ of the original (free-space) momentum. The wave-momentum retained by 'negative' photon, inside NRM is $(-1/\sqrt{3})$ times the original momentum, pointing in opposite direction to wave-momentum of original photon.

Well originally I can say the 'inductive' reactive energy of this wave-momentum is changed to 'capacitive' type reactive energy inside NRM. Refer figure 20 A and C. This also factually matches that inside NRM phase velocity is opposite to the energy flow or group velocity. The case where $n_p = -1; \quad n_g = 1$, (hypothetically if it exists) the wave momentum transferred (43) to the medium is twice the original wave-momentum, and no mechanical momentum gets transferred to the media, well this is case of total internal reflection. For a medium $n_p = 1$ and $n_g = 1$, the wave and mechanical momentum transferred to the medium is zero, that is all the momentum is retained by photon. The concept of corpuscular mechanical, and wave momentum with active energy and reactive energy concept will be elaborated in subsequent next sections.

15. Electromagnetic pulse a photon it's Energy-Momentum in free space

This section I will elucidate the choice of my definition of wave momentum for photon as in (20). Let a photon pulse be travelling in free space. Observer sitting on the crest and another observer sitting on the envelope, travelling in free space they will find themselves at rest with respect to each other, while the packet enters the NRM, the two observers will find that they are moving away from each other. This is this nature of wave-momentum (hidden momentum, pseudo momentum) that is generator of infinitesimal spatial translations, and the infinitesimal translations of the 'waves' corresponds to motion of its crests and troughs, and in NRM 'opposes' the direction of motion of radiation. It is for this reason the wave-momentum points in the opposite direction to the mechanical momentum inside NRM. Perhaps due to this reason one may state that photon is transformed to 'negative' photon inside NRM, its characteristics is different than that of original photon.

Consider photon travelling in free space with mechanical energy $\mathrm{E}_m = mc^2$ that is energy associated with its corpuscular part, and with phase or wave- momentum as $p = \hbar\omega_0 / c$ having wave energy as $\mathrm{E}_w = pc$, thus total energy is E, having relation as (45) below. Refer figure-17.

$$\mathrm{E}^2 = p^2c^2 + m^2c^4 \tag{45}$$

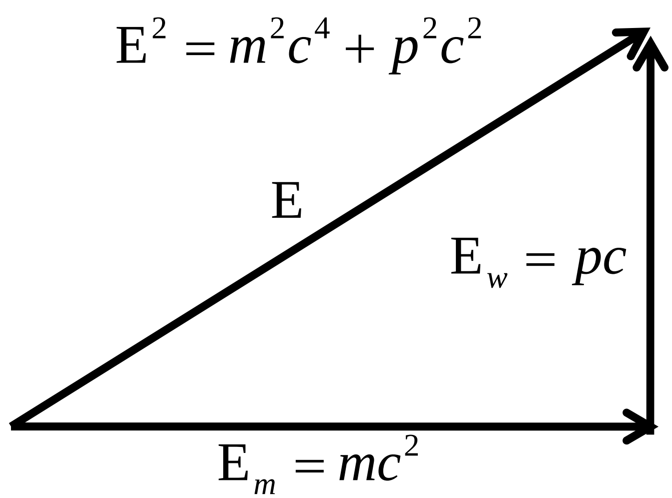


Figure-17 The Energy Diagram for corpuscular and phase (wave) energy in free space

Call v_p as phase velocity and v_g as group velocity of monochromatic EM signal travelling in the region $0 < z < (d/2)$, where the $n_p(\omega) = +1$, with relative permeability $\mu_{r+} = 1$, and relative permittivity as $\varepsilon_{r+} = 1$. Conventionally, I can write for the dispersion less ideal region that;

$$v_p v_g = c^2 \tag{46}$$

This I am assuming that $v_p = (\omega / k) = c$; $(\mathrm{d}\omega / \mathrm{d}k) = c$ in a vacuum where EM waves are travelling is ideal condition. Now I pose a question as, how am I writing (46) that is square of velocity of EM wave equal to product of the phase velocity and group velocity? The answer to that I addressed in following description.

16. What is c^2 just a multiplier or something else?

I consider a space between radiator and receiver is filled by vacuum that carrying between them electromagnetic radiation with energy E and to that I assign a linear momentum (due to wave) as $p_w = \mathrm{E}_w / c$, is also accompanying by a mass (corpuscular nature) $m = \mathrm{E}_m / c^2$.

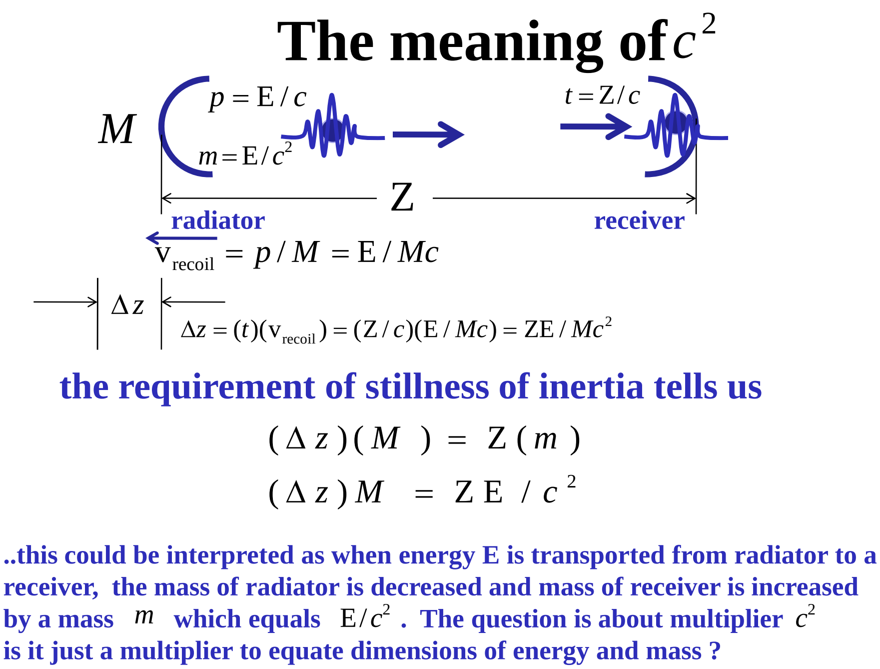


Figure-18: What is c^2 ?

The subscripts m and w distinguishes mechanical and wave energy. Well, the wave particle duality states $\mathrm{E}_m = \mathrm{E}_w = \mathrm{E}$. Really radiator after emitting wave-packet recoils with velocity $v_{\mathrm{recoil}} = p/M = \mathrm{E}/Mc$, where M is the mass of radiator. The wave packet reaches receiver sitting at distance Z after time $t = \mathrm{Z}/c$, and the radiator moves a distance $\Delta z = t v_{\mathrm{recoil}} = (\mathrm{Z}/c)(\mathrm{E}/Mc) = \mathrm{ZE}/Mc^2$. The requirement of stillness of inertia of entire system gives moment balance as $(\Delta z)M = \mathrm{ZE}/c^2$. This description could be interpreted as, when energy E is transported from radiator to receiver; the mass of radiator gets decreased, but the mass of receiver gets increased by m equal to E/c^2 !

The question is for the multiplier as c^2 , which is numerically equal to square of velocity of light in vacuum, which is used to justify the dimensions of the Energy Mass equation that is $mc^2 = \mathrm{E}$! Well can this multiplier have different physical meaning? Let me associate c_g as group velocity of the wave-packet, and then in above paragraph the expression for time will be $t = \mathrm{Z}/c_g$. Let the phase velocity be associated to crest and trough be identified as wave-velocity as c_p then wave momentum correlation will be $p_w = \mathrm{E}/c_p$, this makes the accompanying mass as $m = \sqrt{\mathrm{E}_m^2/(c_p c_g)^2}$. I have kept this expression as under root instead writing $E_m/c_p c_g$ to state that even if $c_p c_g < 0$; I do not land to a 'negative mass'. This validates my choice of multiplier $c^2 = v_p v_g$, and this could be new physical interpretation also.

Now for negative indexed material NRM (lossless and ideal case, with $n_p = -1$) I can write, an approximate relation (46), for region, where I have assumed perfect condition as $\mu_{r-} = \varepsilon_{r-} = -1$; with refractive index as $n_p(\omega_0) = -1$, and $n_g(\omega_0) \cong +1$ this enables the propagating modes inside the LHM slab, with (47). In (47) where I assume that; $v_g \cong c$, inside LHM.

$$v_p v_g \cong -c^2 \qquad (47)$$

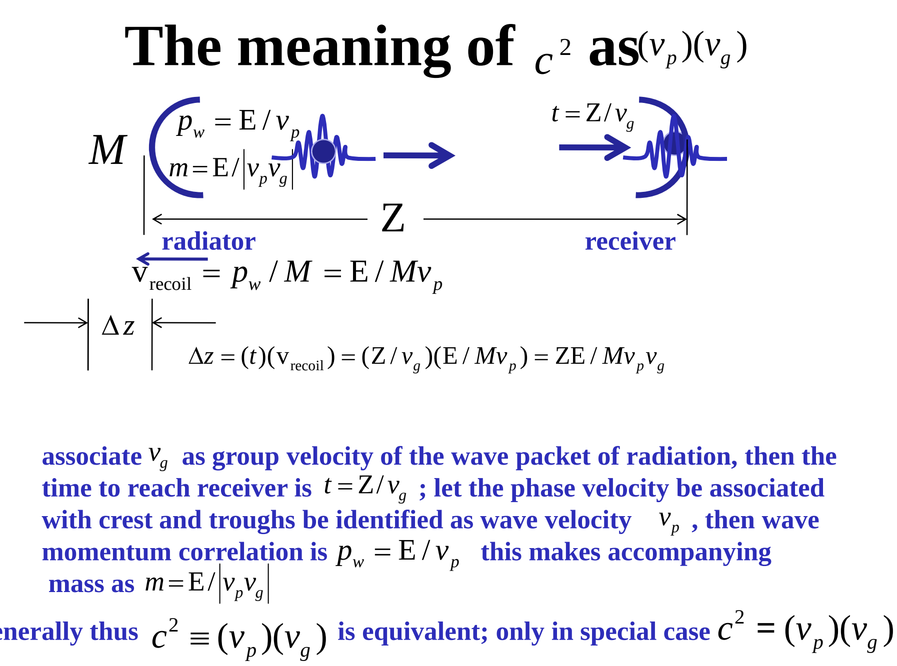


Figure 19: Can we have $c^2 = v_p v_g$

I will now highlight a special case of propagation of radiation inside wave guide where c^2 is exactly equal to $v_p v_g$. In wave guide Electric Field $\bar{E}$ must agree with all Maxwell's equations in the free space inside the guide. Along with divergence of $\bar{E}$ must be zero in the free space inside the guide since there are no charges there. That is the same thing as saying that it must satisfy the wave equation, which is in 3-D;

$$\frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}+\frac{\partial^2 E_y}{\partial^2 z^2}-\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}=0$$

The wave guide of my example guides the waves in z direction with $x-y$ plane as its cross section having y dimension ($b-\mathrm{cm}$) shorter than x dimension ($a-\mathrm{cm}$). Electric field $\bar{E}$ has only a $y-$component, and it doesn't change with y. This gives principal propagating mode with $k_x a=\pi$ as $E_y=E_0\sin(k_x x)\exp[\mathrm{i}(\omega t-k_z z)]$. In above wave equation, where E_y doesn't depend on y I can write as following

$$k_x^2 E_y+k_z^2 E_y-\frac{\omega^2}{c^2}E_y=0$$

Unless E_y is zero everywhere (which is not very interesting) this above expression is correct if

$$k_x^2+k_z^2-\frac{\omega^2}{c^2}=0$$

I have already fixed $k_x=a/\pi$, as for principal mode, so the above expression tells me that there can be waves of type of principal mode (as I have assumed) if k_z is related to the frequency ω so that same above equation gets satisfied. In other words that implies

$$k_z=\sqrt{(\omega^2/c^2)-(\pi^2/a^2)}$$

The waves I assumed and described in the wave-guide are propagated in $z-$direction with value of wave number k_z given by above expression. This wave number from above relation tells me that, for a given frequency ω the speed with which nodes (or antinodes) of waves propagate down the guide, thus giving 'phase velocity' $v_p=\omega/k_z$. The cut-off frequency of wave guide is $\omega_c=\pi c/a$ below which waves do not propagate down the guide. Using these facts and above equation I get, expression for phase velocity as the following

$$v_p=\frac{c}{\sqrt{1-(\omega_c/\omega)^2}}$$

For frequencies above cut-off where travelling waves exists the v_p in wave guide is greater than the speed of EM wave in vacuum c. Therefore, the wave guide simulates a material with refractive index less than unity. In order to know how fast the 'signals' travel, I have to calculate the speed of pulses or modulations made by the interference of waves of one frequency with one or more waves of slightly different frequencies. The speed of the envelope of such group of waves is the group velocity, it is $v_g=\mathrm{d}\omega/\mathrm{d}k$. Taking derivative of $k_z=\sqrt{(\omega^2/c^2)-(\pi^2/a^2)}$ and utilizing the definitions of cut-off frequency, $\omega_c=\pi c/a$ I get the following for 'group velocity'

$$v_g=c\sqrt{1-(\omega_c/\omega)^2}$$

This is less than the speed of EM waves in vacuum c .Therefore geometric mean of v_p and v_g in this special case is just equal to c , or $v_p v_g = c^2$

I take a detour again to show that relation $v_p v_g = c^2$ similarity with Quantum Mechanics. For a particle with any velocity (even relativistic) the momentum p and energy E are related by

$$\mathrm{E}^2 = p^2c^2 + m^2c^4$$

But in the quantum mechanics the energy is $\hbar\omega$ and the momentum is $\hbar/\lambda\!\!\!^-$, that is $\hbar k$ so I write above energy 'right triangle' expression as

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2c^2}{\hbar^2}$$

From above I get $k = \sqrt{(\omega^2/c^2) - (m^2c^2/\hbar^2)}$, which looks very similar to wave guide principal propagation equation, that is $k_z = \sqrt{(\omega^2/c^2) - (\pi^2/a^2)}$, an interesting observation! The equation of total energy, $\mathrm{E}^2 = p^2c^2 + m^2c^4$ has two parts a corpuscular part represented by $\mathrm{E}_m = mc^2$ and the wave-energy momentum part represented by $\mathrm{E}_w = pc$. These two components are represented by right angle triangle of figure-17. So I get total energy as $\mathrm{E}^2 = \mathrm{E}_m^2 + \mathrm{E}_w^2$. In the next sections I shall use this relation and see how, equivalence of c^2 that is product of v_p and v_g , is utilized see have energy and momentum transport, for a photon!

17.Energy-Momentum of photon polariton in Negative Indexed Material

This negative sign in right hand side (47) is representing that group velocity and phase velocity are 180° apart from each other, magnitude being c . Energy mass momentum expression for particle at speed of light in relativistic approach is (45), and substituting (47) I get

$$\mathrm{E}^2 = p^2c^2 + m^2c^4 = p^2(v_p v_g) + m^2(v_p v_g)^2 \tag{48}$$

This is depicted in figure-17. The corpuscular energy (momentum) is orthogonal to wave energy (momentum). Where E is total energy p is momentum of the wave which is present inside the meta-material; m is (rest) mass of the particle carrying the energy packet. Well the rest mass of photon is zero, but I can always associate a mass $m = \sqrt{\mathrm{E}_m{}^2/(c^2)^2}$, for the Electro Magnetic Energy carrying mechanical (corpuscular) energy E_m . This mechanical energy is responsible for radiation positive radiation pressure. While the other part of energy I should associate to phase wave-momentum, hidden momentum, pseudo momentum energy due to the wave nature associated with photon-movement or translation of phases 'crests' and 'trough's' motion, in the media. Manipulating (48) I get as follows:

$$\begin{aligned} \mathrm{E}^2 &= p^2(v_p v_g) + m^2(v_p v_g)^2 \\ \mathrm{E}^2 &= m^2 v_p v_g \left[v_p v_g + \frac{p^2}{m^2} \right] \end{aligned} \tag{49}$$

The equation (49) is for free-space, medium with positive phase and group velocity and both equal to c. That is $v_p = v_g = c$. Now I use (49), for NRM medium and manipulate as below:

$$\begin{aligned} \mathrm{E}^2 &= p^2c^2 + m^2c^4 \\ &= p^2(-v_pv_g) + m^2(-v_pv_g)^2 = m^2(v_pv_g)^2 - p^2(v_pv_g) \\ &= m^2\left|v_pv_g\right|\left[\left|v_pv_g\right| - \frac{p^2}{m^2}\right] \end{aligned} \tag{50}$$

Put in the equation (50) $\left|v_pv_g\right| \cong c^2$, I get

$$\mathrm{E}^2 = m^2c^2\left[(c^2) - \left(\frac{p}{m}\right)^2\right] = (m^2c^2)\left[c^2 - \frac{p^2}{m^2}\right] = m^2c^4 + (-p^2c^2) \tag{51}$$

The expression of (51) I split into two parts, the mechanical (corpuscular) energy part $(\mathrm{E}_m^2 = m^2c^4)$ and the energy transport by wave-momentum part $(\mathrm{E}_w^2 = -p^2c^2)$ part. The (51) show that particle energy is retained itself by the particle, inside NRM where the phase velocity is opposite to group velocity. In this case no (mechanical-corpuscular) energy is transferred to the NRM medium. This I have derived from the part of rest mass-energy that is the first part of expression $\mathrm{E}_m = mc^2$; meaning that corpuscular energy by photon is retained. But the intriguing question is the energy due to wave-momentum part is imaginary, inside NRM! That is equal to $\mathrm{E}_w = \text{-i}(pc)$ (considering the positive root). Note the imaginary wave energy in free space figure-17 is $E_w = +\mathrm{i}(pc)$. I can ascribe to this imaginary 'negative'- photon' a wave-momentum a value $-\hbar\omega_0 / c$. This is depicted in figure-20 (B). Compare the figure 20 A and B, the perpendicular of right angle triangle is opposite as one is positive indexed media and another is negative indexed media of refractive index (phase and group) as unity. The energy E_w associated with the 'wave-energy' is reversed, while mechanical energy remains the same. The 'reactive' nature of E_w opposite sign in both media gives the wave momentum opposite.

Now I retard the group velocity to $v_g = c/3$, and have phase reversal with phase velocity inside NRM (with $n_p = -1; \quad n_g = +3$) as $v_p = -c$ then $\left|v_pv_g\right| = c^2/3$, and put the same in (48) to get

$$\begin{aligned} \mathrm{E}^2 &= m^2\left(\frac{c^2}{3}\right)\left[\frac{c^2}{3} - \frac{p^2}{m^2}\right] \\ &= \frac{1}{9}(m^2c^4 - 3p^2c^2) = \frac{1}{9}m^2c^4 + \frac{1}{3}(-p^2c^2) \end{aligned} \tag{52}$$

Here the particle inside the NRM has less total corpuscular energy; the difference of energy has been absorbed by the media itself. Expression (52) suggests one third of the corpuscular energy $\mathrm{E}_m{}^{\mathrm{NRM}} = (1/3)mc^2$ is retained by the 'photon' inside the NRM slab, and the two thirds of its corpuscular energy are given to the slab!! Well the energy due wave momentum of the photon manifests as imaginary energy in this case as $\mathrm{E}_w{}^{\mathrm{NRM}} = \text{-i}(1/\sqrt{3})pc$, (again retaining the positive root). I ascribe to this imaginary 'negative'- photon' a wave-momentum a value $p_c^{\text{NRM}} = -(1/\sqrt{3})\hbar\omega_0 / c$. This is depicted in figure-20 C. The momentum transfer cases I have discussed in earlier section also and maps correctly with the total energy argument cases as described here. In the figure-17 it is assumed that $|\mathrm{E}_m| = |\mathrm{E}_w| = |\mathrm{E}|$.

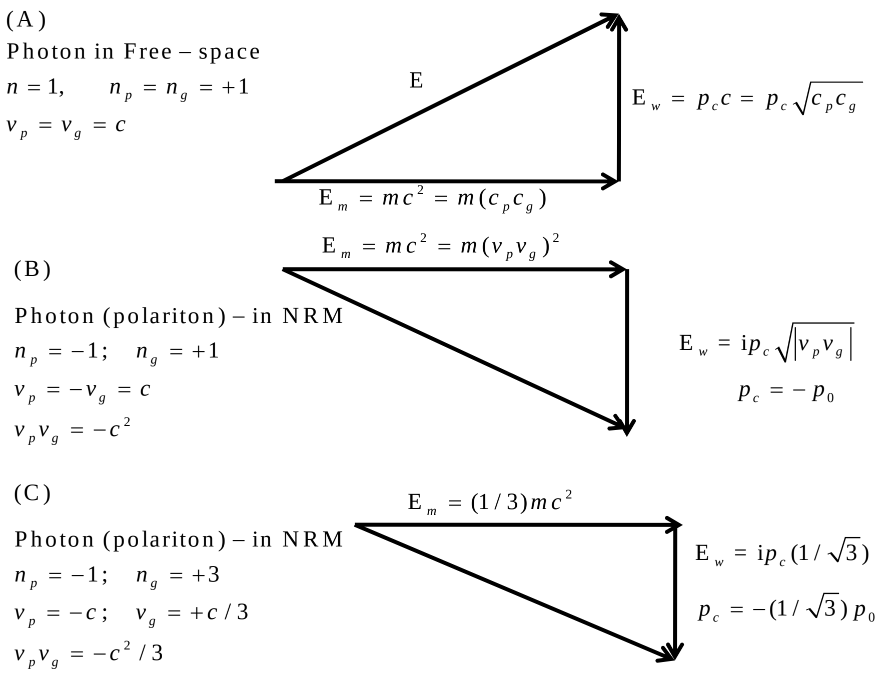


Figure-20 Energy diagrams of corpuscular and wave (phase) energy in NRM

18.A thought experiment

Refer figure-21; let me consider the length of NRM slab, as Z, with $n_p = -1$, and $n_g = 3$. The photon (polariton) is retarded in comparison to its position in absence of medium by distance z, which is

$$z = (c - v_g)\frac{\mathrm{Z}}{v_g} = (n_g - 1)\mathrm{Z} \tag{53}$$

The relativistic form of Newton's first law of motion requires that the centre-of-mass energy of a system not subjected to any external force should be stationary or in uniform motion. My medium is isolated from such external influence then the relevant total energy is sum of photon energy $\hbar\omega_0$ and the rest mass energy of the medium Mc^2, where M is mass of medium. Medium is on zero friction surface. The fact that photon has been retarded by the medium means the centre-of-mass-energy can only have been in uniform motion if the medium has itself moved to the right by a distance Δz, then the moments are (about vertical axis)

$$(\Delta z)(Mc^2) = (z)(\hbar\omega_0) \tag{54}$$

Substituting value of z from (53) I get

$$\Delta z = \frac{\hbar\omega_0 \mathrm{Z}}{Mc^2}(n_g - 1) \tag{55}$$

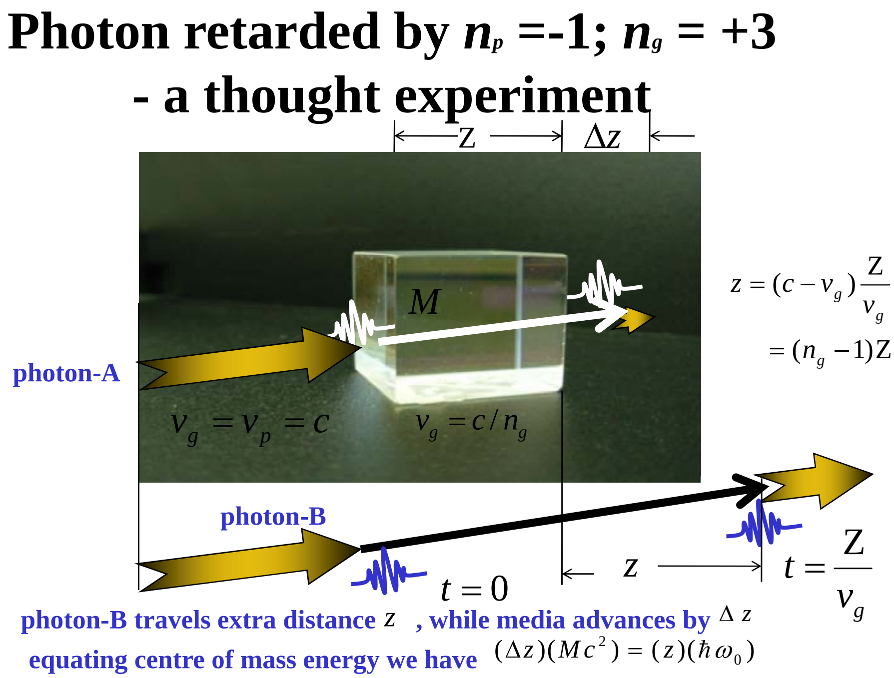


Figure-21 Thought experiment for mechanical momentum transfer

This motion can only take place if energy transfer takes place from photon whilst inside the medium. The required velocity of the medium is $v_g(\Delta z)/Z$, from which I can readily obtain momentum

$$p^{\text{medium}} = Mv_g\frac{\Delta z}{Z} = \frac{\hbar\omega_0}{c}\left(1 - \frac{v_g}{c}\right) = \frac{2}{3}\frac{\hbar\omega_0}{c} = \frac{2}{3}p_0 \tag{56}$$

Where $p_0 = \hbar\omega_0/c$ is the initial momentum of the photon in free space. Momentum conservation suggests that I ascribe the difference between the initial momentum and this medium's momentum to the photon's momentum inside the medium. From previous section the mechanical momentum of photon in this NRM would be

$$p^{\text{NRM}}{}_{m1} = n_p^2\hbar\omega_0/n_g c = v_g n_p^2\hbar\omega_0/c^2 = \frac{1}{3}\frac{\hbar\omega_0}{c} = \frac{1}{3}p_0 \tag{57}$$

$$p^{\text{NRM}}{}_{m2} = \hbar\omega_0/n_g c = v_g\hbar\omega_0/c^2 = \frac{1}{3}\frac{\hbar\omega_0}{c} = \frac{1}{3}p_0 \tag{58}$$

These mechanical momentum expressions when I use in radiation pressure for photons inside NRM of figure-5 (section-11), does make me uncomfortable with negative photon's pressure.

The wave momentum of photon-polariton inside this NRM slab is

$$p^{\text{NRM}}{}_{c} = \frac{\text{sgn}(n_p)}{\sqrt{|n_p n_g|}}\frac{\hbar\omega_0}{c} = -\frac{1}{\sqrt{3}}p_0 \tag{59}$$

The (57) (58) states that; $(1/3)$ of the mechanical momentum is retained by the 'photon' inside this NRM. This is well equating as if 1/3 of 'particular' photon corpuscular energy is retained by photon inside NRM, whereas the wave-momentum retained by photon inside NRM (59) is $-(1/\sqrt{3})$ times the original wave momentum, this part of wave-momentum I have not got from (56), that is by this thought experiment; but via reflection transmission probabilities as I derived earlier! This is because if for the thought experiment comprises of only wave momentum without any corpuscles part or mechanical components-the waves that is translation of phases carrying E_w energy just passes the medium without making mechanical displacement. What this E_w part does is exactly like phonons of 'sound' waves; that is while E_w part gives the atomic molecular vibrations and also (for EM signals) this part makes the medium polarized (magnetized)-without doing mechanical displacement. Therefore I am not making the thought experiment on this wave-momentum part. Unfortunately, unlike phonons (figure-22) the photons require mechanical E_m as well as wave part E_w of the Electromagnetic energy; whereas for 'sound' phonons no mechanical part exist only the wave part does the translation hence for sound phonon we can associate only wave momentum concept.

19. Imaginary 'Reactive Energy' and 'Wave-Momentum' inside medium

In the previous section I could balance the retardation effect stating that the corpuscular energy that comprising of mechanical photon momentum is transferred to the medium thereby inside NRM the retardation of photon takes place. What was intriguing was imaginary energy of the photon inside the NRM, what I had termed as 'reactive' energy. This reactive energy of photon inside NRM is making the waves of phases travel backward inside NRM as contrary to positive indexed material. Could I reframe the wave- momentum inside a media be it positive indexed or be it negative indexed as I have defined in (20); re written as in (60)? Well the discussion suggests yes why not!

$$p_c \overset{\text{def}}{=} \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}} \frac{\hbar\omega_0}{c} = \frac{\operatorname{sgn}(n_p)}{\sqrt{|n_p n_g|}} p_0 \tag{60}$$

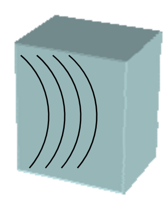
A new way to define canonical (wave pseudo, hidden) momentum inside slab, be it positive refractive indexed or negative refractive indexed system-also this agrees with what I derived from total energy balance description in the previous section. The depiction of energy diagram for NRM is in figure-20.

The electromagnetic pulse (a photon) in free space requires both the mechanical E_m corresponds to particle nature and wave part E_w orthogonal to each other. This is like transmission of electrical energy from generating station to load destination, which has active part (base of right triangle) and reactive part (perpendicular of the right angle) in figure-17. The electrical motor if we want to run it to get mechanical work, that will come from the active part of this power, and the reactive part will magnetize the motor's magnetic circuits; and no mechanical output will be carried out by this reactive part of electricity; but the total power supplied by the power station is the hypotenuse that is $\mathrm{E}^2 = \mathrm{E}_m^2 + \mathrm{E}_w^2$. That is motor draws total energy reactive as well as active from the electrical grid-to deliver 'horse-power'. The example is same as my energy diagram figure-17 and 20.

The free space carries the total energy (active plus reactive) and while it enters the medium the wave part polarizes the system then corpuscles part does work on the media. The difference I can see the NRM media requires a 'capacitor' type leading wave (reactive) energy, while positive or free space media requires an inductor type lagging wave (reactive) energy. The analogy of photon with sound wave is depicted in figure-22. Well the sound waves also can have backward wave (phase) translation, if we have material with negative Elastic Modulus, and negative density-analogous to ENG and MNG. Physical significance of this wave-momentum is to be looked at for wave nature interaction, like conservation in atomic recoil due to spontaneous emission, Doppler' effect, Cerenkov's effect and phase matching in non-linear optics. Thus in the description of thought experiment-what took part was mechanical momentum and I saw that 'single-photon' representation of Minkowski and Abraham behaved similarly in conserving centre of mass energy of the thought experiment; while this new-wave momentum is not considered in the thought experiment. The wave-interaction with NRM because of this new-wave momentum will also reverse the Doppler's effect, reverse the Cerenkov's effect and will do opposite effects to 'wave-interaction' with atoms and electrons.

Analogy with sound waves

an acoustic wave propagates in a crystal



it carries no mass along with it, thus,
since *p*=*m*v, it carries no momentum either

$\hbar k$ **is the pseudo-momentum of a phonon in the medium, or what we have called 'WAVE-MOMENTUM'**

sound, being entirely a material-based field, has only pseudo-momentum

Electromagnetic pulse or photon has both momentum and pseudo-momentum or 'wave-momentum' in the medium

Figure-22 Photon and Phonon

Thus wave momentum, pseudo momentum phase momentums or hidden momentums are infinitesimal spatial translations with crests and troughs moving forward or backward-while mechanical momentums are always positive; are 'pushing momentum' a corpuscular nature.

20. Wave Equation Explanation-and it's modified proposal for Left Handed Maxwell Systems

I can identify the motion of the photon pulse with mechanical momentum but the wave momentum corresponds rather to motion of the phase fronts. The difference is analogous to that between phase and group velocities for a wave; the phase velocity is that at which the phase font propagate, while the pulse and its associated energy propagate at group velocity, thus the phase velocity does not appear in mechanical momentum expressions used above, only v_g appears. I now resort to classical wave as photon and see if I can distinguish between positive refractive indexed media and negative refractive indexed media, through wave equation.

Total energy of system is expressed as Kinetic plus potential as

$$T+V=\frac{p^2}{2m}+V=\mathrm{E} \tag{61}$$

By putting standard Q prescriptors that is $p \to \mathrm{i}\hbar\nabla$ and $\mathrm{E} \to \mathrm{i}\hbar(\partial/\partial t)$, and in addition asking these prescriptors to operate on wave function ψ, the standard Schrodinger wave equation is obtained as

$$-\frac{\hbar^2}{2m}\nabla^2\psi+V\psi=\mathrm{i}\hbar\frac{\partial\psi}{\partial t} \tag{62}$$

The plane wave solution in vector form is $\psi = A\exp\left(-\mathrm{i}\frac{1}{\hbar}p.r\right)$

With $p=\hbar k$ as photon's momentum vector linked with its wave vector, and $\mathrm{E}=\hbar\omega$, without any potential the wave travels in straight line and we have $\mathrm{E}=p^2/2m$ (as $V=0$) and we obtain potential free wave equation as

$$\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+\mathrm{E}\psi=0 \tag{63}$$

This has two solutions

$$\psi(x)=Ae^{\mathrm{i}x\sqrt{2m\mathrm{E}/\hbar^2}}+Be^{-\mathrm{i}x\sqrt{2m\mathrm{E}/\hbar^2}} \tag{64}$$

Case for positive E propagating case

$$\psi(x)=Ae^{x\sqrt{2m\mathrm{E}/\hbar^2}}+Be^{-x\sqrt{2m\mathrm{E}/\hbar^2}} \tag{65}$$

Case for negative E bounded case. This bounded case is for surface wave happens for ENG or MNG only.

Let me take the Q prescriptors modified as

$$p \to -\mathrm{i}\hbar\exp(-\mathrm{i}\theta)\frac{\partial}{\partial x}; \qquad \mathrm{E}\to\hbar\omega; \qquad p\to\hbar[k\exp(\mathrm{i}\theta)]$$

Then I put them in potential free energy expression $\mathrm{E}=p^2/2m$, when I operate this on wave function ψ, we get a new Schrodinger equation as

$$e^{-2i\theta}\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi+\mathrm{E}\psi=0 \qquad (66)$$

Well the solutions are for this wave equation then:

$$\psi(x)=Ae^{\mathrm{i}x\sqrt{2m\mathrm{E}/\hbar^2}\exp(\mathrm{i}\theta)}+Be^{-\mathrm{i}x\sqrt{2m\mathrm{E}/\hbar^2}\exp(\mathrm{i}\theta)}=Ae^{\mathrm{i}xk\exp(\mathrm{i}\theta)}+Be^{-\mathrm{i}xk\exp(\mathrm{i}\theta)} \qquad (67)$$

(67) is case for propagating case

$$\psi(x)=Ae^{x\sqrt{2m\mathrm{E}/\hbar^2}\exp(\mathrm{i}\theta)}+Be^{-x\sqrt{2m\mathrm{E}/\hbar^2}\exp(\mathrm{i}\theta)}=Ae^{xk\exp(\mathrm{i}\theta)}+Be^{-xk\exp(\mathrm{i}\theta)} \qquad (68)$$

(68) is case for bounded case

A quick verification shall state that for $\theta=0$ I get wave equation for normal media where the Right Handed Media (RHM), while $\theta=\pi$ gives a wave propagation in Left Handed Media (LHM) with NRM. This also opens up a possibility of having a system in between RHM and LHM. This gives a wave description of RHM and LHM where in the later case the phase is opposite the energy flow can be represented as different Quantum prescriptors and different Schrodinger wave equations. At least mathematics hints so; well physical consequences are far from reality, at present for these new Q-prescriptors. The rotational component $\exp(\mathrm{i}\theta)$ may be personified as demarcation between phase velocity and group velocity and their relation to the phase and group indices, a future work! The future work shall also relate the relation between this rotational component with that of $N=\mathrm{sgn}\,n_p/\sqrt{|n_p n_g|}$ in new formulation of the canonical (wave or hidden) momentum.

21. Why for Negative Index take negative root of product of two negative quantities?

Let me first draw attention that the refractive index what I derived earlier with electrodynamics principles is a complex quantity; as explained comes because of 'damping' term in equation in motion (oscillator) of charges. This I elaborated in the section origin of refractive index, and thereafter. The basic property of media that is dielectric permittivity and magnetic permeability gives me the index of refraction, that is, $n=\sqrt{\varepsilon\mu}$ for the medium. Well in the resonance of epsilon and mu, near electric and magnetic plasma frequency, I get negative epsilon and negative mu. So I say my index of refraction is; $n<0$, a negative number. That is $n=-\sqrt{(-\varepsilon)(-\mu)}$; surprising! The mathematicians will scold me on this issue, how should I take a negative square root of product of two negative numbers?

Let me start with a naïve approach. In actual cases I have loss tangents for dielectric and for magnetic permeability; signifying amount of losses in those material. Therefore actually I can write negative values of epsilon and mu as $|\varepsilon|e^{\mathrm{i}\pi}$ and $|\mu|e^{\mathrm{i}\pi}$, which are actually complex numbers. With this representation I can go ahead and say that $n=\sqrt{|\varepsilon||\mu|e^{\mathrm{i}\pi}}=-\sqrt{\varepsilon\mu}$. A very raw explanation, that I must take negative root. Mathematicians are not satisfied. I say this is a very raw and naïve approach, but opens up possibility of further arguments, to consider negative roots.

Well, I must have second naïve argument, with respect to the 'radiation' of power, as my wave propagates. The power radiated inside any media (be it NRM) depends on the 'wave-impedance' Z, and is always shall be positive for power radiating away, that is $Z = \sqrt{\mu / \varepsilon} > 0$.

Let me write $Z = \sqrt{\mu / \varepsilon} = \sqrt{\mu\mu / \varepsilon\mu} = \mu / n > 0$; for $\mu > 0$ and $n > 0$, is normal refractive index positive. But it also tells me that for negative epsilon and mu I should have $n < 0$, the index I should have negative sign! Also writing the same as $Z = \sqrt{\mu\varepsilon / \varepsilon\varepsilon} = n / \varepsilon > 0$, tells me that I should choose the sign of refractive index as 'negative', for negative epsilon. Therefore, for a system which must radiate power away, the wave impedance indicates that for DNG material (epsilon and mu both negatives) I must choose the sign of refractive index as negative. Still mathematicians are not satisfied. However, I must state that these two naïve approaches have given me free hand to put a minus sign in front of refractive index!

The complex wave vector and wave propagation

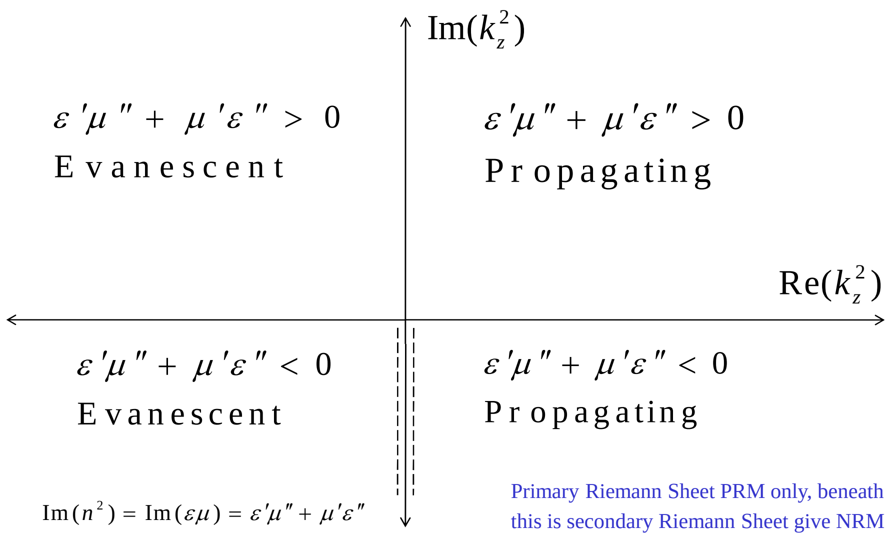


Figure-23 Complex plane for wave vector square

First let me write the Maxwell' equation for electric field as $\nabla^2 E + k^2 E = 0$, for a source free region; is nothing but a wave equation. Note here that the Maxwell's equation does not state about the sign of wave vector (wave number k). Where, $k^2 = \omega^2 \varepsilon\mu = \omega^2 n^2$, is the wave number, what physicists call, and wave vector an engineer will state as, to specify the propagation of EM waves. The solution to the Maxwell's equation is $E = E_0 e^{-\mathrm{i}k \bullet r}$, a case of plane wave as none of its variable change in the plane perpendicular to wave vector k. The wave vector is also $k^2 = k_x^2 + k_y^2 + k_z^2$, and also $k = \beta - \mathrm{i}\alpha$; with β as propagation coefficient, and α the attenuation constant. The plane wave travelling in $+z$ direction I have thus, the electric field as $E = E_0 e^{-\mathrm{i}\beta z} e^{-\alpha z}$, a decaying spatially oscillating wave in a lossy media. Well in lossless case I have $\alpha = 0$ and thus $k = \beta$. This was general revisit to concept of travelling wave, nevertheless the associated perpendicular H field does travel similarly (with magnitude divided by impedance of media), that is $Z = E / H = \sqrt{\mu / \varepsilon}$.

Let me now develop this concept of the 'complex wave vector' while radiation propagates inside the media positive or be it negatively refracting. Consider an EM wave vector $(k_x, 0, k_z)$ with propagation in $+z$ direction incident from free space, that is $(-\infty < z < 0)$. At $z = 0$, I have a semi infinite media from $(0 < z < \infty)$, other than free space with material properties as ε and μ. Due to invariance in x-direction, I preserve the k_x across the boundary. The propagating component k_z is found from rule given as following

$$k_z = \pm\sqrt{\varepsilon\mu\frac{\omega^2}{c^2} - k_x^2}; \qquad k_z^2 = \varepsilon\mu\frac{\omega^2}{c^2} - k_x^2$$

Now the physical choice is to be made for sign of the square root above. It may so happen that the second media the semi infinite one, could be propagating media if the $k_x^2 < \mathrm{Re}(\varepsilon\mu\omega^2 / c^2)$, or $\mathrm{Re}\, k_z^2 > 0$ which gives me k_z as real; or the media may be supporting decaying evanescent waves, while k_z is imaginary, that is when I have condition of $k_x^2 > \mathrm{Re}(\varepsilon\mu\omega^2 / c^2)$ or $\mathrm{Re}\, k_z^2 < 0$. This enables me to draw a plane depicting plane for k_z^2 as $k_z^2 = (\mathrm{Re}\, k_z^2) + \mathrm{i}(\mathrm{Im}\, k_z^2)$.

If I look at expression of k_z^2, I find the real and imaginary part comes from $\mathrm{Im}(\varepsilon\mu) = \mathrm{Im}\left[(\varepsilon' + \mathrm{i}\varepsilon'')(\mu' + \mathrm{i}\mu'')\right] = \varepsilon'\mu'' + \mu'\varepsilon''$, when I take these epsilon and mu as complex quantities. The four quadrants based on propagating and evanescent I have depicted in figure-23. Let me now write the dielectric permittivity and magnetic permeability of a 'lossy' medium as $\varepsilon \equiv \varepsilon' + \mathrm{i}\varepsilon''$ and $\mu \equiv \mu' + \mathrm{i}\mu''$; complex quantities indeed, with real part with prime and imaginary part as double-prime. The convention I take for absorbing media when $\varepsilon'' > 0$ and $\mu'' > 0$; for 'amplifying' media I take the imaginary parts of both epsilon and mu as less than zero (negative). I consider a 'plane wave' proportional to $e^{\mathrm{i}(nk_0)z}$, traveling in 'absorbing' media, (k_0 is free space wave vector) and do the following arithmetic

$$\begin{aligned}
n &= \pm\sqrt{\varepsilon\mu} = \pm\sqrt{(\varepsilon' + \mathrm{i}\varepsilon'')(\mu' + \mathrm{i}\mu'')} \\
&= \pm\sqrt{(\varepsilon'\mu' - \varepsilon''\mu'') + \mathrm{i}(\varepsilon'\mu'' + \mu'\varepsilon'')} \\
&\cong \pm\sqrt{\varepsilon'\mu' + \mathrm{i}(\varepsilon'\mu'' + \mu'\varepsilon'')} \\
&\cong \pm\sqrt{\varepsilon'\mu'}\left(1 + \frac{\mathrm{i}(\varepsilon'\mu'' + \mu'\varepsilon'')}{2\varepsilon'\mu'}\right) \\
&\cong \pm\left(\sqrt{\varepsilon'\mu'} + \frac{\mathrm{i}}{2}\frac{\varepsilon'\mu'' + \mu'\varepsilon''}{\sqrt{\varepsilon'\mu'}}\right)
\end{aligned}$$

I get, from above $n = \pm(\mathrm{Re}\,n + \mathrm{i}\,\mathrm{Im}\,n)$, what I got in the section of refractive index, a complex one (that one due to damped harmonic motion of charges).

If the media is absorbing, with $\varepsilon'' > 0$ and $\mu'' > 0$ also has negative epsilon and negative mu, that is $\varepsilon' < 0$, and $\mu' < 0$, I have the index for NRM as

$$n = \pm\left(\sqrt{\varepsilon'\mu'} - \frac{\mathrm{i}}{2}\frac{\varepsilon'\mu'' + \mu'\varepsilon''}{\sqrt{\varepsilon'\mu'}}\right) = \pm\mathrm{Re}\,n \mp \mathrm{i}\,\mathrm{Im}\,n$$

If I choose positive sign of above, then the plane wave in semi infinite absorbing NRM get the form as following (I have only written the complex part)

$$\exp(\mathrm{i}nk_0 z) = \exp(\mathrm{i}k_0 z\sqrt{\varepsilon'\mu'}) + \exp\left(\frac{\varepsilon'\mu'' + \mu'\varepsilon''}{2\sqrt{\varepsilon'\mu'}}\right)k_0 z$$

The above states that as the distance, z grows, the spatially oscillating plane waves 'grows' in amplitude inside negative refractive indexed material (NRM)! On contrary, waves should decay in the dispersive media; thus I cannot select positive root; this enables me to select the negative root (and only negative root) for refractive index of DNG, NRM. I write negative refractive index, for doubly negative material therefore as, with negative sign as following

$$n = -\left(\sqrt{\varepsilon'\mu'} - \frac{\mathrm{i}}{2}\frac{\varepsilon'\mu'' + \mu'\varepsilon''}{\sqrt{\varepsilon'\mu'}}\right) = -\sqrt{\varepsilon'\mu'} + \frac{\mathrm{i}}{2}\frac{\varepsilon'\mu'' + \mu'\varepsilon''}{\sqrt{\varepsilon'\mu'}}$$

Could I have satisfied mathematicians now, for selecting negative root for negative epsilon and negative mu?

Now I draw attention towards wave vector k_z, which inside a medium for travelling wave is actually nk_0. Therefore, the imaginary part of the refractive index that is mainly from the quantity $\varepsilon'\mu'' + \mu'\varepsilon''$, determines the nature of propagation (properties) in any medium (absorbing, amplifying, positively refracting or negatively refracting). This quantity is also $\mathrm{Im}(n^2) = \mathrm{Im}\left[(\varepsilon' + \mathrm{i}\varepsilon'')(\mu' + \mathrm{i}\mu'')\right] = \varepsilon'\mu'' + \mu'\varepsilon''$. The quantity $\mathrm{Im}\,k_z^2$ in a media is thus proportional to $\mathrm{Im}(n^2) = \varepsilon'\mu'' + \mu'\varepsilon''$. This I have placed in four quadrants of k_z^2 plane depicted in figure-23

For 'absorbing' medium the wave amplitudes at infinities has to disappear. For 'amplifying' medium one should carefully form the discussion. The only conditions are that evanescent waves remain decaying, propagating ones remains propagating and no 'information' can flow from infinities towards source! This ensures that the 'near field' features of a source cannot be probed at a large distance merely by putting the source in an amplifying medium.

Now if I take square root of $k_z{}^2$ plane, I will divide the plane into two Riemann sheets. The figure-23 depicts one plane, that is primary Riemann sheet, with propagating region and evanescent region of EM waves (that depends on $\mathrm{Re}(k_z{}^2)$, as I have explained earlier in this section), with regions of positive $\mathrm{Im}(k_z{}^2)$ indicating absorbing media, and negative

$\mathrm{Im}(k_z^{\ 2})$ indicating region of amplifying media. If I say that $\mathrm{Re}(\varepsilon)$ and $\mathrm{Re}(\mu)$ are positive, I can say that the primary Riemann sheet corresponds to positively refracting media (PRM). That is what I have printed in figure-23 too. Also this figure show a branch cut by 'dotted' lines which I have to unfold to take square root of this plane $k_z^{\ 2}$; rather 'inconvenient' branch cut!

This inconvenient branch cut -90^0 and 270^0 in the plane of $k_z^{\ 2}$ gives me range of arguments (angles) for primary Riemann sheet as $\angle k_z^{\ 2} = \theta$, with $-\pi/2 < \theta < 3\pi/2$, beneath this sheet the secondary Riemann sheet with range of angles as $\angle k_z^{\ 2} = \theta$, with $3\pi/2 < \theta < 7\pi/2$. If I take square root of this plane of figure-23, I write the following

$$k_z = \pm\sqrt{k_z^2} = \pm\sqrt{\left|k_z^2\right| e^{\mathrm{i}\theta}} = \begin{cases} \sqrt{k_z^2}\, e^{\mathrm{i}\theta/2} \\ \sqrt{k_z^2}\, e^{\mathrm{i}\theta/2 + \mathrm{i}\pi} \end{cases}$$

The second Riemann sheet corresponds to NRM with angles of $3\pi/4 < \angle k_z < 7\pi/4$. This is depicted in figure-24

Complex wave vectors in two Riemann sheets first sheet PRM second sheet NRM

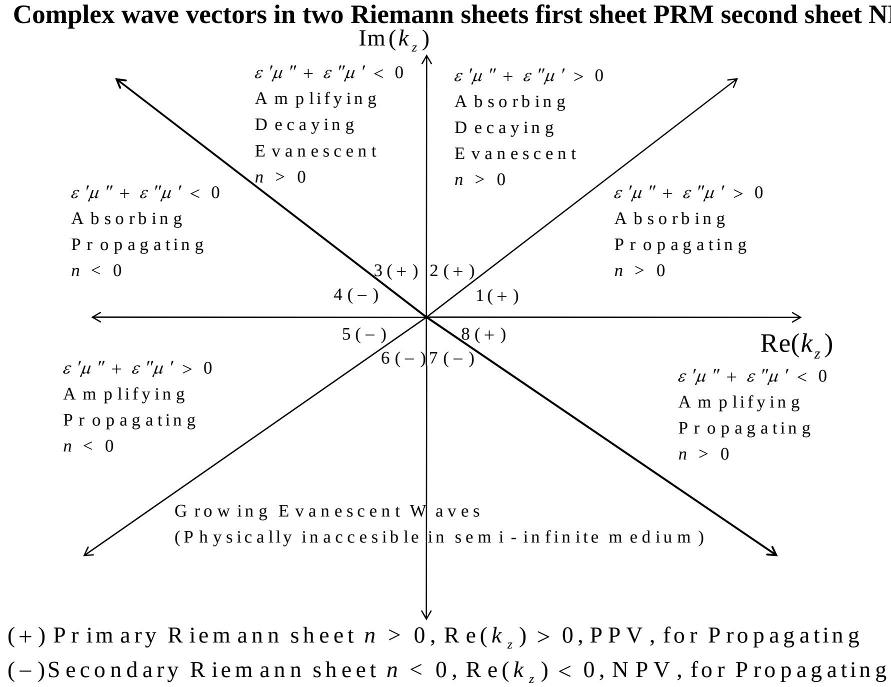


Figure-24: Taking square root of plane of $k_z^{\ 2}$ to get plane of k_z

I now give explanation for the two Riemann sheet zones, as it appeared in figure-24. Region 1 and 8 corresponds to propagating waves in (positively refracting media) PRM that are absorbing or amplifying respectively. Region 6 and 7 corresponds to growing evanescent waves that build up at infinities, which are unphysical in the semi-infinite medium (though important in truncated NRM slab!). Decaying evanescent waves fall with region 2 if

$$\mathrm{Im}(n^2) = \mathrm{Im}(\varepsilon\mu) = \varepsilon'\mu'' + \varepsilon''\mu' > 0$$

and in the region 3 if $\varepsilon'\mu''+\varepsilon''\mu'<0$. Note the Poynting vector points away from the source (interface) if medium are absorbing overall and actually towards the source (interface) if media is amplifying overall. For the case of evanescent waves in amplifying media my choice of Poynting vector those points towards the source (interface in this case). This however does not violate the causality as the Poynting vector energy flow decays exponentially to zero at infinity and no information flows from infinity. The counter-intuitive behavior does not imply that source has turned into sink-rather indicates that there would be large (infinitely large unsaturated linear gain) accumulation of energy density (intense field enhancements) near a source. Now propagating waves in ENG MNG, simultaneously, that is DNG in region 4 and 5 depending on whether $\varepsilon'\mu''+\varepsilon''\mu'<0$ or $\varepsilon'\mu''+\varepsilon''\mu'>0$ respectively corresponding to absorbing and amplifying media respectively. In both cases negative square root need be chosen, this is start of second Riemann sheet. In case of normal incidence the sign of wave vector (k_z) and sign of index of refraction (n), are same. The quantity $\varepsilon'\mu''+\varepsilon''\mu'$ determines the energy flow. In dissipative media $\mathrm{Im}(k_z)<0$ for propagating waves, which reduces to $\mathrm{Im}(n)>0$ for normal incidence. Thus one can reasonable talk of Negative Phase Velocity (NPV) rather Negative Group Velocity (NGV). In PRM I have positive phase velocity (PPV) and positive group velocity. The definition of absorbing and amplifying gets reversed in the secondary Riemann sheet, which I summarized in figure-25.

For NRM in secondary Riemann sheet absorbing and amplifying reversed

$$n \cong \pm[(\varepsilon'\mu')^{1/2} + \{i/2\}\{\varepsilon'\mu'' + \mu'\varepsilon''\}/(\varepsilon'\mu')^{1/2}] = \pm[\mathrm{Re}(n) + i\,\mathrm{Im}(n)]$$

$$n = \begin{cases} \mathrm{Re}(n) + i\,\mathrm{Im}(n) \\ -\mathrm{Re}(n) - i\,\mathrm{Im}(n) \end{cases}$$

$$\mathrm{Im}(n) \cong \varepsilon'\mu'' + \mu'\varepsilon''$$

$$\mathrm{Im}(k_z) \cong \varepsilon'\mu'' + \varepsilon''\mu' = \begin{cases} >0 & \text{dissipative PRM} \\ <0 & \text{amplifying PRM} \end{cases}$$

$$\mathrm{Im}(k_z) \cong \varepsilon'\mu'' + \varepsilon''\mu' = \begin{cases} <0 & \text{dissipative NRM} \\ >0 & \text{amplifying NRM} \end{cases}$$

For dissipative , absorbing NRM with ENG and MNG simultaneously, we get , $\mathrm{Re}(n)<0$, $\mathrm{Re}(k_z)<0$ $\varepsilon'\mu''+\varepsilon''\mu'<0$ $\mathrm{Im}(k_z)<0$ NPV , opposite definitions of absorbing and amplifying as compared to PRM. Thus we get $\mathrm{Im}(n)>0$

Figure 25: The secondary Riemann sheet details for NRM

When I plot the $\omega-k$ or $\omega-\beta$ diagram ($k=\beta-i\alpha$), the dispersion diagram, the anomalous dispersion I call as negative slope indicating $\mathrm{d}\omega/\mathrm{d}k<0$ or $\mathrm{d}\omega/\mathrm{d}\beta<0$, gives me to talk about negative group velocity (NGV). Well, I must say the anomalous dispersion as observed in figure-12, states about negative group delay or feel of negative group velocity-since the diagram is made in first quadrant (k and β positives). The above on plane of k_z (figure-24) tells me that in NRM, the $k_z < 0$, to have NPV. Thus this figure-12 giving idea of NGV and Negative Delay is actually in the third quadrant where k and β are negative-take image of this diagram about vertical axis thus to settle this anomaly.

Conclusion

Can I conclude? I cannot, and also what I discussed I think and opine are ad hoc schemes. Though I tried to debate the controversy regarding the 'photon's' momentum a corpuscular and wave nature, tried to explain the reactive and active energy of the same; and especially inside media and media with negative index of refraction. At least I cannot put value of index of refraction as negative in existing physics formulas! I need careful attention when there is media with refractive index negative, and reformulate the existing physics formulas. Experimental realization of negative index of refraction has as a result raised important questions about the validity of this negative value in well known formulas of physics. The question of corpuscular energy transport inside negative indexed material, formation of reactive (imaginary) energy inside the negative indexed substances, well the character of photon pulse especially its momentum (corpuscular and wave) is addressed along with duality of particle-wave nature of photon. Few new concepts regarding new wave-momentum inside slab and reactive energy inside negative indexed material, and new generalized wave equation is proposed; to meet the future theoretical advances on these realized negative indexed materials. I have several light years to go!

Acknowledgement

This work is supported fully by Board of Research in Nuclear Science (BRNS) Department of Atomic Energy (DAE): the project is called **LEFT HANDED MAXWELL SYSTEMS**. I acknowledge the encouragement received by **Sri B B Biswas** Head Reactor Control Division, **Dr B N Jagtap** Head AMP Div. BARC, **Sri G P Srivastava** Dir. E & I Group, **Dr R K Sinha** Director BARC, and **Dr S Banerjee** Chairman AEC; for inspiring me to do counterintuitive science along with Emeritus Professor of Univ. of Jadavpur, **Dr Deepak Ranjan Poddar**.
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Prof. Subal Kar (subalkar@hotmail.com) Institute of Radio Physics & Electronics (IRPE) University of Calcutta, Kolkata, I acknowledge them all.

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Well, I have several light years to go before I know what exactly a photon is!