Plasmoid Illumination by Electrostatic Resonance formed by Negative Permittivity & Imaginary Refractive Index Media in Microwave Field

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Abstract

In this paper electrodynamics of plasmoid lighting is explained, which we are generating via creation of localized hot spot by application of microwave energy. The microwave is applied via co-axial applicator, the monopole antenna; and the 'near field' of the radiation causes local hot spots and thereby thermal runaway causing ejection of small particles from the base substrate, within 5-10 seconds. These particles interacts with the Electromagnetic field, and due to electrostatic resonances occurring from negative dielectric permittivity (thereby giving imaginary refractive index) of these small particles, giant electromagnetic energy fields are locally accumulated; making local discharge thus giving illuminated plasmoid ball. This paper explains the formation of giant electric field at electrostatic resonance, which is primary cause of localized discharge thus plasmoid illumination.

Key words:
Electrostatic resonance, negative permittivity, Fredholm's integral equation, meta-material

1. Introduction

The enigmatic natural phenomena usually occur after a lightning strike that may lead to plasma formation and a source of considerable electromagnetic (EM) radiation. If the frequency of this spectrum of EM radiation is such that the di-electric permittivity of the formed plasma is negative (equivalently the refractive index an imaginary value) then electrostatic resonances may occur. The nucleation of electrostatic resonance and the spatial growth of resonance region may be facilitated by scale invariance resonance frequencies. Electrostatic resonances may produce considerable accumulation of EM energy, giving 'giant' local fields, which may visually manifest as plasmoid lighting. This is plausible explanation of the 'fire-balls' observed in nature. In our experiment of 'material processing with microwave', we are using a near field co-axial applicator (Figure-1) through a monopole antenna, to an object (Bone, cement, concrete, thin copper sheet, thin aluminum sheet, glass etc), in order to melt via dielectric heating and thermal runaway principle; through 'resonating near field' region of monopole. The molten object (which takes 5-10 second, after application of microwave power of 150-200 Watts radiating at 2.54 G Hz) when pierced by the monopole itself in order to drill the object; gets exposed to microwave radiation and absorbs more microwave to form plasma, which behaves exactly as natural fire-ball. The difference is the plasmoid ball in our case carries the original substrate material composites (bone, glass silica, aluminum, copper glass particles) where as the atmospheric fire-ball contains composites of air breakdown. In this paper the electrodynamics aspect of plasmoid lighting is explained, as the particles which comes out from molten substrate behaves as negative epsilon material, accumulating large surface charges (surface Plasmon polaritons SPP), the phenomena of surface Plasmon resonances, causes local giant fields, making local discharges; causing illumination.

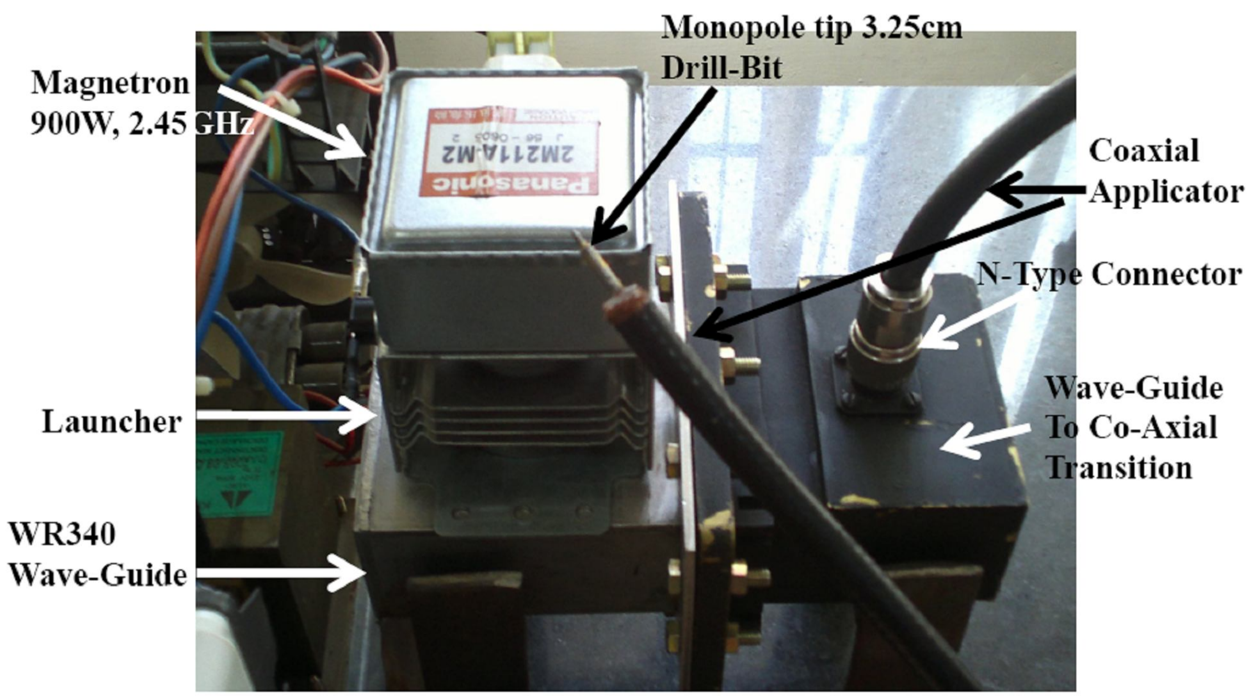

Figure-1: The set up of microwave near field applicator with monopole (IIT Roorkee)

Figure-2 and figure-3 gives the result of drilling of wet bone and aluminum sheets via co-axial near field applicator. The same has been tried on glass, concrete, cement, wood, and while doing the experiment, illuminated plasmoid formation is observed, depicted in figure-4. We shall be dealing with electrodynamics aspect of the plasmoid illumination aspect and will give plausible reasoning. Why we are calling the plasmoid instead a plasma ball is, due to seemingly lesser number density of sparsely placed particles which accumulated surface charges; as compared to large number density and free charges in the case of real plasma. This plasmoid may be a case of dusty plasma.

Figure-2 Bone drilled with co-axial applicator near field (IIT-Roorkee)

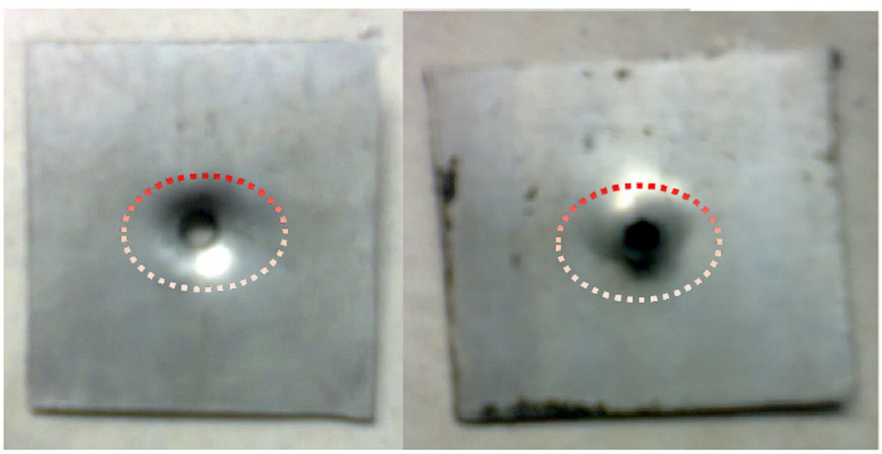

Figure-3 Aluminum plate 0.7mm drilled via near field (IIT-Roorkee)

2. Review of wave mechanism and boundary conditions

2.1 . Propagating Electro-Magnetic wave

In this part we review the fundamentals of plane-wave propagation and boundary conditions. Let an EM radiation (TE or TM) travel in media number-1, from $-\infty < z < 0$, in $+z$ direction with media properties dielectric permittivity and permeability as ε_1, μ_1; encounters a boundary at $z = 0$. The media number-2 extends from $0 < z < \infty$, with properties ε_2 and μ_2. The boundary conditions at the interface $z = 0$, where t is tangential component, and n is the normal component at the boundary. The following expression (1) formulation ignores surface currents and surface charges. The expression (1) states that tangential components of electric and magnetic field and normal components of electric and magnetic flux densities are continuous at the boundary.

$$\begin{aligned} H_{1t} &= H_{2t} \\ E_{1t} &= E_{2t} \\ \varepsilon_1 E_{1n} &= \varepsilon_2 E_{2n} \\ \mu_1 H_{1n} &= \mu_2 H_{2n} \end{aligned} \tag{1}$$

Figure-4: Formation of plasmoid and its illumination

Let us take a case of TM polarization, the magnetic field's $y-$component is:

$$H_{y1} = A\mathrm{e}^{-j(k_{z1}z+k_{x1}x)} + B\mathrm{e}^{j(k_{z1}z-k_{x1}x)} \tag{2}$$

The number A is incident and B is reflected amplitude in medium-1. The imaginary number $j=\sqrt{-1}$; physicist calls it iota $i=\sqrt{-1}$. The wave vectors satisfy the following condition in the medium-1, [1]-[10], [14], [19], [27], [29], [35], and [38].

$$k_{z1}{}^2 + k_{x1}{}^2 = k_1{}^2 = \omega^2 \mu_1 \varepsilon_1 \tag{3}$$

Applying Maxwell's curl condition, we get Electric field in medium-1 as:

$$E_{x1} = -\frac{1}{j\omega\varepsilon_1}\frac{\partial H_{y1}}{\partial z} = \frac{k_{z1}}{\omega\varepsilon_1}\left[A\mathrm{e}^{-j(k_{z1}z+k_{x1}x)} - B\mathrm{e}^{j(k_{z1}z-k_{x1}x)}\right] \tag{4}$$

In medium-2 only transmitted component appears with amplitude C, we write $y-$component of magnetic field as:

$$H_{y2} = C\mathrm{e}^{-j(k_{z2}z+k_{x2}x)} \tag{5}$$

The curl of which is:

$$E_{x2} = -\frac{1}{j\omega\varepsilon_2}\frac{\partial H_{y2}}{\partial z} = \frac{k_{z2}}{\omega\varepsilon_2}C\mathrm{e}^{-j(k_{z2}z+k_{x2}x)} \tag{6}$$

At the boundary $z=0$ matching is possible if the fields vary in the same manner in the $x-$direction, which is possible if $k_{x1}=k_{x2}$, meaning phase velocity along $x-$direction must be same on both sides of the boundary. It follows from geometry that $k_{x1}=k_1\sin\theta_1$, where θ_1 is the incident angle, and $k_{x2}=k_2\sin\theta_2$, where, θ_2 is angle of refraction, and $k_1=\omega\sqrt{\varepsilon_1\mu_1}$, $k_2=\omega\sqrt{\varepsilon_2\mu_2}$ and $\omega=2\pi f$, the angular temporal frequency, in radian/second. From which we get usual Snell's law that is (7), with $n_{1,2}$ indicating refractive index of the medium, 1 and 2.

$$\omega\sqrt{\mu_1\varepsilon_1}\sin\theta_1=\omega\sqrt{\mu_2\varepsilon_2}\sin\theta_2$$
$$n=\frac{n_1}{n_2}=\frac{\sin\theta_1}{\sin\theta_2}=\sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} \qquad (7)$$

Further matching conditions, from tangential field components at the boundary are $A+B=C$, from the condition $H_{y1}=H_{y2}$, at $z=0$, and $(k_{z1}/\varepsilon_1)(A-B)=(k_{z2}/\varepsilon_2)C$ from condition $E_{x1}=E_{x2}$ from $z=0$. Here we can define reflection and transmission coefficients, as $R=B/A=(1-\zeta_e)/(1+\zeta_e)$, $T=C/A=2/(1+\zeta_e)$ where, $\zeta_e=(\varepsilon_1k_{z2})/(\varepsilon_2k_{z1})$. If instead of TM polarization, it were TE polarized wave, then with incident Electric Field in $y-$ direction, then above derivation remains same, but ζ_e, needs be replaced with ζ_m, with $\zeta_m=(\mu_2k_{z1})/(\mu_1k_{z2})$ [1]-[10], [14], [19], [27], [29], [35], and [38].

2.2. Bounded Wave (Eigen-Solution Resonance)

Let us now explore another aspect of wave mechanics, namely that the wave can stick to the boundary. When it does so it is called the 'surface' wave, [1]-[10], [14], [19], [27], [29], [35], and [38]. Assuming again TM wave as in earlier section, the new feature is that, waves can propagate along the boundary, but their amplitudes decline exponentially away from $z=0$ the boundary, (in $z-$direction). This is "bounded wave", can happen only if k_x is sufficiently high, so that, k_{z1},k_{z2} are imaginary numbers and thus replaced by $-j\kappa_1,-j\kappa_2$ where κ_1,κ_1 are real (positive). Hence the propagation coefficient in the $x-$direction is obtained as: $k_x^{\,2}=k_1^{\,2}+\kappa_1^{\,2}=k_2^{\,2}+\kappa_2^{\,2}$.We are looking for a wave that can exist on surface without an input. In more pretentious language we are looking for "eigen-solutions" (resonances). That is in terms of (2) and (5) with input $A=0$ the components B, and C exists and are finite numbers; that is amplitude of wave that declines away from boundary in negative z-direction, in medium-1 (B); and decays exponentially from the boundary in positive $z-$direction in medium-2, (C) respectively, (without input that is A). This is eigen-solution (resonance). In terms of meta-materials we call this process as Surface Plasmon Polariton (SPP), SPP-Wave, Surface Resonance, or Surface Plasmon resonance; [1]-[10], [14], [19], [27], [29], [35], and [38].

The equation of magnetic and electric field in medium 1 and 2, for the SPP or resonance case is

$$H_{y1}=Be^{\kappa_1z}\mathrm{e}^{-jk_xx} \qquad H_{y2}=Ce^{-\kappa_2z}\mathrm{e}^{-jk_xx} \qquad (8)$$

Apply curl expression to get, electric field in x-direction

$$E_{x1}=\frac{-\kappa_1}{j\omega\varepsilon_1}B\mathrm{e}^{\kappa_1z}\mathrm{e}^{-jk_xx} \qquad E_{x2}=\frac{\kappa_2}{j\omega\varepsilon_2}C\mathrm{e}^{-\kappa_2z}\mathrm{e}^{-jk_xx} \qquad (9)$$

Apply again curl expressions to get electric field in $z-$direction

$$E_{z1}=\frac{-k_x}{\omega\varepsilon_1}B\mathrm{e}^{\kappa_1z}\mathrm{e}^{-jk_xx} \qquad E_{z2}=\frac{-k_x}{\omega\varepsilon_2}C\mathrm{e}^{-\kappa_2z}\mathrm{e}^{-jk_xx} \qquad (10)$$

2.3. Resonance (Eigen-Solutions) are satisfied for Negative Dielectric Permittivity and Imaginary Refractive Index

Here we see that if one media is having negative dielectric permittivity, the bounded waves appear as surface Plasmon resonance. In order to satisfy boundary conditions of the eigen-solution that is bounded wave sticking to the surface, we need to match H_y and E_x at $z=0$, which gives; $B=C$ thereby we obtain

$$-\frac{\kappa_1}{\varepsilon_1}=\frac{\kappa_2}{\varepsilon_2} \tag{11}$$

Note that we have taken, κ_1,κ_2 both real positive. Therefore the condition (11) is satisfied only if; the dielectric permittivity of medium-2 is negative, that is

$$\varepsilon_2<0 \tag{12}$$

This is the condition of existence of electric surface waves or electric resonance. The (12) is also equivalent to having $\zeta_e=-1$ from $\zeta_e=(\varepsilon_1 k_{z2})/(\varepsilon_2 k_{z1})$.

Now we substitute, in $k_x^{\,2}=k_1^{\,2}+\kappa_1^{\,2}=k_2^{\,2}+\kappa_2^{\,2}$, the value of κ_1 and κ_2 above to have relation in k_x and ω, by using the free space expressions $k_1=\varepsilon_{r1}\mu_{r1}k_0$, $k_2=\varepsilon_{r2}\mu_{r2}k_0$, $\mu_{r1}=\mu_{r2}=1$, $k_0=\omega/c$; to get dispersion of SPP that is $k_x=(\omega/c)\sqrt{(\varepsilon_{r1}\varepsilon_{r2})/(\varepsilon_{r1}+\varepsilon_{r2})}$. The SPP wave is travelling along the surface, and like travelling wave the wave-vector in k_x, in $x-$direction should be real positive (note this is bounded in $z-$direction). With $\varepsilon_{r2}<0$ we may write

$$\varepsilon_{r1}\varepsilon_{r2}<0 \qquad\qquad \varepsilon_{r1}+\varepsilon_{r2}<0 \tag{13}$$

The (13) is more general to (12), where relative permeability is used $\varepsilon=\varepsilon_r\varepsilon_0$ $\mu=\mu_r\mu_0$, where ε_0 and μ_0 are free space values. Write the negative relative permittivity as $\varepsilon_{r2}=-n^2$, where n indicates refractive index (imaginary refractive index). Then in terms of refractive index we get, $k_2^{\,2}=\mu_{r2}\varepsilon_{r2}k_0^2=-n^2k_0^2$ gives $\kappa_1=\sqrt{k_x^2-k_0^2}$, and $\kappa_2=\sqrt{k_x^2-k_2^2}=\sqrt{k_x^2+(nk_0)^2}$.

3. Appearance of Giant Fields at Electrostatic Resonance with negative dielectric permittivity (imaginary refractive index)

Having seen the basics of surface waves resonance phenomena, in earlier section we go further with electrodynamics of imaginary refractive index. Let us consider metal, a flat metal surface is almost a perfect reflector of electromagnetic waves in visible region, and application of metal films as mirrors has a long history. Metal films become 'semi-transparent' at 'resonant' wavelengths (frequencies), allowing the excitation of EM waves propagating on the surface. Consider a thin metal film surface, in the optical and IR spectral ranges the collective excitation of the free electron density coupled to EM fields result in SPP. SPP can be excited when we have negative dielectric permittivity, as described in previous section. The metal film in visible and IR ranges has $\varepsilon_m=\varepsilon_m'-j\varepsilon_m''$, with $\varepsilon_m'<0$ negative, and loss factor very less $(\varepsilon_m''/\varepsilon_m')\ll 1\sim 0$. Let the medium-1 is free space at $(-\infty<z<0)$, with $\varepsilon_{r1}=\mu_{r1}=1$ and the metal be the medium-2 placed at $z=0$, and the property for $0<z<\infty$ is $\varepsilon_{r2}=-n^2<0$, $\mu_{r2}=1$. The magnetic field decaying in z direction and travelling in x direction for a bounded wave (SPP) is

$$H_{y1} = H_0\, \mathrm{e}^{(jk_x x + k_{z1} z)}; \quad z < 0 \qquad H_{y2} = H_0\, \mathrm{e}^{(jk_x x - k_{z2} z)}; \quad z > 0 \tag{14}$$

The expression (14) has SPP wave vector as k_x where $k_{z1} = \sqrt{k_x^2 - k_1^2} = \sqrt{k_x^2 - k_0^2}$ and $k_{z2} = \sqrt{k_x^2 - k_2^2} = \sqrt{k_x^2 - \varepsilon_m k_0^2} = \sqrt{k_x^2 + (nk_0)^2}$, with $k_0 = \omega / c$ the free space wave vector and $n = \sqrt{-\varepsilon_m} = j\sqrt{\varepsilon_m}$. The continuity of tangential component of magnetic field at the boundary $z = 0$, are satisfied as $H_{y1}(x, z = 0) = H_{y2}(x, z = 0)$. The Electric field is found from Maxwell's expression $\mathrm{curl} H = -jk\varepsilon E$ with $\varepsilon = 1$ for $z < 0$, and $\varepsilon = \varepsilon_m = -n^2$ for $z > 0$ has components E_{x1} and E_{x2}, the continuity requirements for these tangential components gives $E_{x1} = E_{x2}$, resulting in following

$$\frac{\partial H_{y1}}{\partial z} = -\frac{1}{n^2}\frac{\partial H_{y2}}{\partial z}; \quad \text{at} \quad z = 0 \tag{15}$$

For $n > 1$ this equation is satisfied and leads relation of wave vector of SPP and refractive index ($\varepsilon_m < 0$), that is $k_{SPP} = k_x = (k_0 n) / \sqrt{n^2 - 1}$. Note for $n > 1$ the wave vector for SPP is real so that magnetic field H, decays exponentially in metal and free space. The component perpendicular to propagation of SPP the surface waves that is E_z, takes the following values at the interface, $E_z(0-) = -(k_x / k_0) H_0\, \mathrm{e}^{(jk_x x)}$ on the free space side ($z < 0$); and
$E_z(0+) = -(k_x / k_{z2}) H_0\, \mathrm{e}^{(jk_x x)} = (k_x / n^2 k_0) H_0\, \mathrm{e}^{(jk_x x)} \neq E_z(0-)$
on the metal side ($z > 0$).

These are obtained from $\mathrm{curl} H = -jk\varepsilon E$ at $z = 0^-$ and $z = 0^+$ sides. The discontinuity of the electric filed as seen the normal components $E_z(0^+) \neq E_z(0^-)$ manifests as surface charge density, which propagates together with electric and magnetic field along the metal film surface

$$\sigma(x) = \frac{1}{4\pi}\left[E_z(0+) - E_z(0-)\right] = \frac{(1 + n^2)}{4\pi n \sqrt{n^2 - 1}} H_0\, \mathrm{e}^{(jk_x x)} \tag{16}$$

The surface wave which consists of EM field coupled to surface charge propagates by rearrangement of charge density, not surprising that its speed is always less than c, that is $c_x = \omega / k_x = c\sqrt{n^2 - 1} / n < c$.

When the refractive index has a property of approaching unity $-n^2 = \varepsilon_m \to -1$, the SPP speed approaches zero, the surface wave SPP stops on the surface. In this ideal case, the surface charge diverges (16), as $(n^2 - 1)^{-1/2}$ blows up; so does the normal component of Electric field-gives production of 'giant fields' at electrostatic resonance.

In this section we have justified that permittivity value negative, for infinite sheet will have resonances at its surface as surface charge density, which can be very high and thus it's associated transverse electric field are giant fields. We can have any arbitrary shaped particle with negative permittivity that in the uniform electric field will have resonances giving surface charge density as well as associated giant fields. Now we generalize this above concept in following section.

4. Generalization of Electrostatic Resonance theory

Resonant behavior of dielectric object occurs at certain frequency for which object has dielectric permittivity negative value $\varepsilon < 0$, as described in previous sections, and also free space wavelengths of EM radiation is very large compared to the objects dimensions. Say metal bead are embedded in dielectric substance will have effective negative permittivity and the plasma frequency will scale down from visible IR regions (for metals) to Microwave ranges. This is essence of creation of artificial structures of meta-material which behave as negative epsilon meta-material at the microwave region [1]-[10], [14], [19], [27], [29], [35], and [38]. The free space wavelength of the EM radiation needs be very large compared to the dimensions of the object and its lattice dimensions, makes application effective medium theory possible [1]-[10], [14], [19], [27], [29], [35], and [38], and from electrodynamics point of view quasi static [30] approximations are valid, this also suggests the resonance are electrostatic in nature, as they appear at specific negative values of dielectric permittivity for which source free electrostatic field may exist. Plausibly the electrostatic based resonance mechanism is explanation of illumination of the plasmoid. The idea of giant fields associated with resonances has been introduced in the earlier section, now we generalize the concept of electrostatic resonance.

We are interested in $\varepsilon < 0$ for which source free electrostatic field may exist. This source free field is curl free and divergence free inside volume of particle Γ^+ and outside the volume of particle Γ^- of the dielectric object (nm sized particles ejected from solid substrate after application of near field of wave length about 12 cm for 2.45 G Hz), refer figure-5. The potential is continuous across the boundary S of the object, while normal components of electric field follows (1), on the surface boundary S.

$$\varepsilon E_n^+ = \varepsilon_0 E_n^- \tag{17}$$

The electric potential of the source free electric field can be represented as an electric potential of single layer of charge distribution over surface S [30].

$$V(P) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma(Q)}{r_{QP}} \mathrm{d}S_Q \tag{18}$$

In other words, a single layer of electric charge (with surface charge density σ) on S creates the same electric field in the free space as source free electric field may exist in presence of dielectric object [30]. It is apparent that electric field of the surface charge σ is curl free and divergence free in Γ^- and Γ^+ regions, and potential is continuous across S. The normal component of Electric field of a single layer potential is given by [30], [39] and [40].

We now simply assume that the particle in the figure-5 is convex shaped, ellipsoid, spherical etc. There will be surface charge at the surface boundary, when the particle is placed in source free electric field. The surface charge and normal component of electric field for a flat surface and its resonance is described by expression (16). Thus our problem is to determine the Electric fields at point P, in figure-5, normal to the surface. Let us first assume that the particle of figure-5 is having surface charge density $\sigma(P)$ at point P, and at other points such as Q, the charge density is $\sigma(Q)$. In other words let this surface charge be the function of location of points at the surface S, for a very general case.

We can view the electric field at the point P, as electric field in an infinitesimal hole at P as sum of the fields from the rest of the shell plus the field due to an infinitesimal patch with surface charge density equal but opposite to patch cut-out. In other words remove a small patch at P, which then will have surface charge density opposite in sign that is $-\sigma(P)$. This patch is so small that we can consider as plane surface instead of curved surface; and the standard pill-box calculation applied for Gauss law for a surface of charge density $-\sigma(P)$ coulomb/m^2 will give normal electric field from the surface as $-\sigma(P)/2\varepsilon_0$ [30].

Say the convex surface is a sphere of radius R, having charge of q coulombs, and let infinitesimal patch at P having radius a, will have surface charge density $\sigma(P)=(q)/(4\pi R^2)$. The total electric field at P is thus $E(P)=E_{hole}+E_{spherical-shell}$. The electric field of spherical shell, by applying a Gauss surface just outside the shell is; $E_{spherical-shell}(4\pi R^2)=q_{enclosed}/\varepsilon_0=q/\varepsilon_0$; from here $E_{spherical-shell}=(q)/(4\pi\varepsilon_0 R^2)$.

From here we calculate $E(P)=\left(-\sigma(P)/2\varepsilon_0\}+\{(q)/(4\pi\varepsilon_0 R^2)\right)=(q)/(8\pi\varepsilon_0 R^2)$. If we were to calculate electric field at P, from inside the surface, the procedure is same but the field direction of the patch will be opposite to what we considered in above calculations. In the above calculations we assume that coulombs contents of the patch at P that is $q(a^2/4R^2)$ very small fraction of total charge q; and thus we have applied the Gauss law stating that total charge enclosed is q in spite of patch of radius a removed. Due to infinitesimal small patch we can have this assumption; and with all these arguments we write the electric field (normal to a surface S) at point P for just outside and just inside the surface of figure-5 as (19); a general relation for an arbitrary shaped object.

$$E_n^{\pm}(P)=\mp\frac{\sigma(P)}{2\varepsilon_0}+\frac{1}{4\pi\varepsilon_0}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q \tag{19}$$

By putting (19) into (17), and after following steps we arrive at homogeneous boundary integral equation (20); written with kernel of integration $K(Q,P)$.

$$\varepsilon E_n^{+}=\varepsilon_0 E_n^{-}$$

$$-\varepsilon\frac{\sigma(P)}{2\varepsilon_0}+\varepsilon\frac{1}{4\pi\varepsilon_0}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q=-\varepsilon_0\frac{\sigma(P)}{2\varepsilon_0}+\varepsilon_0\frac{1}{4\pi\varepsilon_0}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q$$

$$-\varepsilon\sigma(P)+\varepsilon\frac{1}{2\pi}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q=-\varepsilon_0\sigma(P)+\varepsilon_0\frac{1}{2\pi}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q$$

$$\varepsilon\sigma(P)-\varepsilon_0\sigma(P)=\varepsilon\frac{1}{2\pi}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q-\varepsilon_0\frac{1}{2\pi}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q$$

$$\sigma(P)=\left(\frac{\varepsilon-\varepsilon_0}{\varepsilon+\varepsilon_0}\right)\frac{1}{2\pi}\int_S\sigma(Q)\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3}\mathrm{d}S_Q$$

From above derivation we write the following in compact integral form, that is

$$\sigma(P)=\frac{\lambda_{eigen}}{2\pi}\int_S K(Q,P)\sigma(Q)dS_Q \tag{20}$$

Where the kernel in integral equation of (20) is

$$K(Q,P)=\frac{\mathbf{r}_{QP}\bullet\mathbf{n}_P}{r_{QP}^3} \tag{21}$$

The eigen-value is

$$\lambda_{eigen}=\frac{\varepsilon-\varepsilon_0}{\varepsilon+\varepsilon_0} \tag{22}$$

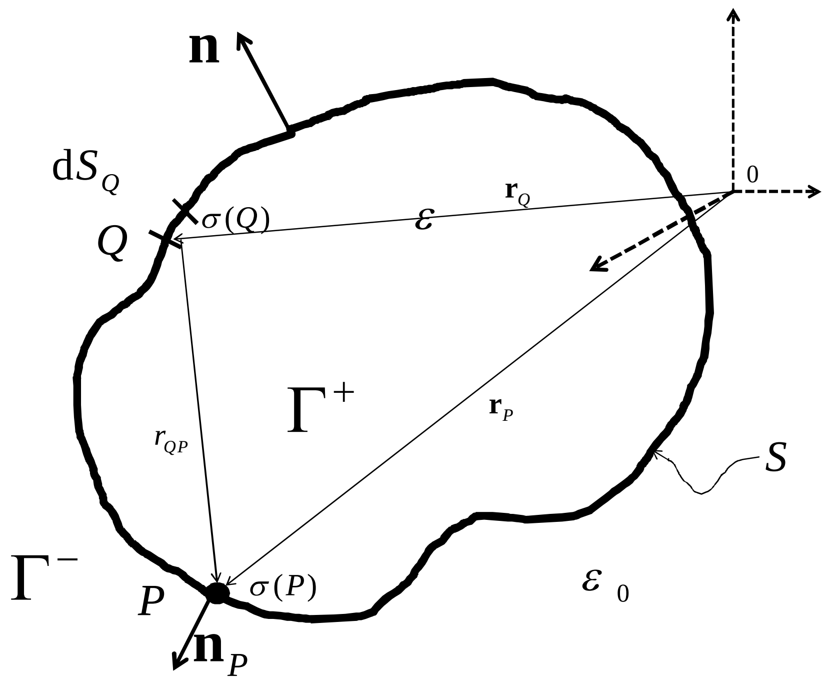

Figure-5: The dielectric particle in curl free divergence free Electric field

From (22) we will have negative ε as $\varepsilon=-\varepsilon_0(\lambda_{eigen}+1)/(\lambda_{eigen}-1)$, for particular shape of S .The (20) is Fredholm's homogeneous integral equation (of second kind) which has form

$$f(x)=\lambda_f\int_a^b K(x,y)f(y)\mathrm{d}y \tag{23}$$

The source free electric fields may exist only for such values of ε, that (20) has non zero solutions. In other words resonant values of ε (and corresponding resonant frequencies) as well as resonant electrostatic modes are embedded in eigen-values and eigen-functions of (20), which should be found. It is shown [41] [42] interesting properties of the eigen-values. First, all the eigen-values λ_{eigen} are real, with $\lambda_{eigen}=1$ as eigen-value; this value corresponds to a case that $\varepsilon\to\infty$; and its respective eigen-function $\sigma(P)$ is distribution of surface charge of a conductor surface (this value is not required for our case). All other eigen-values except unity, corresponds to a source free resonance configuration of electric field and according to (22) these configurations may exists only for negative values of ε . After the negative values of ε are found through (20), the approximate frequency dependency of ε (say Drude model [1]-[10], [14], [19], [27], [29], [35], and [38]) can be employed to find resonant frequencies.

5. Discussion on Electrostatic Resonance and its Properties

It is apparent that mathematical structure of the integral in (20) is invariant with respect to scaling S. This leads to unique property of electrostatic resonance that is 'the resonant frequency depends on object shape but are scale invariant with respect to object dimension, provided they remain well below the free space wave length of excitation EM radiation'.

If $\mathbf{E}_i$ and $\mathbf{E}_k$ are the electric fields corresponding to eigen-function σ_i and σ_k, then [39] the fields are orthogonal, that is

$$\int_{\Gamma^{\pm}} \mathbf{E}_i \bullet \mathbf{E}_k d\Gamma = 0 \tag{24}$$

The orthogonality (24) holds separately for Γ^+ and Γ^- regions. However, the eigen-functions $\sigma_i(Q)$ and $\sigma_k(Q)$ corresponding to eigen-values $\lambda_{eigen-i}$, $\lambda_{eigen-k}$ are not orthogonal on S, because the kernel (21) of the integral equation (20) is not symmetric (non Hermitian).

These orthogonality conditions can be useful in analysis of the coupling of specific 'resonance mode' to the incident electric field. For example we take spherical shaped particle in figure-5; its resonance modes will be different from ellipsoid (elliptical-harmonics). However, there are resonant modes with uniform electric field in Γ^+. This means according to orthogonality, condition that only these uniform resonant modes will be excited by uniform (within Γ^+) incident radiation. The condition of uniformity with in Γ^+ of the incident radiation is some extent natural due to object dimensions smaller to free space radiation as we stressed earlier. For complex shaped objects many resonant modes with appreciable average values of electric field components over Γ^+ may exist. All such modes will be well coupled to the uniform incident radiation and can be excited by such incident fields at respective frequency of resonance.

For a convex S of figure-5 estimate of eigen-values can be derived via estimate [40];

$$\left|\lambda_{eigen}\right| > C = \frac{1}{1-\left(\frac{A}{4\pi Rd}\right)} \tag{25}$$

Where A is area of S; R is maximum radius of curvature of S; d the diameter of Γ^+. Using (25) and (22), the upper bound and lower bound for possible resonance values of permittivity ε can be obtained as

$$\frac{1+C}{1-C} < \frac{\varepsilon}{\varepsilon_0} < \frac{1-C}{1+C} \tag{26}$$

For plasma we have dispersion of permittivity as Drude model; that is $(\varepsilon/\varepsilon_0) = 1-(\omega_p^2/\omega^2)$, substituting into (26) we have

$$\frac{C-1}{2C} < \frac{\omega^2}{\omega_p^2} < \frac{C+1}{2C} \tag{27}$$

Which says that bandwidth for resonance frequency is smaller than $\omega_p/\sqrt{C} = \omega_p\sqrt{1-(A/2Rd)}$

For a unit sphere the surface charge density will be given by spherical harmonics. The standard eigen functions for spherical harmonics is given by [30] $Y_{l,m}(\theta,\varphi)$. This spherical harmonic eigen function is defined in terms of Legendre's polynomial, $P_{l,m}(x)$ as $Y_{l,m}(\theta,\varphi)=P_{l,m}(\cos\theta)e^{im\varphi}$, where $-l \le m \le +l$, θ and φ are co-altitude polar($0 \le \theta \le \pi$; North pole to South pole) angle and longitude azimuth angle ($0 \le \varphi \le 2\pi$) of spherical coordinates respectively; with corresponding eigen values $\lambda_{eigen-l}=2l+1$. From (22) giving ε as $\varepsilon_l=-\varepsilon_0(1+1/l)$; and valid for, $l \ge 1$. The lowest electrostatic resonant mode, for $l=1$ $\varepsilon_1=-2\varepsilon_0$ is uniform in Γ^+, and only can be excited (into three possible spherical harmonic modes) by uniform incident radiation, that is for $l=1$, we have three spherical harmonics with $m\in\{-1,0,1\}$, giving $Y_{1,0}(\theta,\varphi)$, $Y_{1,1}(\theta,\varphi)$ and $Y_{1,-1}(\theta,\varphi)$. The first spherical harmonics, with $m=0$ represents 'zonal mode', where the charges positive and negative are distributed by equatorial symmetry. The north and south poles getting maximum positive charge density and negative charge density. The modes with $m=\pm 1$ are 'sectorial modes', and symmetric with respect to z -axis, where the charge distribution is spread in two sectors eastern and western. The 'tesseral mode' is not existing with $l=1$ and $m=\pm 1$. Thus there exist resonant modes with permittivity value as negative, where the charge density and correspondingly electric filed will be very large, at resonant frequencies, for arbitrary shaped particles ejecting out from solid substrate of our experiment.

6. Conclusion

The discussion in this paper gives a plausible explanation to observation of lit plasmoid; via classical electrodynamics principles. The dielectric permittivity of the particulates ejecting out from molten solid substrate having negative values at the resonances is what causes giant surface charge densities accompanied by giant surface electric fields; after its interaction with the existing EM radiation. These giant fields cause local discharges, thereby glowing the entire plasmoid volume; as long as the EM field remains. This experiment not only opens up possibility of using microwave near field for machining industry and or surgery biomedical applications; but also gives immense potential of 'material identification'; of the substrate composition through optical spectroscopy of the identification of spectral lines of the lit plasmoid. Doing experiments with small angle X-Ray scattering for particulate density, and size, along with optical spectroscopy of the lit plasmoid are promising experiments for future in this field of near field microwave applicator.

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