The Limitations of Bode Stability Criterion

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Tutorial note on Failure of Bode Criterion

Abstract

The Bode stability criterion as studied in classical text books is a sufficient but not necessary condition for instability of a closed loop process. Therefore it is not possible always to use this criterion to make definitive statement about the stability of a given process.

Introduction

The Bode plot is an important tool for analysis of closed loop systems. It is based on calculating the amplitude ratio (i.e. gain) and phase angle for the open-loop transfer function. The Bode stability criterion presented in most of the text books is sufficient but not necessary condition for stability of closed loop processes. Thus it is not always possible to use this criterion to make definitive statement about the stability of a given process. Here we will give examples of Inverse Response System and System with pure delay and control the same via PD or PID controller, and show the ineffectiveness of the Bode stability criterion, and how wrong inferences may be drawn on stability. Later on we shall try and revise the Bode stability criterion.

About Inverse Response Systems

When you push down on your car's accelerator, you expect the car to speed up, right? What if it slows down? Or even worse: You lift your foot off the accelerator and your car speeds up. And the more you lift your foot, the more the car speeds up. These are almost unthinkable and certainly scary situations, yet they occur every day several processes around the world. The phenomenon is called an inverse response.

Consider a transfer function as difference between two positive transfer functions i.e.

$$G(s) = G_1(s) - G_2(s)$$

One such is the difference between two first order transfer functions

$$G(s) = \frac{K_1}{\tau_1 s + 1} - \frac{K_2}{\tau_2 s + 1}; \qquad K_1, K_2, \tau_1, \tau_2 > 0$$

It follows for a step input $U(s) = \frac{1}{s}$, the, the output $Y(s) = G(s)U(s)$ so that

$$
\begin{aligned}
Y(s) &= Y_1(s) - Y_2(s) \\
&= \left(G_1(s)\right)\left(U(s)\right) - \left(G_2(s)\right)\left(U(s)\right) \\
&= \left(\frac{K_1}{\tau_1 s + 1}\right)\left(\frac{1}{s}\right) - \left(\frac{K_2}{\tau_2 s + 1}\right)\left(\frac{1}{s}\right)
\end{aligned}
$$

The final output at $t \uparrow \infty$ is via final value theorem is following

$$
\begin{aligned}
y(\infty) &= \lim_{s \downarrow 0}\left(s\left(Y(s)\right)\right) \\
&= \lim_{s \downarrow 0}\left(s\left(\frac{K_1}{\tau_1 s + 1}\right)\left(\frac{1}{s}\right) - s\left(\frac{K_2}{\tau_2 s + 1}\right)\left(\frac{1}{s}\right)\right) \\
&= K_1 - K_2
\end{aligned}
$$

So depending on cases $K_1 > K_2$ or $K_1 < K_2$; the final settled value of output $y(\infty)$ will be positive or negative; for a unit positive step input. If say $K_1 > K_2$, then in that case the final settled value is positive. This final value is also called the ultimate response.

Applying initial value theorem i.e. $\lim_{t \downarrow 0} f(t) = \lim_{s \uparrow \infty}\left(s\left(F(s)\right)\right)$, where $F(s) = \mathcal{L}\left\{f(t)\right\}$ and applying derivative of Laplace, i.e. $\mathcal{L}\left\{f^{(1)}(t)\right\} = sF(s)$, where $f^{(1)}(t) = \frac{\mathrm{d}}{\mathrm{d}t} f(t)$ assuming $f(0) = 0$, we get following steps

$$
\begin{aligned}
\lim_{t \downarrow 0} \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= \lim_{s \uparrow \infty}\left(s\left(sY(s)\right)\right) = \lim_{s \uparrow \infty}\left(s^2\left(Y(s)\right)\right) \\
&= \lim_{s \uparrow \infty}\left(s^2\left(\frac{K_1}{\tau_1 s + 1}\right)\left(\frac{1}{s}\right) - s^2\left(\frac{K_2}{\tau_2 s + 1}\right)\left(\frac{1}{s}\right)\right) \\
&= \lim_{s \uparrow \infty}\left(\frac{K_1}{\tau_1 + \frac{1}{s}} - \frac{K_2}{\tau_2 + \frac{1}{s}}\right) = \frac{K_1}{\tau_1} - \frac{K_2}{\tau_2}
\end{aligned}
$$

Suppose that

$$
\frac{K_2}{\tau_2} > \frac{K_1}{\tau_1}
$$

Then the initial slope in the opposing response is greater than initial slope of the main response. The response will go in the inverse direction of the ultimate response.

The inverse response system is represented in following diagram (Figure-1)

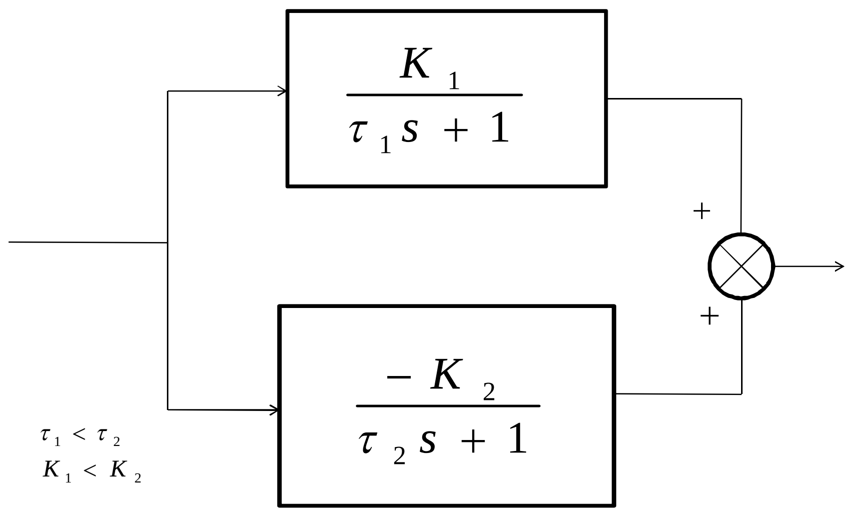

Figure-1: Inverse response system

Inverse Response System a Transfer Function with one Right Half Plane Zero

The Transfer function of an inverse response system can be casted as follows

$$
\begin{aligned}
G(s) &= \frac{K_1}{\tau_1 s+1} - \frac{K_2}{\tau_2 s+1} \\
&= \frac{\left(K_1\tau_2 - K_2\tau_1\right)s + \left(K_1 - K_2\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)} \\
&= \frac{K\left(as+1\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}; \qquad a = \frac{K_1\tau_2 - K_2\tau_1}{K_1 - K_2}; \quad K = \left(K_1 - K_2\right)
\end{aligned}
$$

Where the inverse response will occurs when the transfer function has a positive zero, or a zero at Right Half Plane (RHP). We make $a = -b, \quad b > 0$ and write

$$G(s) = K\frac{\left(1-bs\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}$$

Thus we have a positive zero as $s = \frac{1}{b}$, i.e. at the RHP. We see the dynamic behavior of system with a zero at RHP.

Transfer Function

$$G(s) = K\frac{\left(1-bs\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}$$

Unit step response to input $U(s) = \frac{1}{s}$ is

$$Y(s) = \frac{K\left(1-bs\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}\left(\frac{1}{s}\right)$$

Ultimate response from final value theorem is

$$
\begin{aligned}
y(\infty) &= \lim_{s\downarrow 0}\left(s\left(Y(s)\right)\right) \\
&= \lim_{s\downarrow 0}\left(s\frac{K\left(1-bs\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}\left(\frac{1}{s}\right)\right) \\
&= K
\end{aligned}
$$

The initial rate of change of output (or initial slope) from initial value theorem we write

$$
\begin{aligned}
\left.\frac{\mathrm{d}y(t)}{\mathrm{d}t}\right| &= \lim_{s\uparrow\infty}\left(s^2\left(Y(s)\right)\right) \\
&= \lim_{s\uparrow\infty}\left(s^2\left(\frac{K\left(1-bs\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}\right)\left(\frac{1}{s}\right)\right) \\
&= \lim_{s\uparrow\infty}\left(s^2\left(\frac{Ks\left(\frac{1}{s}-b\right)}{s\left(\tau_1+\frac{1}{s}\right)s\left(\tau_2+\frac{1}{s}\right)}\right)\left(\frac{1}{s}\right)\right) \\
&= -\frac{Kb}{\tau_1\tau_2}
\end{aligned}
$$

Thus we find a transfer function with one RHP zero has initial response in reversed direction.

Boiler Drum Level Control-example of Inverse Response System

One of the most common occurrences of inverse response is found in the control of boiler drum level. In a boiler, water is converted to steam. Steam and water separates in the boiler drum, with the steam then leaving through a pipe at the top of the drum. It is important to keep the level of water in the drum away from this pipe or water will exit with the steam and damage downstream equipment. Even more important is to always have some water in the drum – when the boiler runs dry there is no water to cool it, and this will result in severe damage to the boiler. So the water level in the drum is normally maintained close to its centerline.

The drum level is controlled by adding water to the boiler, called feed-water. A closed-loop controller looks at the drum level and if it is lower than the set-point it opens the feed-water control valve to increase the feed-water flow rate and vice versa (Figure-2).

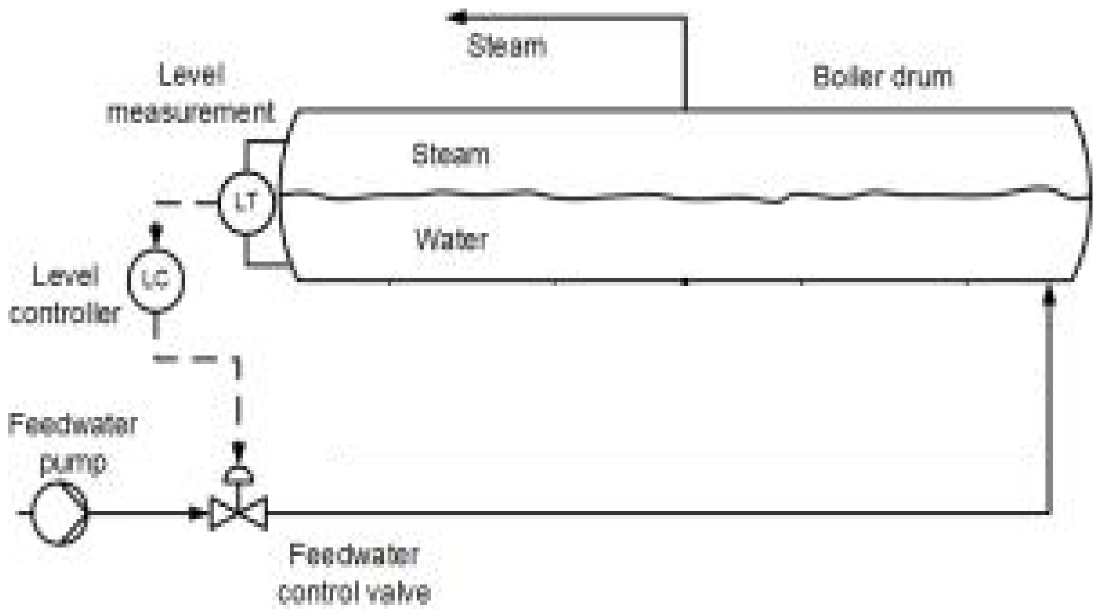

Figure-2: Boiler drum level control diagram

The temperature of the feed-water flowing into the drum is normally below boiling point. When we add more of this colder water to the boiler, some of the steam bubbles in the boiler condense. This makes the steam pressure to fall which measures the level decrease via LT. However, the effect is only temporary. After a while, the higher rate of feed-water flow overcomes the lost volume and the drum level rises (Figure-3). The opposite is also true: when we decrease the flow rate of the colder feed-water, steam production increases, and the additional steam bubbles cause the drum level to rise. But after a while, the drum level begins to fall, as expected. The inverse response is via swell/shrink mechanism that we will not discuss.

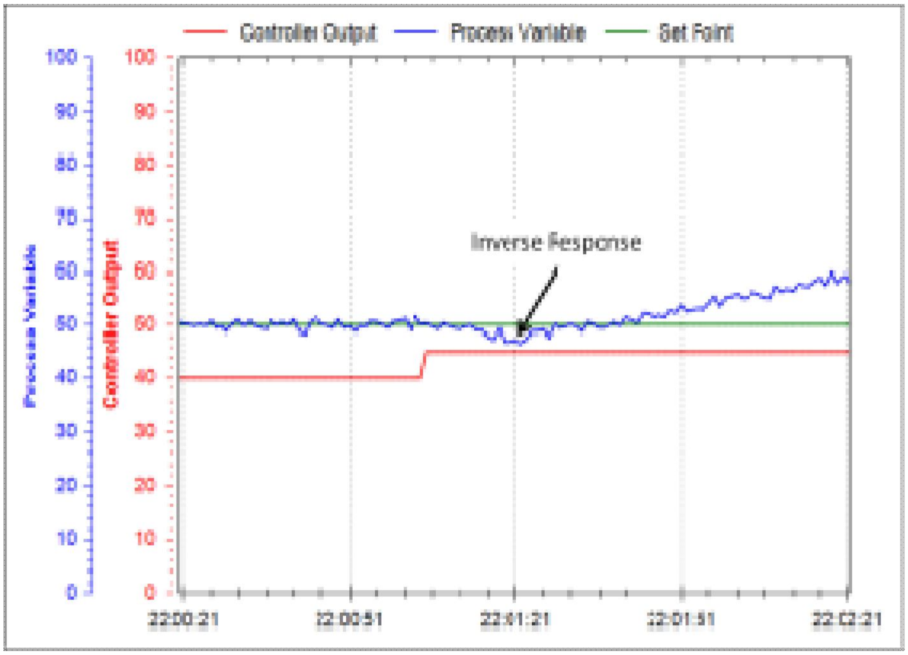

Figure-3: With inverse response, the process first responds in the wrong (inverse) direction, and then in the expected direction.

Controller Tuning Implications with Inverse Response System

Processes exhibiting inverse response can easily cause control loop stability problems. Using derivative control is questionable from a stability perspective, and certainly not useful. Using a high controller gain is not possible since it will "chase" the inversely responding process and create a snowball (runaway) effect. But when you use a low controller gain on an integrating process, you also have to use a long integral time (low integral gain). So you end up with a very slow-responding control loop, and any attempt to speed it up significantly lowers its stability. This is why three-element control is the strategy of choice for drum level control.

When you do step-testing on a process with an inverse response and determine the process characteristics to tune the controller, you should treat the entire duration of the inverse response as dead time (Figure-4). Then you can apply your usual level controller tuning rules using this pseudo dead time.

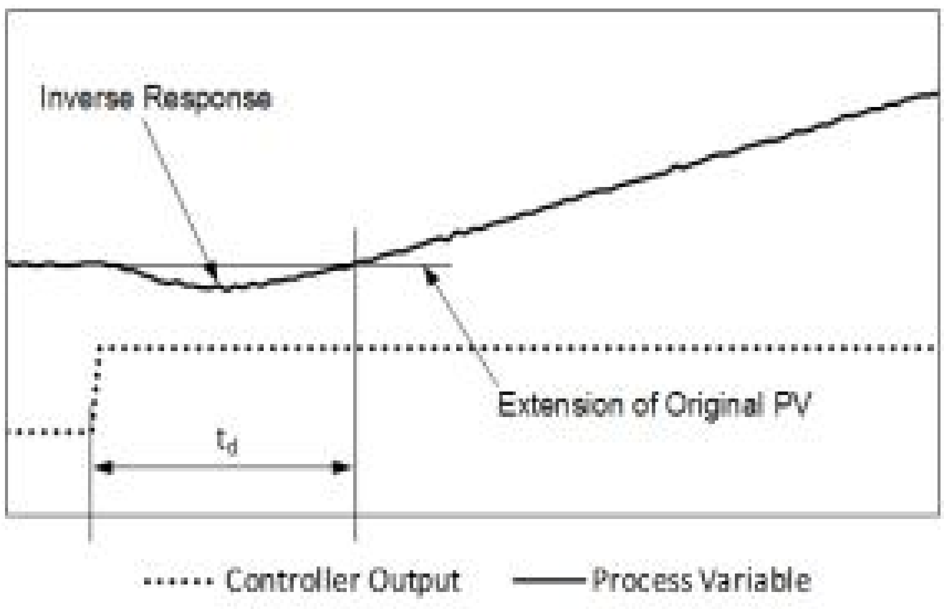

Figure-4: Dead-time (t_d) measurement on an inversely responding process.

Therefore we represent inverse response system transfer function along with a dead time transfer function (a delay transfer function) $e^{-t_d s}$ and say the plant transfer function as following

$$G_P(s) = \left(G(s)\right)e^{-t_d s}$$
$$= \frac{K\left(1-bs\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}e^{-t_d s}$$

The delayed function in time is $f(t-t_d)$, when a function $f(t)$ is passed through a delay block. In Laplace domain we represent $\mathcal{L}\left\{f(t-t_d)\right\} = e^{-st_d}F(s)$, where $F(s) = \mathcal{L}\left\{f(t)\right\}$. The biggest obstacle to handle delay is not in modeling, the problem lies in the fact that delay cannot be expressed as rational polynomial. There are some approximations that have rational polynomials, examples of approximations are

$$e^{-t_d s} \cong 1 - st_d + \frac{(st_d)^2}{2} - \frac{(st_d)^3}{6} + \dots$$

$$e^{-t_d s} \cong \frac{1}{1 + st_d + \frac{(st_d)^2}{2} + \frac{(st_d)^3}{6} + \dots}$$

$$e^{-t_d s} \cong \frac{1 - \left(\frac{st_d}{2}\right)}{1 + \left(\frac{st_d}{2}\right)}$$

$$e^{-t_d s} \cong \frac{1}{\left(1 + \frac{st_d}{k}\right)^k}$$

For the last one chose k large. We note that all these approximation give a zero at Right Half Plane.

Bode Stability Criteria a review

The Bode stability criterion comes from Berkhausen's criterion for oscillations. It states that if A is the gain of the amplifying element (the feed forward path) in circuit and $\beta(j\omega)$ is the transfer function of the feedback path, so βA is the loop gain around the feedback loop of the circuit, the circuit will sustain steady state oscillations only at frequencies for which:

1. The magnitude of loop gain is equal to unity (or zero dB) that is $|\beta A| = 1$
2. The phase shift around the loop is zero or integer multiple of 2π (or 360^0): $\angle \beta A = 2\pi m$, $m = 0, 1, 2, 3, \dots$

The close loop Transfer function of the negative feedback amplifier is thus

$$G_{CL}(j\omega) = \frac{A}{1 + (A)(\beta(j\omega))}; \quad s = j\omega$$

Berkhausen's criterion is necessary condition for oscillations but not a sufficient condition; some circuits satisfy the criterion but do not oscillate. Similarly the Nyquist stability criterion also indicates instability but is silent about oscillations. Apparently there is no definitive formulation of an oscillation that is both necessary and sufficient.

Berkhausen's original formula for self sustained oscillations or self-excitation for determining the oscillation frequencies of the feedback loop involved an equality sign i.e. $|\beta A| = 1$. At that time conditionally-stable non-linear systems were poorly understood; it was widely believed that this gave the boundary between stability i.e. $|\beta A| < 1$and instability i.e.$|\beta A| \geq 1$; and erroneous version found its way into books. However, stable oscillations only occur at frequencies for which equality holds.

The Bode stability criterion i.e. $|\beta A| < 1$ at phase cross over frequency i.e. at $\omega_{C_{180}}$ where phase angle of open-loop transfer function βA i.e. $\angle \beta A = -180^0$ as in text book is a sufficient but not a necessary condition. Thus we cannot make a definitive statement regarding stability of a closed-loop control system of plant by using this criterion.

Tool to analyze stability of a close-loop system is classically by use of Bode-plot of 'open loop' transfer function of Gain and Phase. It is evaluating the amplitude and phase angle of the 'open-loop' transfer function

$$G_{OL}(s) = (G_C(s))(G_P(s)); \qquad s = j\omega$$

Where we call $G_{OL}(s)$ as open loop transfer function, $G_C(s)$ is the controller (PID) transfer function, and $G_P(s)$ is the plant transfer function. We consider unity feedback control system and the close loop transfer function is got by putting the feedback factor as $\beta = 1$ and feed-forward factor $A = G_{OL}(s) = (G_C(s))(G_P(s))$ as following

$$G_{CL}(s) = \frac{G_{OL}(s)}{1 + G_{OL}(s)} = \frac{(G_C(s))(G_P(s))}{1 + (G_C(s))(G_P(s))}$$

The criterion comes from the fact that denominator of $G_{CL}(s)$ is zero for instability, that is $1+G_{OL}(s)=0$ meaning $G_{OL}(s)=-1+j0$ indicating $|G_{OL}(s)|=1$ or 0dB, with the phase angle i.e. $\angle G_{OL}(s)=-180^0$. The negative is practically put as most of control block gives a phase lag.

We often come across the statements in books that this Bode criterion that is though sufficient condition is a necessary condition as well. Is this statement a correct one? Thus we may safely say-that criterion of Bode that is formulated as a necessary condition for stability, but no definitive statements can be made based on a necessary condition alone.

Often some riders are put on this Bode stability criterion; "…the Bode stability criterion only applies to systems that cross $\phi=-180^0$ phase line 'once'; where ϕ is the phase-shift of the open loop transfer function i.e. $(G_C(s))(G_P(s))$. For multiple crossings one must use Nyquist criterion of encirclement of point $(-1,0)$ …" This Bode Stability criterion is from Berkausen's oscillation criterion. The oscillation criterion takes the loop phase i.e. around the loop, while Bode Stability criterion considers 180^0 contribution already given by negative feedback component-and thus considers breaking the loop calling it open-loop.

If we have a system that crosses the phase angle -180^0 and have amplitude ratio (or gain) less than unity (negative dB) at this phase-cross over frequency-still system can be unstable. That is because the open loop transfer function $(G_C(s))(G_P(s))$ can have amplitude ratio greater than unity for other frequency cross over points say -540^0, -900^0, we call as $-180^0-n\times 360^0$, with n positive integers.

Another point get mentioned about Bode stability criterion, that is it cannot be used if the frequency response of the open loop transfer function exhibits a "non-monotonic" phase angles or amplitude ratio (gains) at frequency higher than the first phase cross over frequency of -180^0. So there are cases where Bode stability criterion fails to give conclusion about close loop stability. These two issues we will consider via examples in the next sections.

Failure of Bode Stability Criterion for Inverse Response System with delay

Let us take a plant showing inverse response and with a delay represented by the following transfer function

$$
\begin{aligned}
G_P(s) &= \frac{K\left(1-bs\right)}{\left(\tau_1 s+1\right)\left(\tau_2 s+1\right)}e^{-t_d s}; \qquad K=1, \quad b=1, \quad \tau_1 s+1 \equiv s, \quad \tau_2=\tau \\
&= \frac{(1-s)}{s\left(\tau s+1\right)}e^{-t_d s} \\
&= \frac{(1-s)}{s\left(0.1s+1\right)}e^{-0.4s} \approx \frac{\left(1-s\right)}{s\left(0.1s+1\right)}e^{-0.4s}
\end{aligned}
$$

The parameters chosen are $\tau = 0.1\text{min}\ t_d = 0.4\text{min}$.

This has ultimate value as positive i.e.

$$
\begin{aligned}
y\left(\infty\right) &= \lim_{s\downarrow 0}\left(sY(s)\right) = \left(s\left(G_p(s)\right)\left(U(s)\right)\right); \quad U(s)=\frac{1}{s} \\
&= \lim_{s\downarrow 0}\left(s\left(\frac{(1-s)e^{-t_d s}}{s\left(\tau s+1\right)}\right)\left(\frac{1}{s}\right)\right) = +\infty
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\mathrm{d}y(t)}{\mathrm{d}t}\right|_{t=0} &= \lim_{s\uparrow\infty}\left(s^2 Y(s)\right) = \lim_{s\uparrow\infty}\left(s^2\left(G_P(s)\right)\left(U(s)\right)\right); \qquad U(s)=\frac{1}{s} \\
&= \lim_{s\uparrow\infty}\left(s^2\left(\frac{(1-s)e^{-t_d s}}{s\left(\tau s+1\right)}\right)\left(\frac{1}{s}\right)\right) = -\frac{1}{\tau}\lim_{s\uparrow\infty} e^{-t_d s} = 0
\end{aligned}
$$

The inverse response of the system at start time is getting blocked by the delay unit.

The plant transfer function has already having pole at origin, and is inherently unstable. Therefore integral action in the controller is not chosen for a step or impulse disturbance of set-point. Thus choose a PD controller of $G_{PD}(s) = K_p + sK_D$ type. The PD controller we top with a pre-filter having a Low-Pass filtering with a transfer function $G_{filter}(s) = \left(\frac{1}{\tau_f s+1}\right)$ and make the consolidated controller transfer function as following

$$G_C(s) = \left(G_{filter}(s)\right)\left(G_{PD}(s)\right)$$

$$= \left(\frac{1}{\tau_f s + 1}\right)\left(K_P + K_D s\right) = \frac{K_P + K_D s}{\tau_f s + 1}$$

$$= \frac{K_C + K_C \tau_D s}{\alpha \tau_D s + 1}; \qquad K_P = K_C\,, \quad K_D = K_C \tau_D\,; \quad \tau_f = \alpha \tau_D$$

$$= K_C \left(\frac{\tau_D s + 1}{\alpha \tau_D s + 1}\right) = \frac{0.2\left(s+1\right)}{\left(0.05 s + 1\right)}$$

The parameters chosen are $K_C = 0.2$, $\tau_D = 1\text{min}$ $\alpha = 0.05$

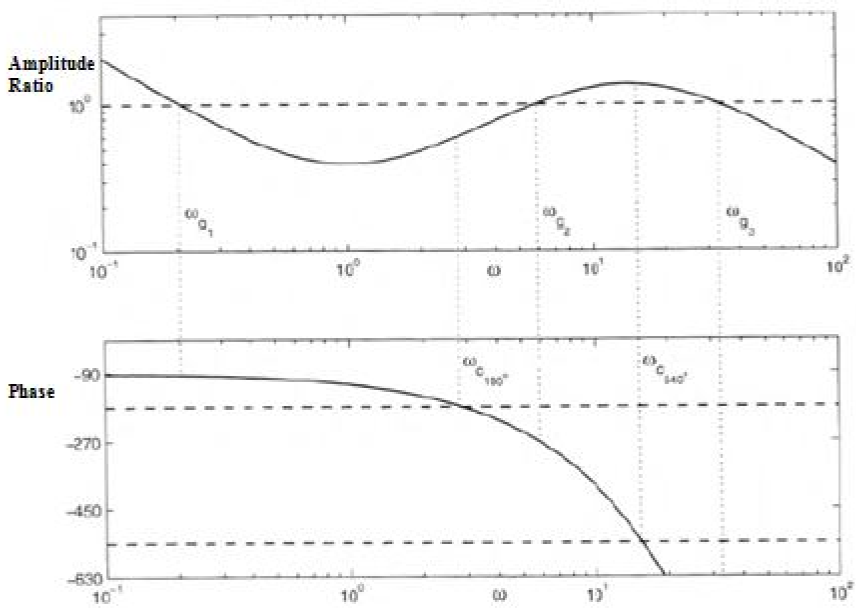

Figure-5: Bode plot of the open loop transfer function for a inverse response process

The resulting Bode plot for the open-loop system $\left(G_P(s)\right)\left(G_C(s)\right)$ is shown in Figure-5. The phase-cross over frequency $\omega_{C_{180}}$ is defined to be the frequency at which the open loop phase angle is -180^0. Furthermore the gain cross-over frequencies ω_g are defined to be frequencies at which the open-loop amplitude ratio is equal to one (or zero dB), and $\omega_{C_{540}}$ corresponds to the frequency where the phase angle crosses -540^0.

The amplitude ratio corresponding to a phase angle of -180^0 is about 0.6 i.e. negative dB value- i.e. having sufficient gain margin. Thus we conclude via Bode stability rule the stable closed loop system. The conclusions we draw is (1): The system is stable and (2): the gain margin is 1.67, i.e. the controller gain can be increased by 67% without making the system unstable.

But observing the amplitude at phase angle of -540^0, the value is 2.0 a positive dB value. At this point we conclude the closed loop system is unstable. Increasing the controller gain makes system even more unstable; instead a reduction in the gain by 50% will result in a stable closed – loop system.

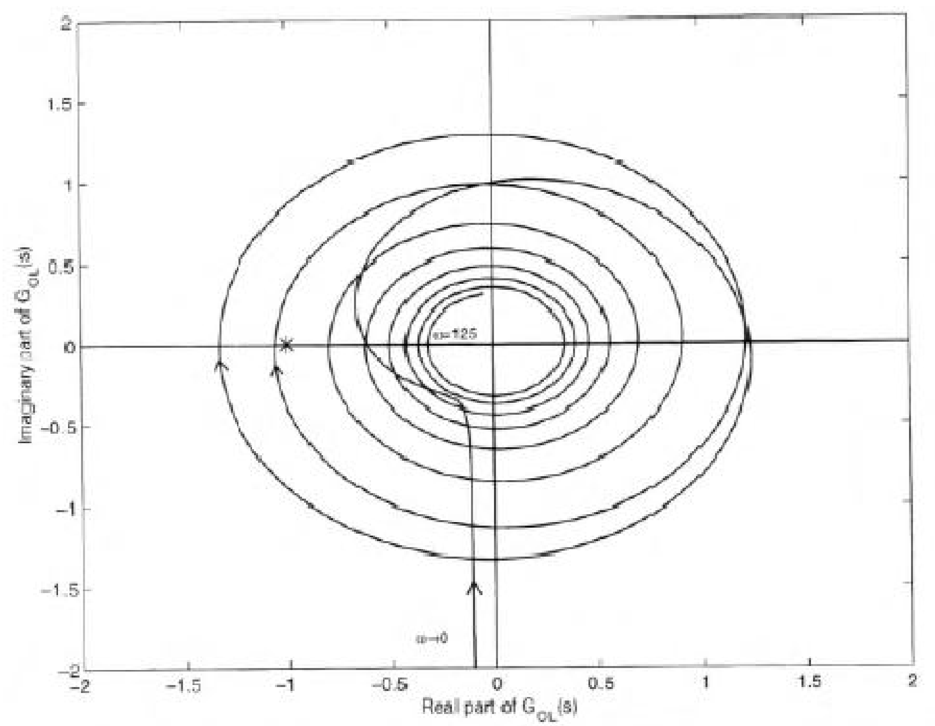

Figure-6: Nyquist plot of the open loop transfer function for a inverse response process

The above stability discussion via Bode plot is validated by Nyquist plot as shown in Figure-6. The figure says the system is unstable due to the observation that curve shows encircling the point $(-1, 0)$ twice in a clock-wise direction.

Failure of Bode Stability Criterion for controlling system of pure delay

Let us consider a plant which is a time pure delay of θ minutes, that is plant transfer is following

$$G_P(s) = e^{-\theta s}$$

We note that approximate representation of delay says a system with positive zero-as indicated above. The controller is PID topped with a low pass filter, that is

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s$$
$$G_{filter}(s) = \frac{1}{\tau_{filter} s + 1}$$

Thus consolidated controller transfer function is following

$$\begin{aligned} G_C(s) &= \left(G_{filter}(s)\right)\left(G_{PID}(s)\right) \\ &= \left(\frac{1}{\tau_{filter} s + 1}\right)\left(K_P + \frac{K_I}{s} + K_D s\right) \\ &= K_C\left(\frac{\tau_1 s + 1}{\tau_1 s}\right)\left(\frac{\tau_D s + 1}{\alpha \tau_D s + 1}\right) \end{aligned}$$

The PID controller and filter variables are following

$$K_P = K_C\left(1 + \frac{\tau_D}{\tau_I}\right); \quad K_I = \frac{K_C}{\tau_1}; \quad K_D = K_C \tau_D s; \quad \tau_{filter} = \alpha \tau_D$$

We take the values $\theta = 0.6\,\text{min}$, $K_C = 0.1$, $\tau_1 = 4\,\text{min}$, $\tau_D = 1\text{min}$ and $\alpha = 0.05$.

Let us draw Bode Magnitude and Phase plots for Transfer Function of a PID, i.e. $G_{PID}(s) = K_P + K_I s^{-1} + K_D s$ as depicted in Figure-7. The upper plot is for $K_P = K_I = K_D = 1$ and lower plot is for $K_P = K_D = 1,\ K_I = 0.5$. These two figures show that by variation of K_P and K_I values we get different 'notch-frequency' i.e. ω_n. The notch frequency in the PID transfer function is the value of frequency at which we get minimum gain in magnitude with zero-phase angle.

$$G_{PID}(s) = K_P + K_I s^{-1} + K_D s \qquad G_{PID}(j\omega) = K_P + j\left(K_D\omega - K_I\omega^{-1}\right)$$

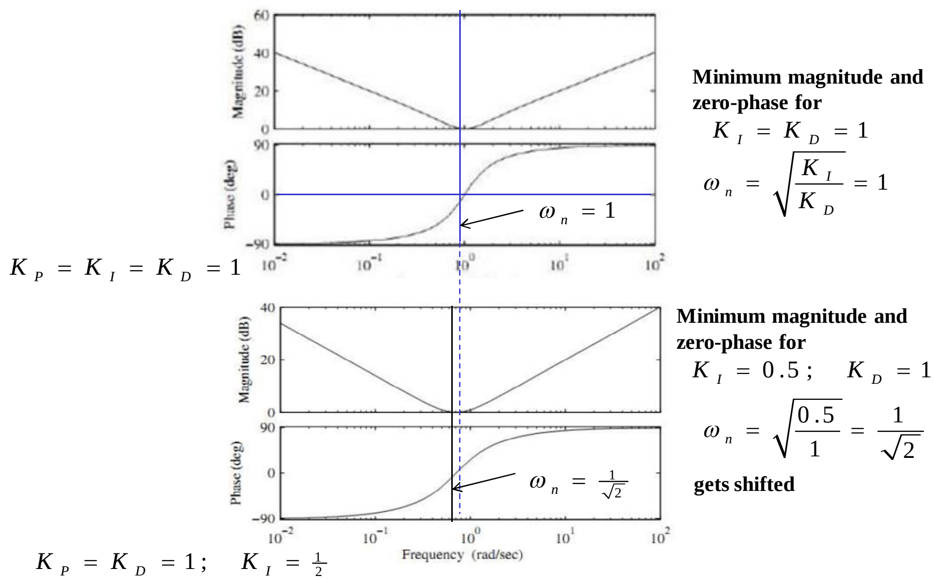

Figure-7: Bode plots for Transfer Function of PID

The plot the magnitude and phase angle Bode plots for open loop transfer function for delay system i.e. $G_{OL}(s) = \left(G_P(s)\right)\left(G_C(s)\right)$ is shown in the Figure-8.

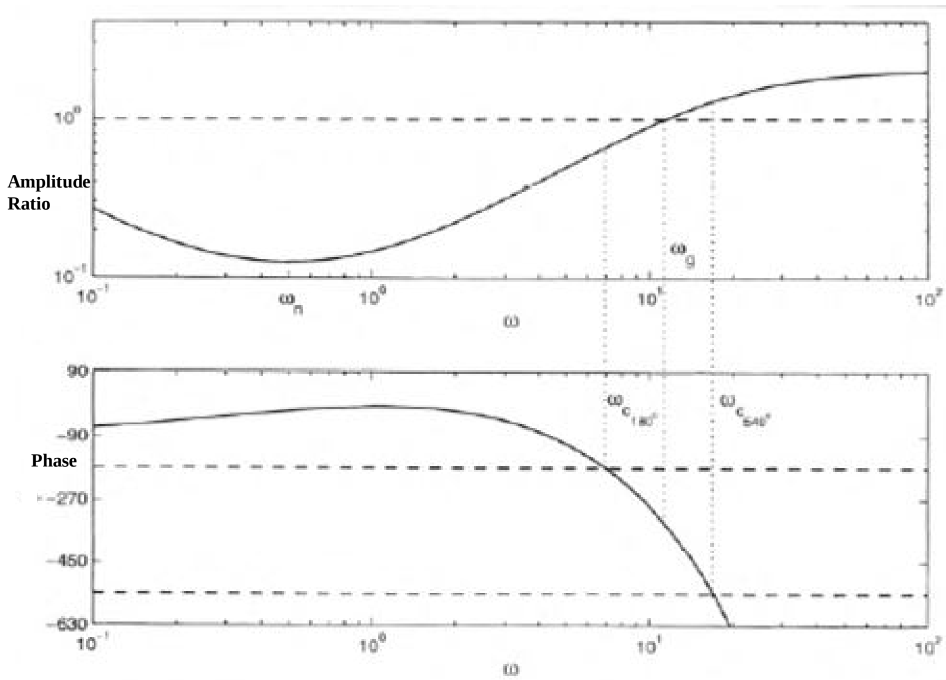

Figure-8: Bode plot of the open loop transfer function for time-delay process

The notch frequency ω_n for this PID controller is $0.5\,\text{min}^{-1}$ and the amplitude ratio of the open-loop process is monotonically increasing at higher frequencies (Figure-8). The phase crossover frequency is located at $\omega_{C_{180}} = 7\,\text{min}^{-1}$ and at higher frequencies, both the amplitude ratio and the phase angles are monotonic. The value of gain (amplitude ratio) at $\omega_{C_{180}} = 7\,\text{min}^{-1}$, i.e. phase cross over frequency is less than unity, indicating stability. The Nyquist plot says differently.

The Nyquist diagram of the open loop function is shown in the Figure-9. It can be concluded that this system is unstable since the point $(-1, 0)$ is encircled an infinite number of times in a clockwise directions. This again contradicts the interpretation that we get from Bode stability criterion.

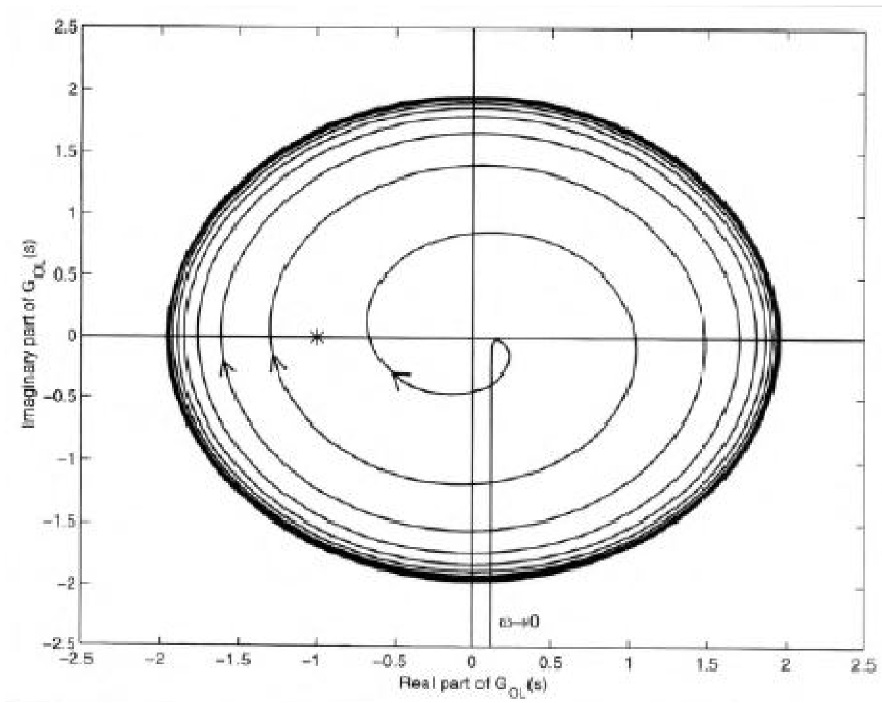

Figure-9: Nyquist plot of open loop transfer function of delay process

Interpretations

The two examples indicate that it is not possible to formulate Bode stability criterion in all cases. Thus we note following points

a. A system should be analyzed for stability using the Bode plot if it has 'at most' one phase cross over frequency. Additionally if it has only one gain cross over frequency and the amplitude ratio as well as the phase angle are decreasing at the gain crossover and afterwards, then the gain and phase margins can be calculated as per classical method as described by Bode stability criterion, in the text books.

b. A system that has only one phase crossover frequency but multiple gain crossover frequencies is stable if the amplitude ratios corresponding to corresponding to frequencies where $\phi = -180^0 - n \times 360^0$, are less than unity then the closed loop system is stable. The gain margin is calculated from the crossover frequency or a frequency corresponding to a larger n, whichever exhibits the larger amplitude ratio.

c. If the Bode plot information is inconclusive, the Nyquist stability analysis should be applied for stability of closed loop system.

Revised Bode Stability Criterion

A closed loop system is stable if the open loop system is stable and the frequency response of the open loop transfer function has an amplitude ratio of less than unity at all frequencies corresponding to $\phi = -180^0 - n \times 360^0$, where $n = 0, 1, 2, \ldots.\infty$.

The logic of above revised Bode stability criterion follows directly from the Nyquist criterion. When this definition of a stability criterion is recast in a form for use in Nyquist plot, the resulting set of closed loop stable system is given by the curves that do not cross the real axis to the left of point $(-1, 0)$ and are open-loop stable. Thus all these curves do not encircle $(-1, 0)$ in either direction, and this set is a subset of all closed loop systems described by the Nyquist stability criterion.

Conclusion

This revised stability criterion of Bode is sufficient condition for stability. It is not necessary condition, since a system can have multiple phase crossover frequencies (some of them with amplitude ratios larger than unity) and still be stable. If a case arises that is not covered by the revised criterion, then Nyqust stability criterion should be used for stability.

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