Energy/Fuel Efficient and Iso-Damped Robust Control Systems with Developed Fractional Order PID-a brief report

Concept document for possible "Start Up" by Private/Public Sector units or Entrepreneurs on the developed controllers based on fractional calculus for possible sponsoring from Govt. of India agencies under "Make in India" Program

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Abstract

In this brief report, the design and implementation of fractional order proportional-integral-derivative (FOPID) controller is presented in analog and digital domains, with the measured results to show the energy/fuel efficiency and enhanced robustness obtained, as compared to classical PID controls. The FOPID controller is tested with DC-Motor, Magnetic Levitation System, and Brushless DC Motor. The results that are presented show advantages of using fractional calculus in control system. Several of experiments that are developed (including the ones presented here) are kept in 'Fractional Calculus Engineering Laboratory' at VNIT-Nagpur; (https://timesofindia.indiatimes.com/city/nagpur/VNIT-develops-energy-minimizing-controller-transfers-tech-to-BARC/articleshow/55444793.cms). This laboratory is the first of its kind in globe that was dedicated to Nation on 15 November 2016, for using as a platform for advancement in 'Fractional Order Control Science' and to develop further industrial applications.

Keywords: *Fractional calculus, PID, Fractional Order PID, Fractional order Laplace variable, Pole-Zero approximation, Iso-Damping, Performance Indices, Energy/Fuel efficient control system.*

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Introduction

Classical controls are in form of PID (Proportional Integral Derivative) controllers that exist since 1910. The PID was invented in 1910 by Elmer Sperrys for ship auto pilot. However the 'electronic' circuit based conventional classical controllers such as PD, PI and PID have been applied in industry for over half-a-century to control linear and nonlinear systems. The tuning methods for PID controllers i.e. "Ziglers-Nichols" is well proven and exists since 1942.

Recently, such control schemes have been extended to their generalized form using fractional calculus (differentiation and integration of an arbitrary order). The 'Fractional Calculus Engineering Laboratory' (https://timesofindia.indiatimes.com/city/nagpur/VNIT-develops-energy-minimizing-controller-transfers-tech-to-BARC/articleshow/55444793.cms), is the first of its kind in globe to have experiments/demonstrations to show the advantages of using fractional calculus in control science [1] .The FOPID controller has fractional order differ-integration operations [2]-[5]. In applications, where these non-integer order controllers (i.e. FOPID) are used there is added flexibility in adjusting the gain and phase characteristics as compared to integer order controllers as shown in Figure-1, [2], [3]. The Figure-1a gives the transfer function (in frequency domain) of FOPID controller as $C_{\mathrm{FOPID}}(s)=\mathrm{K_p}+\mathrm{K_i}s^{-\alpha}+\mathrm{K_d}s^{\beta}$, *with parameters* α *and* β *the non-integer values (greater than zero). These parameters* $\alpha,\beta\in\mathbb{R}^{+}$ *give two extra degrees of freedom in tuning as compared to three in number for the classical PID whose transfer function is* $C_{\mathrm{PID}}(s)=\mathrm{K_p}+\mathrm{K_i}s^{-1}+\mathrm{K_d}s$ *[2], [3]. Note that classical PD, PI, and PID take only three points namely* (0, 1), (1, 0) and (1, 1) *in the entire* $\alpha-\beta$ *plane, whereas the FOPID is having domain of operation in the entire first quadrant of* $\alpha-\beta$ *plane (Figure-1a). This flexibility makes fractional order control more versatile tool in designing robust and precise control systems. The fractional Laplace operators in the transfer function (i.e.* $s^{-\alpha}$ *and* s^{β}*) of FOPID corresponds to fractional integration of order* α *and fractional differentiation of order* β*, respectively [2], [3]. Thus, in the time domain the controller output i.e.* $u(t)$ *which is obtained via operation on real time error signal i.e.* $e(t)$ *is* $u(t)=\mathrm{K_p}\left(e(t)\right)+\mathrm{K_i}\left(D_t^{-\alpha}e(t)\right)+\mathrm{K_d}\left(D_t^{\beta}e(t)\right)$*. This has operation of fractional integration and fractional differentiation (Figure-1b). The operation* $D_t^{\upsilon}x(t)$ *is fractional derivative/integration with respect to variable* t *for a function* $x(t)$*, for fractional order* υ; $\upsilon\in\mathbb{R}$ *[2], [3], [4], [5]. When* $\upsilon=1$*, the operation is one-whole classical derivative, and with* $\upsilon=-1$ *the operation is classical one-whole order integration. This structure of FOPID controller is depicted in the Figure-1b. A better understanding of the potential of fractional calculus and the increasing number of studies related to the fractional order controllers led to the importance of studying aspects such as the analysis, design, implementation, tuning, and application of these* *controllers in diverse applications. Some of the results we briefly report here. For detailed study the readers may view the articles, notes etc. listed in references-with links.*

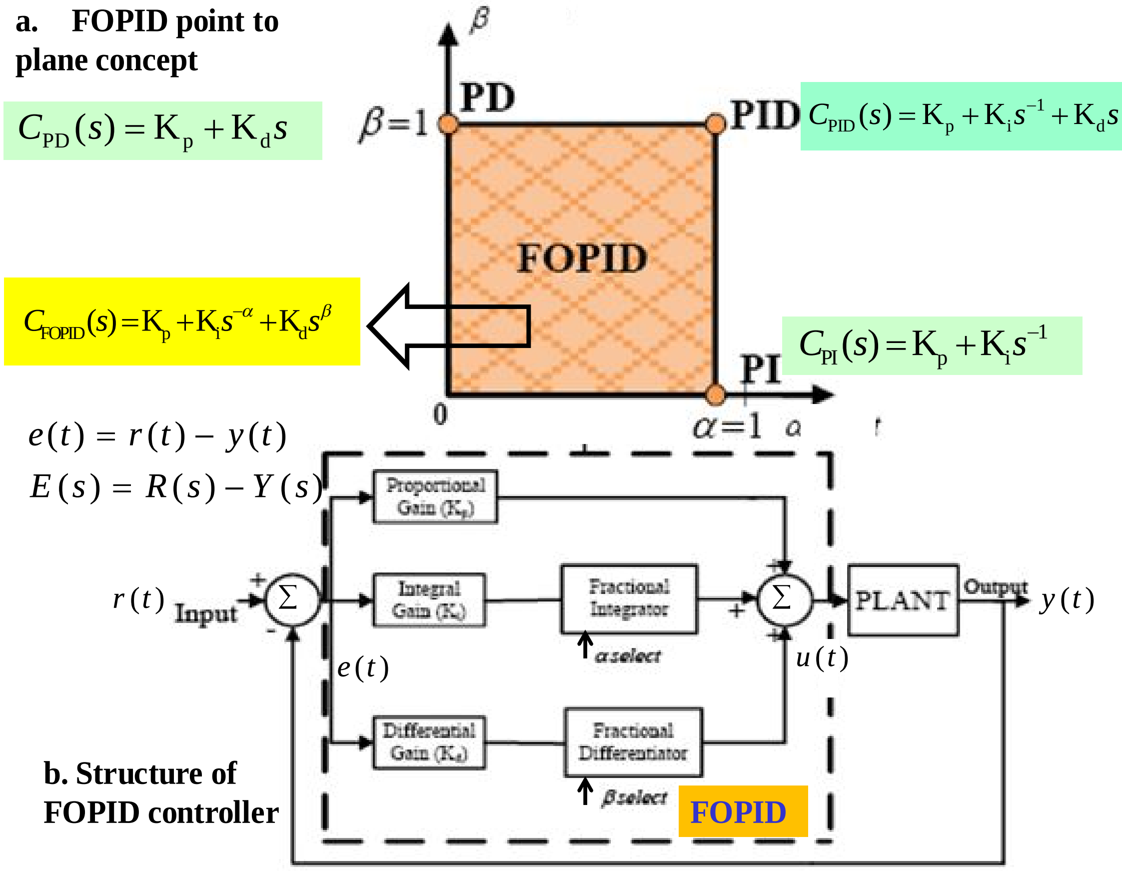


Figure-1: The concept of FOPID and structure of FOPID Controller

Practical realization of the fractional order differ-integrations-the algorithm

Fractional order differentiators and integrators are not available commercially. The fractional order Laplace element i.e. $s^{\pm\alpha}$; $\alpha \in \mathbb{R}$ *when implemented into analog or digital circuits is called 'fractance' [6]. Implementation of these 'fractance' is by using a band-limited integer order transfer function approximation i.e.in frequency band* $\omega_l \le \omega \le \omega_h$. *Thus, the fractional order Laplace operator i.e.* s^{α}; $\alpha \in \mathbb{R}$; *gets approximated by ratio of rational polynomials as-band limited integer order transfer function depicted as following*

$$s^{\pm\alpha} \approx \frac{(s+z_1)(s+z_2)...(s+z_N)}{(s+p_1)(s+p_2)...(s+p_N)}$$

in the band $\omega_l \le \omega \le \omega_h$ *[2], [3]. This rational approximation is then implemented by using analog circuit technique [6], [7], and then using digital techniques [8], [12], [13], [14].*

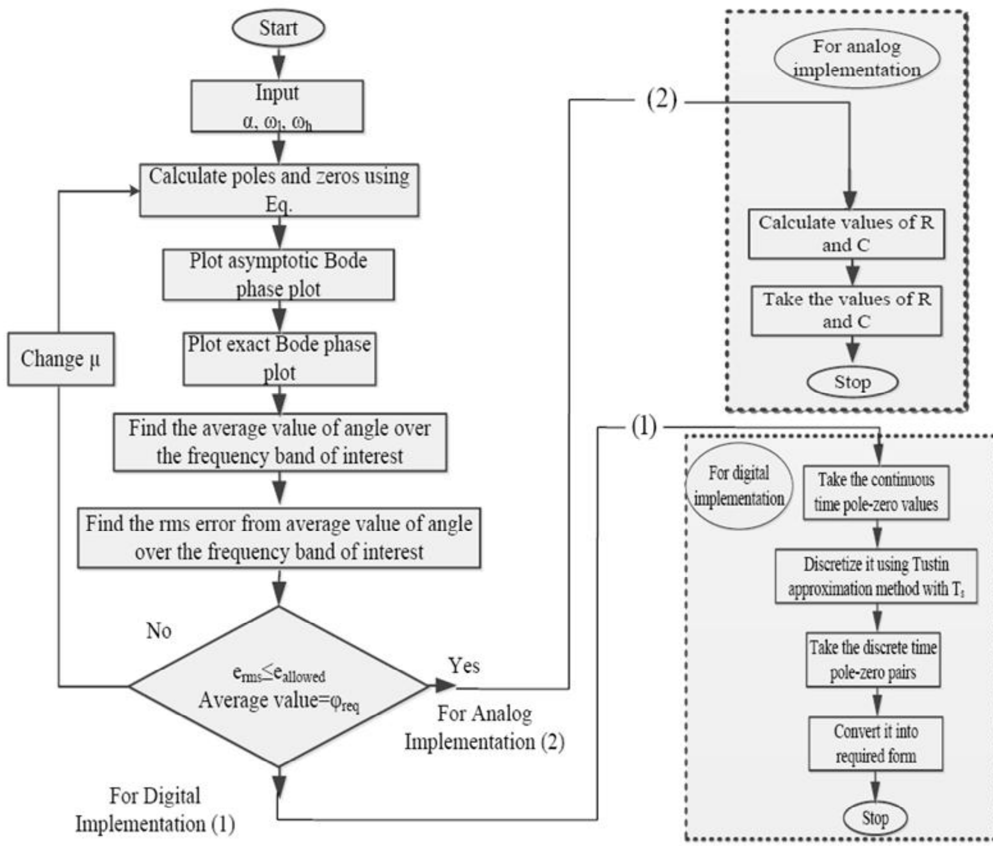
A novel pole-zero interlaced approximation method (a new proprietary algorithm) is developed to approximate the fractional order Laplace operators [6] [7]. The basic idea is of getting a *constant phase is by slope cancellation of asymptotic phase plots for zeros and poles [2], [3], [6], [7]. For fractional order integrator, first pole* p_1 *is selected such that its asymptotic phase plot passes through point* $(\omega_l, \phi_{\mathrm{req}})$*, then* z_1 *i.e. the first zero gets selected; and thereafter subsequent poles and zeros are selected so as to keep the asymptotic plot constant at* $\phi_{\mathrm{req}} = \alpha \times 90^0$*. For semi-integration operation* $\alpha = -0.5$ *so we get* $\phi_{\mathrm{req}} = -45^0$*. Thus our objective is to have phase angle* equal to ϕ_{req} *with an error less than less than or equal to* $\mathrm{e}_{\mathrm{allowed}}$*, i.e.* $\mathrm{e}_{\mathrm{rms}} \le \mathrm{e}_{\mathrm{allowed}}$ (*Figure-2*) *for all frequencies* ω *such that* $\omega_l \le \omega \le \omega_h$*. For a* n *number pole zero-pair we have the first pole* p_1 *as*

$$p_1 = 10^{\left(\frac{\phi_{\mathrm{req}} + 45\log\omega_l}{45} + 1\right)}$$

From this we calculate first zero as $z_1 = 10\omega_l$*; the second pole as* $p_2 = 10^{\log p_1 + 2 - \mu}$*, the second zero is at* $z_2 = 10^{\log z_1 + 2 - \mu}$ *continuing this we get* $n-$*th pole as* $p_n = 10^{(2-\mu)} p_{n-1}$ (*i.e. also* $p_n = 10^{(2-\mu)(n-1)} p_1$) *and* $n-$ *th zero as* $z_n = 10^{(2-\mu)} z_{n-1}$ *i.e. also* $z_n = 10^{(2-\mu)(n-1)} z_1$*); till* $p_n \ge \omega_h$*. This set of recurring relation is tagged as Eq. in Figure-2. After this we define RMS error in phase angle as following, which is required in the algorithm* (*Figure-2*) *with* x *defined as* $x = \log\omega$*, and* ϕ_{avg} *described as* $\phi_{\mathrm{avg}} = \frac{1}{x_h - x_l}\int_{x_l}^{x_h} \phi(x)\mathrm{d}x$*.*

$$\mathrm{e}_{\mathrm{rms}} = \sqrt{\frac{1}{x_h - x_l}\int_{x_l}^{x_h} \left(\phi(x) - \phi_{\mathrm{avg}}\right)^2 \mathrm{d}x}$$

We note here the values of band limits i.e. ω_l, ω_h *is based on system-identification, that is carried out a priori with the known dynamic model of the system [8], [12], [13], [14].*



<u>Figure-2: Algorithm of pole-zero approximation method for realizing fractional Laplace operator as ratio of rational polynomials</u>

In the described recurring relation tagged as Eq. we have a trim-parameter μ which is normally zero but is selected as to adjust density of p_i, z_i in the phase plot to have ϕ_{avg} as close to ϕ_{req}. The details of its usage with several examples are in [6], [7] and the range of μ is $0 \le \mu < 2$. The approximation with six-pole zero pair with $\omega_l = 10^2 \, \text{radian/sec}$, $\omega_h = 10^4 \, \text{radian/sec}$ with error $\pm 1^0$ for $\alpha = -0.5$ gives values of p_i and $z_i (i = 1-6)$ is listed in Table-1, with plot depicted in Figure-3c.

i	**1**	**2**	**3**	**4**	**5**	**6**
z_i	-7.0795	-51.286	-371.54	-2691.5	-19,498	-141,250
p_i	-2.6128	-18.928	-137.12	-993.34	-7191.6	-52,131

Table-1: The pole and zero of the rational approximation of semi-integration on band limited range of 100-10,000 radian/sec.

The rational approximation of semi-integration operation is thus as follows

$$s^{-0.5} \approx \frac{(s+7.0795)(s+51.286)(s+371.54)(s+2691.5)(s+19,498)(s+141,250)}{(s+2.6128)(s+18.928)(s+137.12)(s+993.34)(s+7191.6)(s+52,131)}$$

For a fractional order differentiator in the band limit of $\omega_l \le \omega \le \omega_h$ *we do first selection of first zero* z_1 *and then first pole* p_1*; and rest is same recurring method that is described for fractional integrator. Therefore for semi-differentiator i.e.* $s^{0.5}$ *the approximation will be reciprocal of what we obtained for semi-integration i.e.* $s^{-0.5}$ *as above.*

The algorithm gives the rational approximation in form of band limited transfer function for the fractional order Laplace variable $s^{\pm\alpha}$ *as indicated above. The digitization is done from* s *to* z *domain by using Tustin formula (Figure-2), i.e.* $s=\frac{2}{T_S}\left(\frac{z-1}{z+1}\right)$*. With* $T_S = 0.0001\sec$ *[8] we obtain digitized representation of semi-integration (of Table-1) as following transfer function.*

$$s^{-0.5}\Big|_{z-\text{domain}} = \left(0.008941\right)\frac{(z+0.752)(z-0.763)(z-0.964)(z-0.995)(z-0.999)(z-0.013)}{(z+0.446)(z-0.471)(z-0.905)(z-0.986)(z-0.998)(z-1)}$$

The discrete transfer function is implemented by using standard digital-filter algorithm [8], [12], [13] [14]. With the obtained approximated fractional Laplace operator for s^{α}, s^{β} *the FOPID transfer function i.e.* $C_{\text{FOPID}}(s) = \text{K}_\text{p} + \text{K}_\text{i}s^{-\alpha} + \text{K}_\text{d}s^{\beta}$ *is dicretized to* $z-$*domain (Figure-1). Thereafter choosing the standard digital-filter formula a difference equation for discretized* $C_{\text{FOPID}}(s)$ *is obtained. This difference equation relates discretized* $e(t)$ *stated as* e_k *to get discretized controller output* $u(t)$ *stated as* u_k *which is given as* $u_k = \sum_{i=1}^{m} a_i u_{k-i} + \sum_{j=1}^{n} b_j e_{k-j}$ *[8], [12], [13], and [14].*

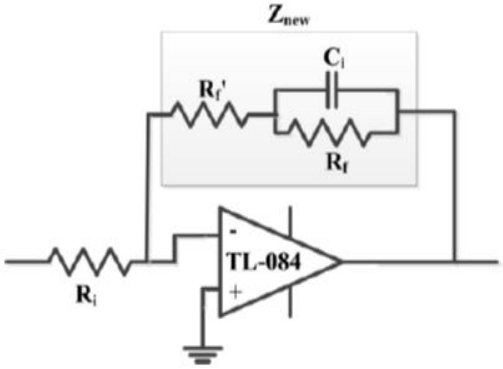
Analog FOPID Controller

The fractional order impedance or 'fractance' circuit is realized with two port network having passive components resistor (R) *and capacitor* (C) *[6] [7], along with operational amplifiers (Figure-3-i-a).This analog circuit of Figure-3-i-a is designed to generate the pole-zero pairs by use of available* R-C *components designed for a given fractional order* $(\alpha;\ \alpha \in \mathbb{R})$*. The asymptotic phase plot of fractional order integrator is shown in Figure-3-i-c for* $\alpha = -0.5$*; for frequency band of* $10^2 \le \omega \le 10^4$*. The Figure-3-i-b shows a photograph of developed 'fractance'* *circuit with six poles-zero pair i.e. having six circuits* $\mathrm{i} = 1 - 6$ *of Figure-3-i-a connected in series with different values of* $\mathrm{R_i}, \mathrm{R_{f'}}, \mathrm{C_i}; \mathrm{R'_f}$. *Refer Table-2 for the values of the six circuits [7].*

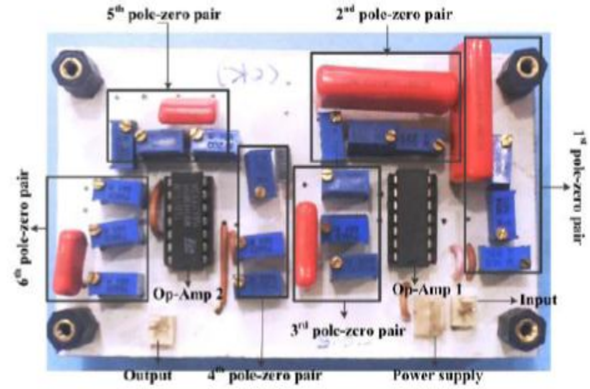
Circuit Section $\mathrm{i} \rightarrow$	**1**	**2**	**3**	**4**	**5**	**6**
$\mathrm{C_i}$	1μF	1μF	0.47μF	0.068μF	10nF	2.2nF
$\mathrm{R'_i} = \mathrm{R'_f}$	382.7kΩ	52.8kΩ	15.52kΩ	14.8kΩ	13.8kΩ	8.71kΩ
$\mathrm{R_i} = \mathrm{R_f}$	223.9kΩ	30.9kΩ	9.08kΩ	8.66kΩ	8.13kΩ	5.1kΩ

Table-2: R-C values of six circuits for realizing the half order integrator

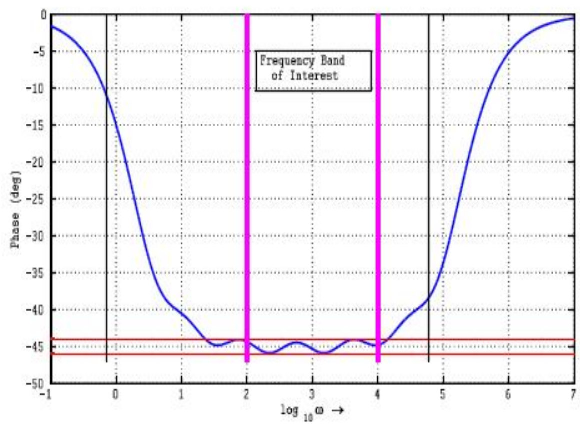
A semi-integrator for the required specifications of band-limited approximation is realized by six poles-zero pairs as per algorithm of Figure-2, presented in Table-1. The resistors and capacitor values of the circuit of Figure-2a given in Table-2 is by formula (*a*): *for fractional Integrator* $\mathrm{R_i} = \frac{1}{p_i \mathrm{C_i}}; \quad \mathrm{R'_i} = \frac{1}{(z_i - p_i)\mathrm{C_i}}$ *and* (*b*): *for fractional differentiator* $\mathrm{R_i} = \frac{1}{z_i \mathrm{C_i}}, \quad \mathrm{R'_i} = \frac{1}{(p_i - z_i)\mathrm{C_i}}$ *[7]. In the formula we select first the available value of capacitor and then the resistor value is calculated. The exact resistor value is adjusted by using standard E48 series resistors with a potentiometer in series. The algorithm, as shown in Figure-2, is developed to determine the actual values of resistor and capacitor components. Then these 'fractance' circuits are organized* (*with operational amplifiers*) *as shown in Figure-1b, to get analog FOPID controller. The analog FOPID controller is shown in Figure-3-ii and 4a, it has got facility to select fractional order of differentiation* β, *and fractional order integrator* α *as* $0.2, 0.5$ *and* 0.8*; and also via potentiometer one can select the gain values of the 'gains'* $\mathrm{K_p}$, $\mathrm{K_i}$ *and* $\mathrm{K_d}$.



a. Proposed circuit implementation of one p-z pair



b. Photograph of developed Fractance



c. Phase plot for $\phi_{req} = 45^{o}$ on band of (100, 10000) rad/sec

Figure-3-i: Analog realization of band limited fractional differ-integral operator

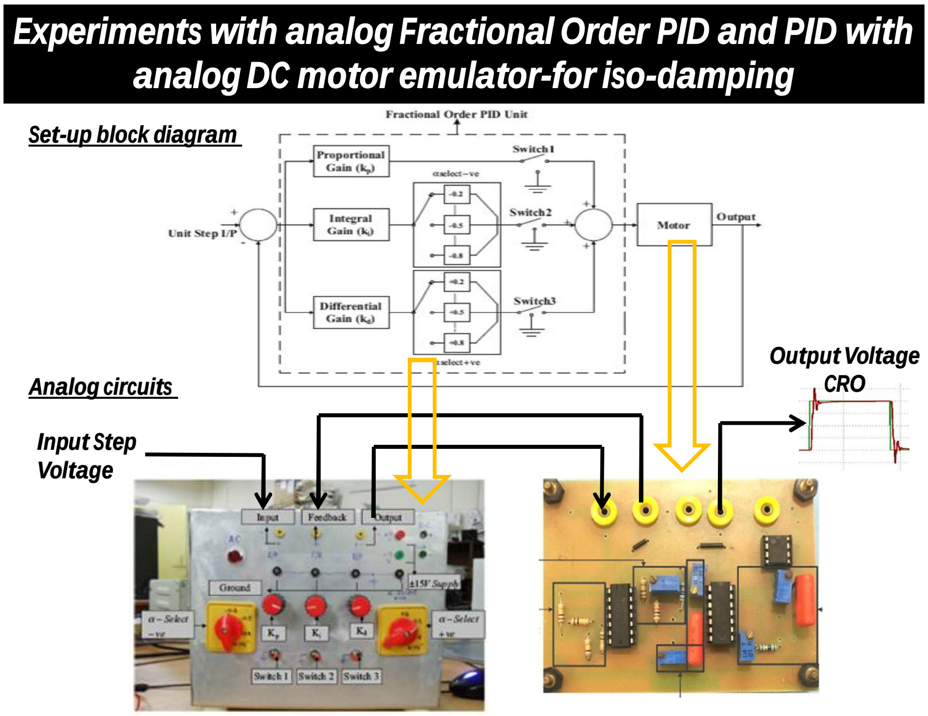
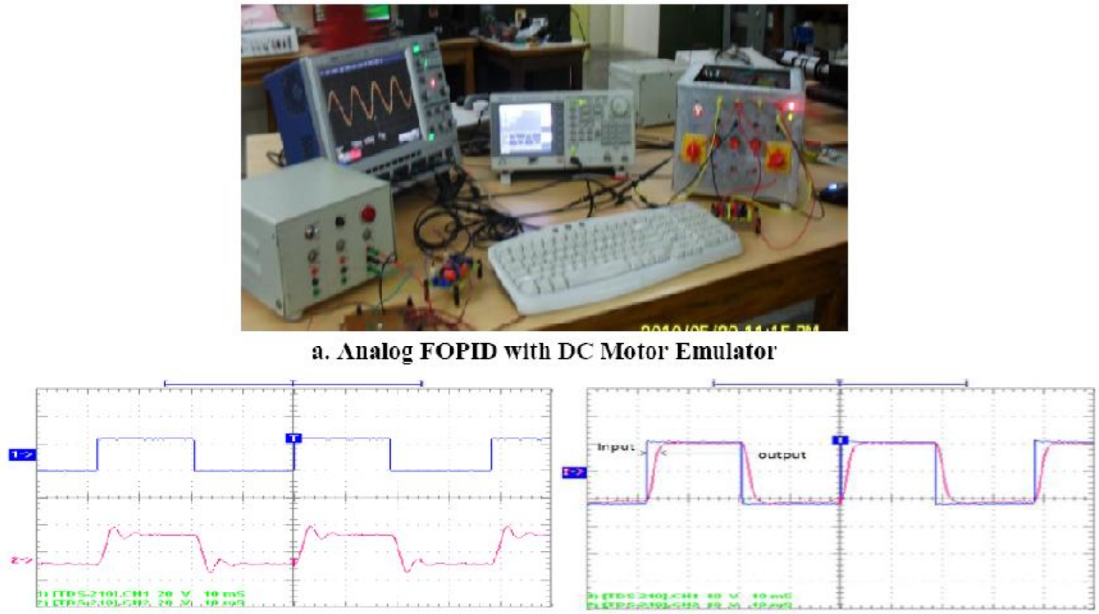


Figure-3-ii: Experiments set-up for demonstrating iso-damping robustness on analog circuits FO-PID with DC Motor Emulator

To verify the analog FOPID controller performance, a hardwired emulator circuit of DC motor (i.e. plant) is developed. Plant consists of a DC motor and a load with specification as: Speed $\mathrm{N} = 2000\mathrm{rpm}$*, the armature resistance as* $\mathrm{R_a} = 2\Omega$*, armature inductance as* $\mathrm{L_a} = 3\mathrm{mH}$*, rotor inertia* $\mathrm{J} = 1.78 \times 10^{-4}\,\mathrm{Kg-m^2}$*, motor constant* $\mathrm{K_v} = 1.02$*,DC armature voltage* $\mathrm{V}_a = 24\mathrm{V}$ *[7].The plant transfer function* $G(s)$ *of DC motor a second order stable function realized by circuit is*

$$G(s) = \frac{\omega(s)}{\mathrm{V}(s)} = \frac{\mathrm{K_v}/\mathrm{L_a J}}{s^2 + \frac{\mathrm{R_a}}{\mathrm{L_a}} s + \frac{\mathrm{K_v^2}}{\mathrm{JL_a}}}$$

The complete hardware setup is shown in Figure-3-ii. Performance test result of DC motor model emulator circuit is shown in Figure- 4b.The performance of the realized analog FOPID controller is tested with the DC motor emulator. The FOPID performance indicates that the controller with $\alpha, \beta = 0.5, \quad \mathrm{K_i} = \mathrm{K_d} = 1$ *could make it possible to maintain the desired control on the output speed. The response of DC motor model with FOPID controller is shown in Figure-4c.*



a. Analog FOPID with DC Motor Emulator

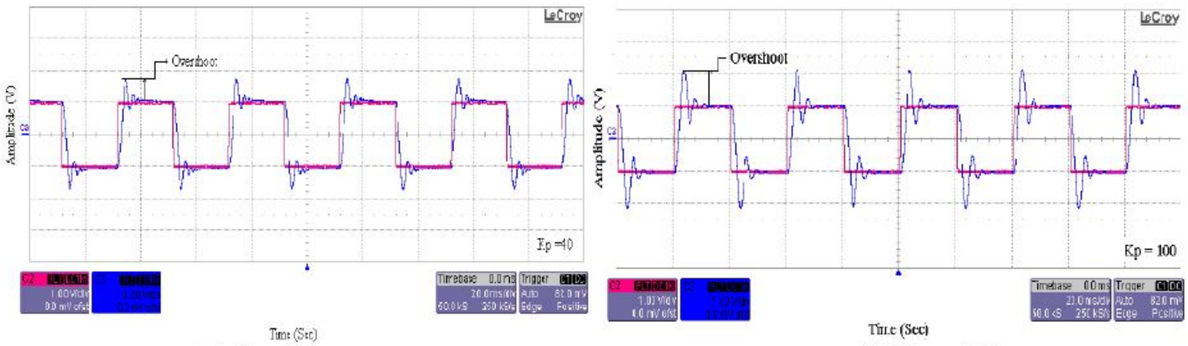
b. Result of DC Motor Circuit Emulator (the plant)

c. Result of DC Motor with FOPID controller

<u>Figure-4: Analog FOPID controlling Speed of DC Motor Emulator</u>

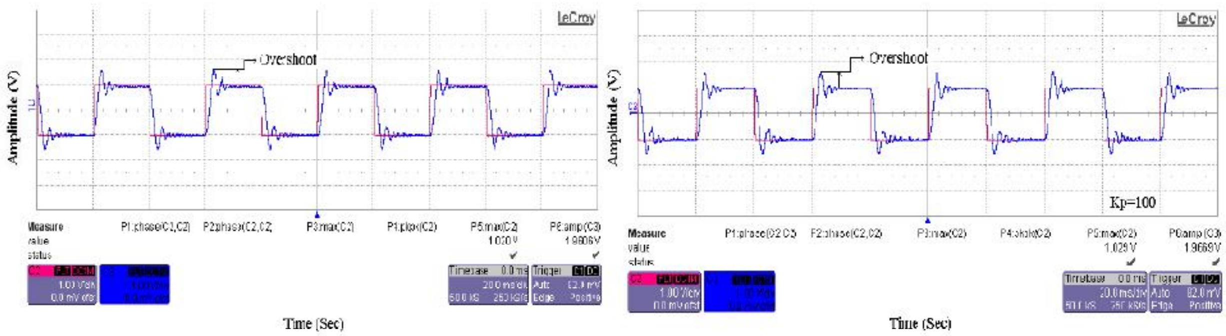
Observed Robustness- 'iso-damping' by using FOPID as compared to classical PID controller

The variation in gain $\mathrm{K_p}$ from 40 to 100 with $\alpha, \beta = 0.5, \quad \mathrm{K_i} = \mathrm{K_d} = 1$ is set with PID and FOPID controller. The recorded response is given in the CRO traces of Figures-5 and 6. From CRO traces of Figures-5(a) and (b), it is observed that the peak overshoot with a PID controller is varying widely, when the gain $\mathrm{K_p}$ is changed from 40 to 100. Whereas, from CRO traces of Figures-6(a) and (b), the peak overshoot is almost constant in FOPID controlled system, while the gain $\mathrm{K_p}$ is changed from 40 to 100. This phenomena of overshoot remaining constant over wide parametric spread is called 'iso-damping', is a feature what we get via using FOPID; thus we have enhanced robustness in controls [2], [3], [7].



(a) K_p=40 (b) K_p = 100

Figure-5: Hardware results of PID with DC motor emulator at different parametric gain variation



(a) K_p=40 (b) K_p = 100

Figure-6: Hardware results of FO-PID with DC motor emulator at different parametric gain variation

The Table-3 gives in gist the values of overshoot with parametric spread for PID and FOPID, experimentally measured and with simulations. The Table-3 show that with variation in gain from 40-100 the closed loop response is having overshoot constant at about 25%. This is 'iso-damping', that is got by FOPID while the overshoot varies from 22-65% in case of PID. This is enhancement in robustness experimentally demonstrated by use of FOPID.

Values of K_p (gain)	Close-loop PID		Close-loop FO-PID	
	% Peak overshoot simulation results	% Peak overshoot hardware results	% Peak overshoot simulation results	% Peak overshoot hardware results
40	22	30	25	24
50	30	36	25	24
60	45	44	25	24
70	53	52	25.5	25
100	65	60	25.5	25

Table-3: Comparison of peak overshoots with variation of gain for PID and FOPID

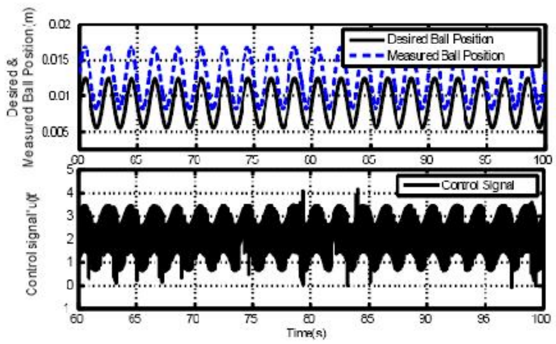
Digital FOPID Controller

a) Magnetic Levitation System controlled by PID and FOPID controller

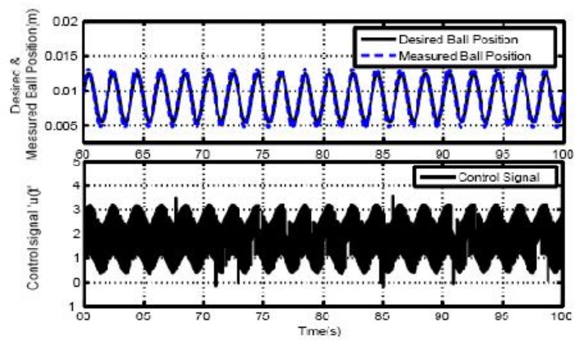
This designed digital FOPID controller is used to control highly non-linear and inherently unstable Magnetic Levitation System (Mag-Lev) in hardware-in-loop mode as shown in Figure-7a. Mag-Lev is basically an electromagnetic system which levitates ferromagnetic objects in space by the magnetic force induced due to the electric current flowing through the coils around a solenoid [8]. The tuned controllers that are compared are having transfer functions, for PD as $C_{\mathrm{PD}}(s) = 4 + 2s$ *, PID as* $C_{\mathrm{PID}}(s) = 5.5 + 0.2s^{-1} + 2s$ *and FOPID as* $C_{\mathrm{FOPID}}(s) = 7 + 12s^{-0.8} + s^{0.4}$*.*

The continuous time pole-zero interlaced approximation method (Figure-2) to get rational approximation for $s^{\pm\alpha}$ *is discretized, using Tustin formula (with* $T_S = 0.01\sec$*) converting* $s-$*domain to* $z-$*domain, with sampling time* $T_S = 0.01\sec$*; in the band* $10^2 \le \omega \le 10^4$ *and then a digital FOPID controller is developed [8].*

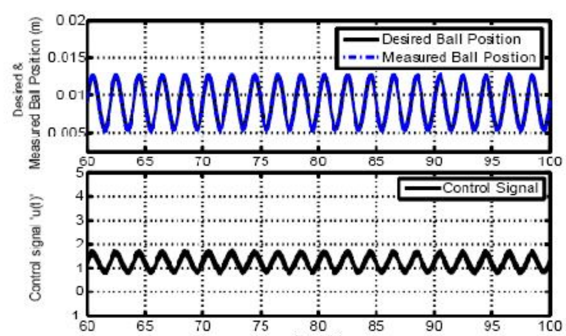
a. Experimental set up of Mag-Lev System



b. Controlling with PD



c. Controlling with PID



d. Controlling with FOPID

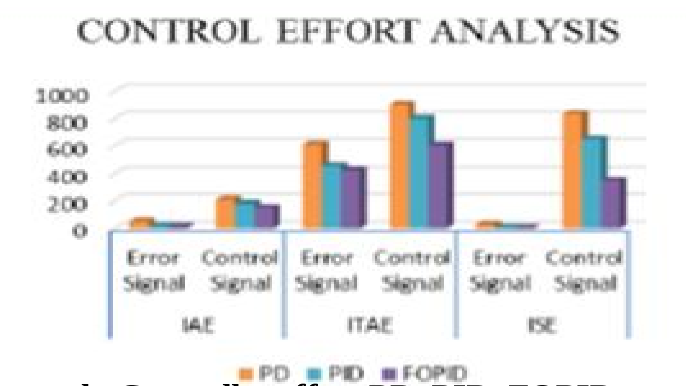
Figure-7: Controlling Mag-Lev System by PD, PID and FOPID

The performance analysis for digital classical PID, PD, and FOPID is carried out. The results in CRO traces of Figures-7b, c, d that show a better control over position accuracy with lesser control efforts $u(t)$ *is achieved with FOPID over the conventional methods i.e. PID, PD [8]. In practical terms, this improvement of controlling with lesser effort translates to better energy/fuel efficiency [2], [3], [8], [9], and [10].*

The control signals $u(t)$ *along with measures as Performance Indices (P.I) those are Integral Absolute (IA), Integral Time Absolute (ITA), and Integral Square (IS) [11] are the indicator of energy utilized by the controller, an important factor in the industrial control paradigm. In this* *context, the fractional order controller proves to be superior to the classical controller. The comparison is presented between PD, PID, and FOPID controls in Figure-8, as for ball position error (Figure-8a) and controller performances (Figure-8b) via comparing performance indices IA, ITA, and IS, calculated on (i) error signal $e(t)$ and (ii) control signal $u(t)$ i.e. output of the controller.*

Acual and desired ball position				
Ball Position (m)		**PD**	**PID**	**FOPID**
Actual Position	**Max.**	16.8X10^{-3}	13.1X10^{-3}	12.6X10^{-3}
	Min.	8.3X10^{-3}	4.85X10^{-3}	5.24X10^{-3}
Desired position	**Max.**	12.5X10^{-3}	12.5X10^{-3}	12.5X10-3
	Min.	5.5X10^{-3}	5.5X10^{-3}	5.5X10^{-3}
Error		29.66%	8.95%	5.75%

a. Error in ball position in controlled Mag-Lev system with PD, PID and FOPID



b. Controller effort PD, PID, FOPID

Figure-8: Comparison of controls PD, PID, and FOPID in Mag-Lev System

b) DC Motor Speed Control by PID and FOPID controller

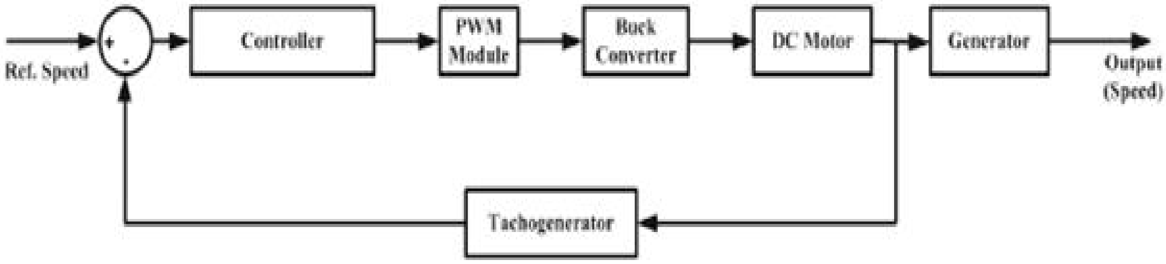
The digital FOPID controller is tested for 1.5kW industrial DC motor drive [12], [13]. Speed control scheme of buck converter fed DC motor drive is shown in Figure-9a. Here, a reference speed is given as set point for FOPID controller, which provides the control signal computing the error generated in the control scheme. The PWM pulses are generated at frequency of 25 KHz with corresponding duty-ratio proportional to the controller output [12], [13]. This signal operates the switch of buck converter and regulates the armature voltage of the DC motor.The controller is implemented on Digital Signal Processor (DSP) TMS320F28027 and TMS320F28377s. An industrial DC motor drive is developed as shown in the Figure-9b.

Here, it is recorded that tuned PID used for this DC motor is $C_{\mathrm{PID}}(s) = 115.6 + 0.22s^{-1} + 1.6s$ *and tuned FOPID used is* $C_{\mathrm{FOPID}}(s) = 15.2 + 0.04s^{-1.4} + 2.4s^{1.2}$*. This tuning resulted in minimizing the Performance Indices (P.I) as shown in the Table-4, with respect to error signal* $e(t)$*. The Table-5 gives the values of performance indices of controller effort with respect to control signal of the controller i.e.* $u(t)$*. The discretized difference equation after following the algorithms of Figure-2, for approximating fractional Laplace operators in band limit of* $10^2 \le \omega \le 10^4$*, with* $T_S = 0.0001\mathrm{sec}$ *for the tuned FOPID controller in standard digital filter formulation is following*

$$\begin{aligned} u_k = {} & 8.819u_{k-1} - 32.14u_{k-2} + 57.74u_{k-3} - 35.01u_{k-4} - 58.1u_{k-5} + 135.7u_{k-6} - 91.19u_{k-7} \\ & - 34.11u_{k-8} + 96.87u_{k-9} - 55.16u_{k-10} - 5.602u_{k-11} + 21.75u_{k-12} - 9.468u_{k-13} + 0.0135u_{k-14} \\ & + 1.264u_{k-15} - 0.4057u_{k-16} + 0.04253u_{k-17} \\ & + 1.357e_{k-1} - 12.35e_{k-2} + 47.16e_{k-3} - 92.21e_{k-4} + 76.22e_{k-5} + 49.99e_{k-6} - 186.8e_{k-7} \\ & + 168.5e_{k-8} - 5.914e_{k-9} - 113.9e_{k-10} + 92.89e_{k-11} - 15.65e_{k-12} - 20.24e_{k-13} + 14.01e_{k-14} \\ & - 2.729e_{k-15} - 0.5494e_{k-16} + 0.3192e_{k-17} - 0.0394e_{k-18} \end{aligned}$$

The measurement is done for armature current and armature voltage for various speed settings for DC motor, in no-load condition and with loaded condition (coupled to a Generator and then loading the generator via resistive load banks). The Figure-10 displays the experimental result.

It is observed that averagely 21.3% less power is drawn from DC source at no-load condition and averagely 19.6% less power is drawn at loaded condition for speed settings from 500RPM to 1300RPM. This is direct evidence of having Energy/Fuel Efficiency by using FOPID.



a. Block Diagram of Circuit & System DC Motor Speed Controls

b. Actual implemented circuit for speedcontrol of DC motor

Figure-9: DC motor speed control by FOPID

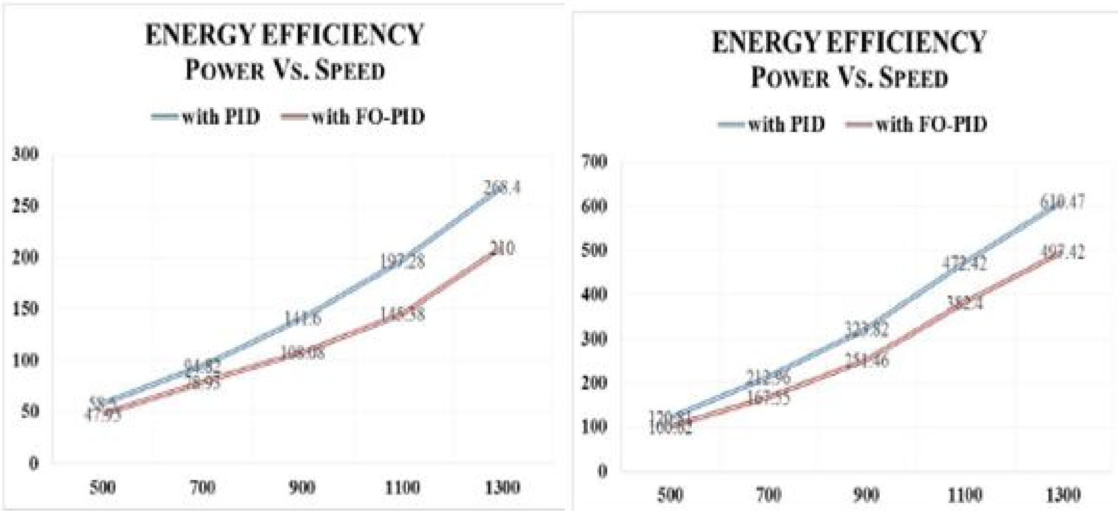
S.No.	Controller	ITAE	IAE	ISE
1	PID	34.21	46.75	7454
2	FOPID	14.94	20.85	6086

Table-4: Minimized performance index for error signal for PID & FOPID controller

S.No.	Controller	ITACE	IACE	ISCE
1	PID	46.56	51.35	10080
2	FOPID	20.85	50.63	6663

Table-5: Minimized performance index for control effort for PID & FOPID controller

Experimental records of Energy/Fuel Efficiency for DC Motor Speed Controls-showing less input power in case of FO-PID for various speeds



No-Load Condition

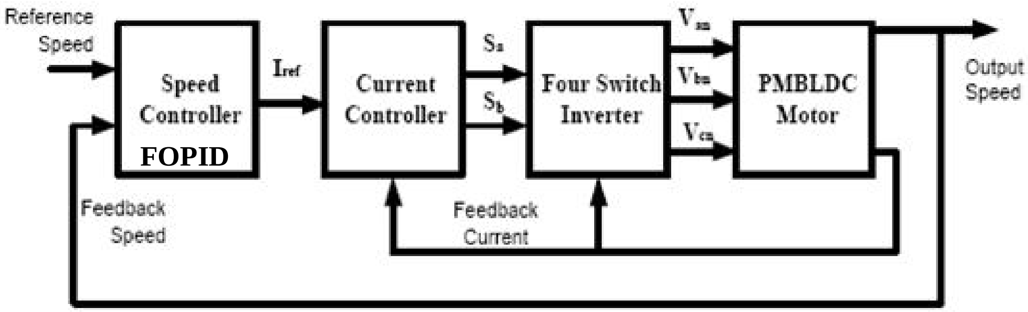
Loaded Condition

The lower performance indices of error and control signal got via tuned FOPID vis-à-vis tuned classical PID manifests as Energy/Fuel efficiency in operation

Figure-10: Measured power input to DC motor for control with tuned PID and tuned FOPID

c) BLDC Motor Speed Control by PID and FOPID controller

The speed control scheme for FOPID controller fed brushless DC motor (BLDC) 0.5kW, 350 RPM [14] drive is shown in Figure-11a. Here, the scheme is having FPGA-in-the-loop. The digital FOPID controller is implemented on Altera FPGA DE2-115 board [14]. Figures-11b, c, d gives the comparison between the tuned PID controller and tuned FOPID controller. The Figure-12 show the comparison of control signal and phase current of BLDC motor controlled by PID & FOPID. We observe significant reduction in the phase current drawn while controlling via FOPID controller as compared with classical PID controller.



a. Block Diagram of BLDC Motor Speed Control

S. N.	Controller	Gain and Fractional Order Value				
		K_p	K_i	K_d	α	β
1.	PID	8	1.295	0.22	1	1
2.	FOPID	4.25	0.2	0.099	1.21	0.6

b. Tuned controller parameters PID & FOPID

S. N.	Controller	Overshoot (%)	Settling Time (s)
1.	PID	1.6	0.35
	FOPID	0.8	0.065

c. Transient performance comparison PID & FOPID

S.N.	Controller	Error Signal			Control Signal		
		ITAE	IAE	ISE	ITACE	IACE	ISCE
1	PID	3.047	25.53	3154	239.1	817.7	994000
2	FOPID	1.145	9.568	1300	5.934	43.1	24350

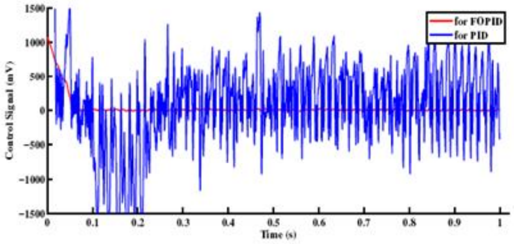
d. Performance Indices Comparison PID & FOPID

Figure-11 BLDC Motor control by PID/FOPID

The tuned FOPID transfer function is $C_{\text{FOPID}}(s) = 4.25 + 0.2s^{-1.21} + 0.099s^{0.6}$. *The fractional Laplace variables* $s^{-1.21}$ *and* $s^{0.6}$ *we approximate as per algorithm described in the form* $s^{\pm\alpha} \approx \frac{\prod_{i=1}^{n}(s+z_i)}{\prod_{i=1}^{n}(s+p_i)}$ *in the frequency band* $10^{-1} \le \omega \le 10^{3}$ *with phase angle error* $\text{e}_{\text{allowed}} = 1^0$ (*Figure-2*)

Then the obtained expression for $C_{\text{FOPID}}(s)$ *we discretize by Tustin method with* $T_S = 0.001\text{sec}$ *to get FOPID in* z*-domain as*

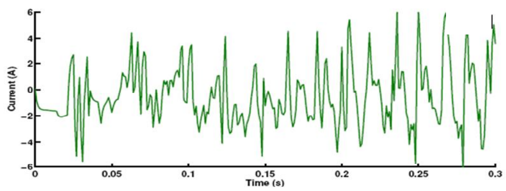
$$C_{\text{FOPID}}(z) = \frac{U(z)}{E(z)} = \frac{\begin{array}{c}116.4z^{17} - 773.7z^{16} + 1797z^{15} - 802.6z^{14} - 3730z^{13} + 6455z^{12} - 1002z^{11} - \\ 6953z^{10} + 6347z^{9} + 914z^{8} - 4282z^{7} - 1860z^{6} + 606.9z^{5} - \\ 730.9z^{4} + 162.1z^{3} + 29.75z^{2} - 15.32z + 1.24\end{array}}{\begin{array}{c}z^{17} - 5.487z^{16} + 8.358z^{15} + 7.387z^{14} - 33.62z^{13} + 20.23z^{12} + 35.87z^{11} - \\ 51.52z^{10} - 2.157z^{9} + 40.67z^{8} - 18.39z^{9} - 11.29z^{6} + 10.83z^{5} - \\ 0.4359z^{4} - 1.979z^{3} + 0.481z^{2} + 0.09213z - 0.03333\end{array}}$$



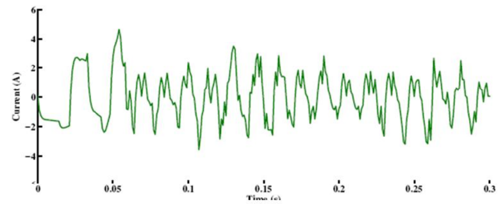
a. Trace of control signals for PID & FOPID for step change in speed demand of BLDC motor

Sr. No.	Controllers	RMS currents (A)		
		I_a	I_b	I_c
1	PID	1.449	2.208	1.736
2	FOPID	0.938	0.886	1.016

b. Comparison of RMS Currents for the three phases of BLDC motor controlled with PID & FOPID



c. Trace of one of the phase currents of BLDC motor control with PID



d. Trace of one of the phase currents of BLDC motor control with FOPID

Figure-12: Performance comparison of BLDC motor speed control with PID and FOPID

Summary

- *FOPID controller is implemented with analog circuit components and tested with DC motor emulator as a plant. It is found to enhance robustness of system for inherent parametric uncertainties and spreads. We get iso-damped response by using FOPID with overshoot remaining constant for wide parametric changes.*
- *Digital FOPID controller is applied to nonlinear unstable Mag-Lev system. The control effort observed is maximum in the case of PD controller and least in the case of FOPID controller which points toward the energy-efficient nature of the FOPID controller. The experiment show RMS voltage exciting the coil amplifier is lesser in case of FOPID case, hence reducing losses and thus increase efficiency in drive.*
- *Digital FOPID controller is implemented on DSP platform as a stand-alone controller and tested with 1.5kW DC motor drive; and was demonstrated to consume lesser energy compared to classical PID controller. This gives experimental validation to Fuel/Energy efficient controls by employing FOPID in drive system control.*
- *Digital FOPID is implemented in the FPGA-in-loop mode for BLDC motor drive demonstrated lesser current drawn as compared to classical PID controller, this enhances Fuel efficiency.*

Fractional Calculus Engineering Laboratory

Several of experiments that are developed (including the ones presented in this article) are kept in 'Fractional Calculus Engineering Laboratory' at VNIT-Nagpur. This lab was covered by Times of India Report: https://timesofindia.indiatimes.com/city/nagpur/VNIT-develops-energy-minimizing-controller-transfers-tech-to-BARC/articleshow/55444793.cms. Figure-13 shows the inauguration of the Laboratory. This laboratory is the first of its kind in globe that was dedicated to Nation on 15 November 2016, for using as a platform for advancement in 'Fractional Order Control Science' and to develop further industrial applications

<u>Figure-13: Inauguration of 'Fractional Calculus Engineering Laboratory' at VNIT-Nagpur Nov, 15, 2016</u>

Conclusions

These practical demonstrations of implementation of fractional calculus in control science, gives a first of its kind a laboratory called 'Fractional Calculus Engineering Laboratory'; where our aim is to have more fractional calculus based controllers and advanced methods developed for industrial usage. Though there is no commercial manufacturer or R&D institute yet, using fractional calculus in controls, perhaps in future we hope this platform will be used for such developments for especially energy/fuel efficient and enhanced robust systems. Presently our aim is to make few more systems on power electronics of electro-mechanical drive and energy conversion systems using fractional calculus; and take this laboratory to industry houses and academic institutes & universities. However, it is satisfying to see the long standing *conjecture/hypothesis of fuel/energy efficient controls is realized via using "Non-Newtonian calculus" or "fractional calculus". Still we have miles to go.*

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