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Application Potential Technology Status & Commercialization

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Super-capacitor Application in Electronic Circuits

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www.shantanudaslecture.com

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Preliminary about capacitor and super-capacitor

Some circuits made with CAG super-capacitors

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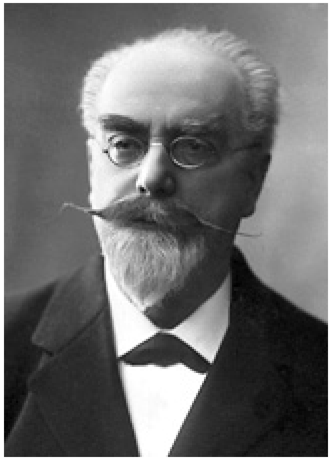
About fixed power pulsed load

Conclusion

Preliminary about capacitor and super-capacitor

Salute to the founder of the Electric Double Layer Capacity (EDLC)



Gabriel Lippmann

We hardly relate to famous electro-capillary experiment of 1875 by Gabriel Lippmann a French physicist who demonstrated that change in surface tension at electrode-electrolyte interface due to change of potential of electrode-electrolyte manifest as surface charges-giving rise to EDLC.

The curves are called famous Lippmann electro-capillary curves, and the curvature gives EDLC-i.e. the capacity

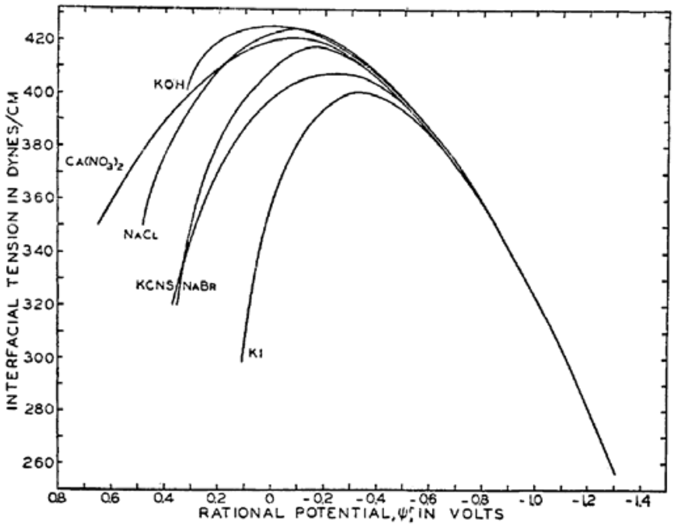
EDLC is basis of super-capacitor technology, where surface excess charges are separated via angstrom distances, manifesting in very high capacity. This is modern electro-chemical phenomena.

Super-capacitors are electrochemical capacitors whereas the others fall into category of electrostatic capacitors. These super-capacitors are of two types faradic and non-faradic. CAG-is non-faradic one.

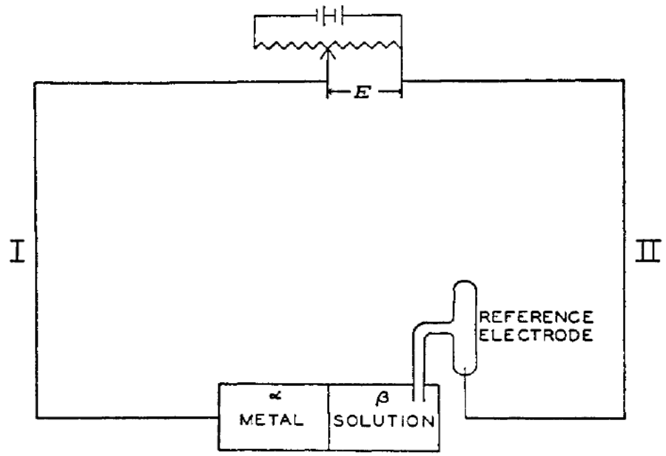
The electro-chemical double layer-from electro-capillary equation

Dates back to 1875 where Lippmann experimented with variable potential to Mercury and electrolyte Hg_2Cl_2 and measured surface tension of mercury via capillary phenomena. The curves he obtained were called Electro-Capillary curves. The Lippmann equation governing charge (rather excess charge) to change in surface tension is also called Electro capillary equations

Lippmann curves



Lippmann experiment scheme



$$\frac{d\gamma}{dE} = -q$$ This is Lippmann equation of electro-capillary

If C is defined as differential capacitance per unit area then

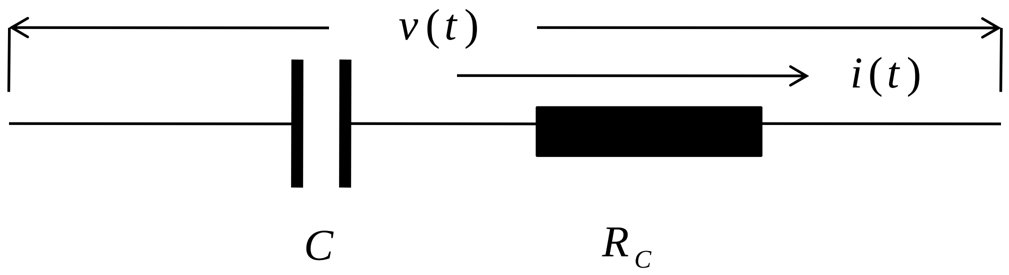
$$C \stackrel{\text{def}}{=} \left(\frac{\partial q}{\partial E}\right) = -\left(\frac{\partial^2 \gamma}{\partial E^2}\right)$$ If C is constant the Lippmann curve is parabola a convex one

Therefore the double layer capacity is due to double rate of change of surface tension w.r.t. potential

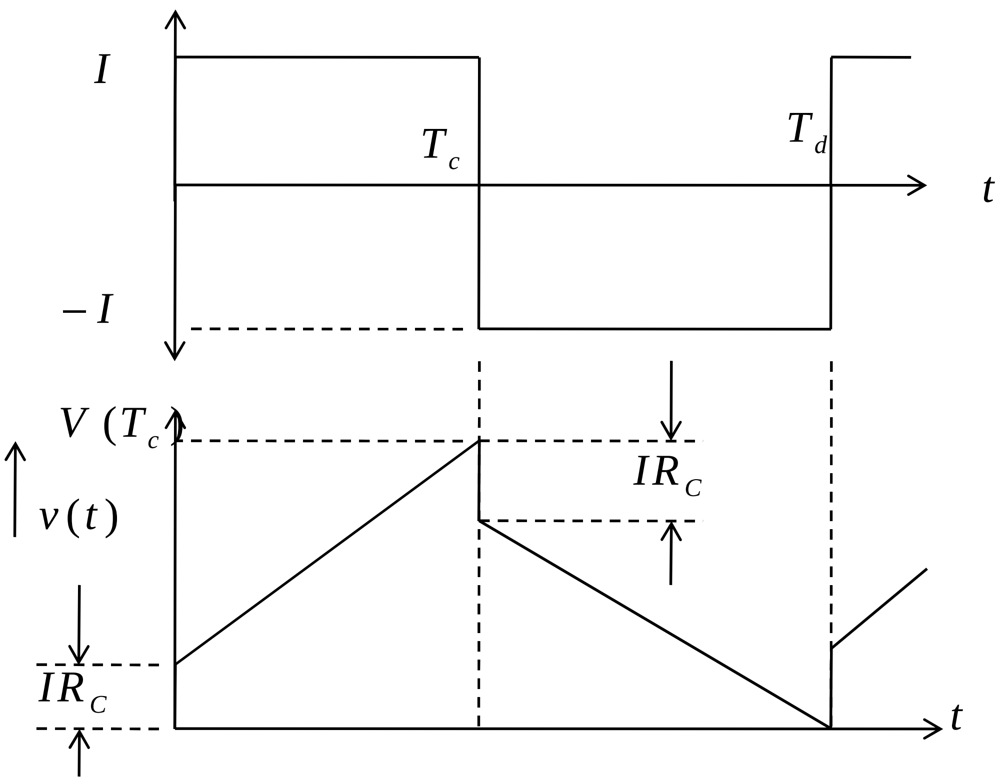
We leave the interesting physics and move to application in electronics

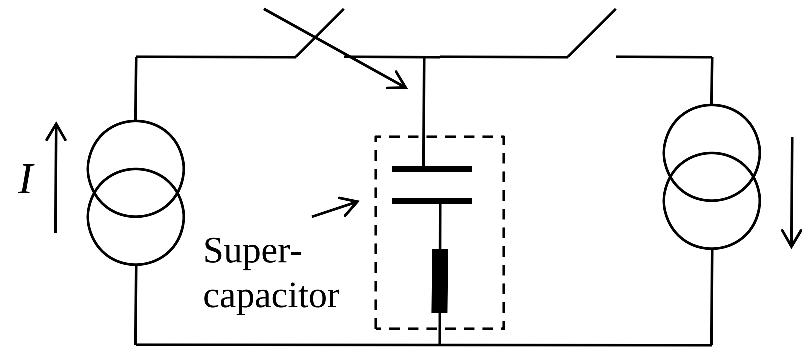
Representing super-capacitor for circuital applications-
and constant current charging/discharging-a very simplified approach

Simply we say super capacitor is composed of a capacity C Farads and Equivalent Series Resistance ESR as R_C ohm.



Ideal charge discharge with constant current source and constant current sink





$$v(t) = i(t)R_C + \frac{1}{C}\int_0^t i(x)\,\mathrm{d}x$$

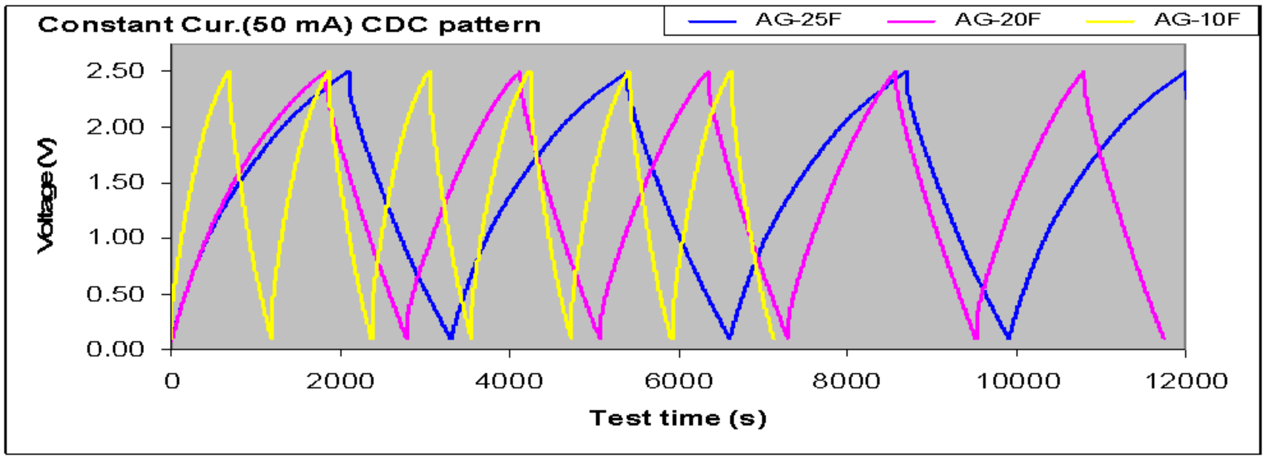
$$V(T_c) = I \times R_C + \frac{I \times T_c}{C}$$

Actually it is ideal representation

$$i(t) = C\frac{\mathrm{d}v}{\mathrm{d}t} \qquad Q(t) = Cv(t)$$

This is ideal text book representation of capacitor, with ESR R_C as zero

The evidence show that we should change the representation by fractional derivative. The voltage is not linearly rising or falling with constant current instead it is behaving as t^n where $n \sim 0.5$



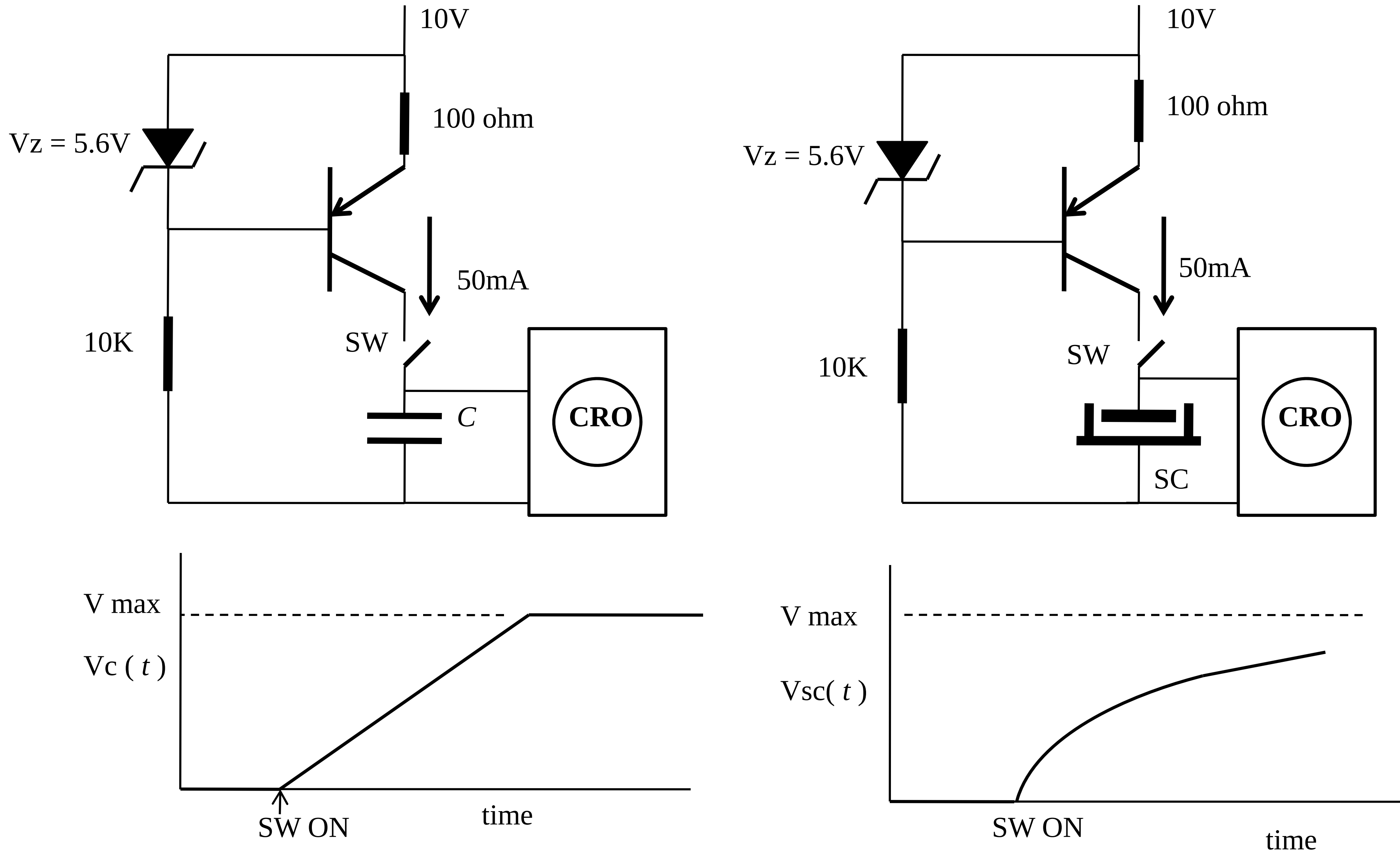
Constant current (50 mA) charge-discharge pattern of 10F, 20 F and 25 F aerogel super-capacitors, studied by using Super Capacitor Test System.
(Courtesy CMET Thrissur)

The new relation is

$$i(t) = C\frac{\mathrm{d}v}{\mathrm{d}t} + C_n\frac{\mathrm{d}^n v(t)}{\mathrm{d}t^n} \qquad Q(t) = Cv(t) + C_n\frac{\mathrm{d}^{n-1} v(t)}{\mathrm{d}t^{n-1}} \qquad 0 < n < 1$$

For super-capacitor case $n \sim 0.5$

Constant current charging of capacitor and super-capacitor-the difference



BRNS Sanction No.2009/34/31/BRNS dated 22.10.2009

Consequence

The new relation is therefore gives us a new theory and new mathematics with to deal with $\frac{d^n}{dt^n}, n \in \mathbb{R}$

The new relation is therefore asking us to modify the existing IEC-62931-2007 Standard to test super capacitor-with $Y_C(s) = sC + s^n C_n + R_C^{-1}$ or $Z_C(s) = (sC)^{-1} + (s^{-n} C_n^{-1}) + R_C$

The new relation is therefore asking for extraction of fractional capacity C_n and fractional order of derivative n

Asking us to have new-capacitor theory and thus new-circuit theory

Here we will not consider the fractional order behavior. Refer www.shantanudaslecture.com for details about fractional order capacitance and its circuit theory via fractional calculus.

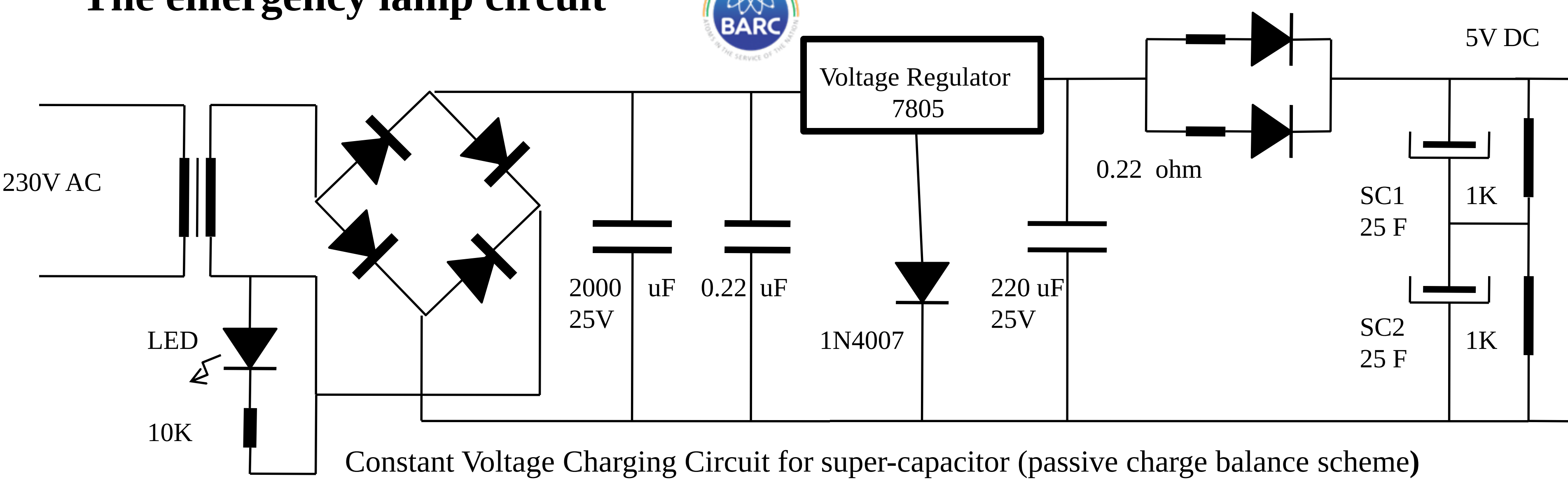
For fractional calculus and its various applications in physical science and engineering refer book: 'Functional Fractional Calculus-2nd Edition, Springer-Germany.'

We have requested funding for: Development of high current Supercapacitor Test System (HCSCTS) for testing/evaluation of Indigenous Supercapacitors (unit cells & capacitor modules)-under consideration at DeitY. The frequency domain extraction method we have formulated and will be submitting.

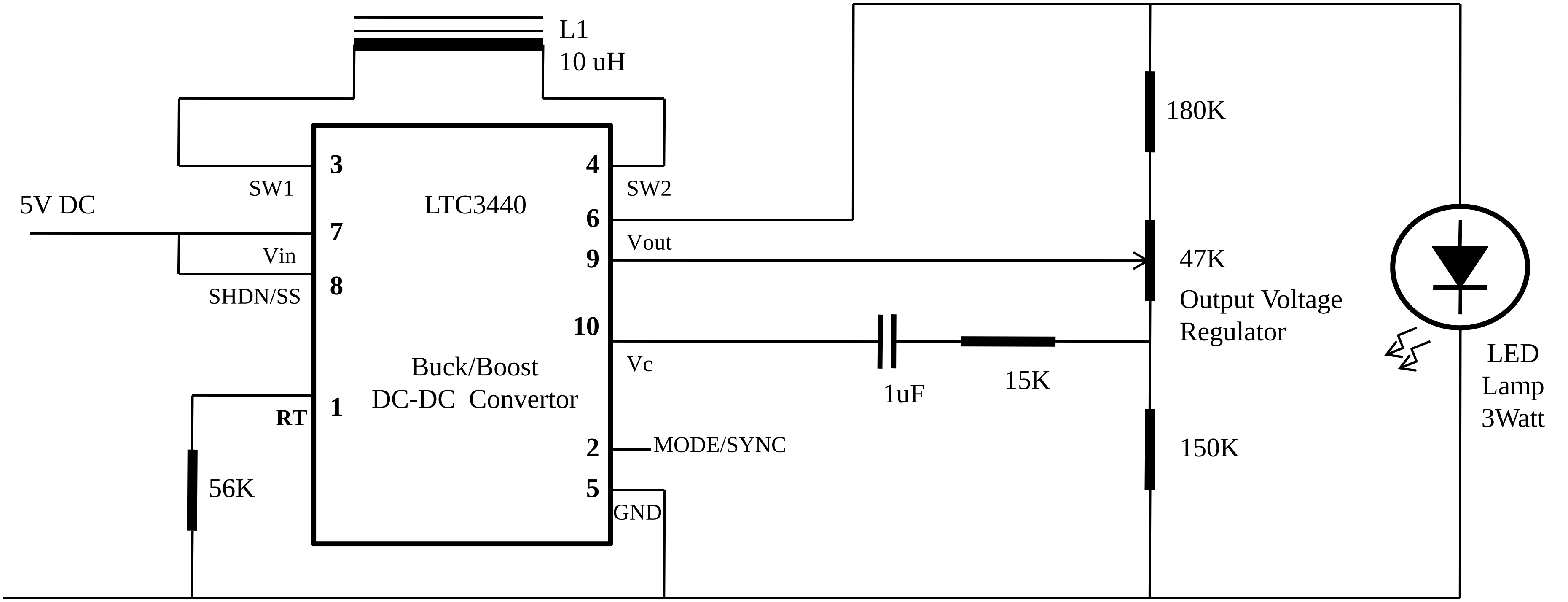
Recently sanction obtained from BRNS/DAE for: Design Development of Power Packs with Super Capacitors & Fractional Order Modeling

Some circuits made with CAG super-capacitors

The emergency lamp circuit



Constant Voltage Charging Circuit for super-capacitor (passive charge balance scheme)



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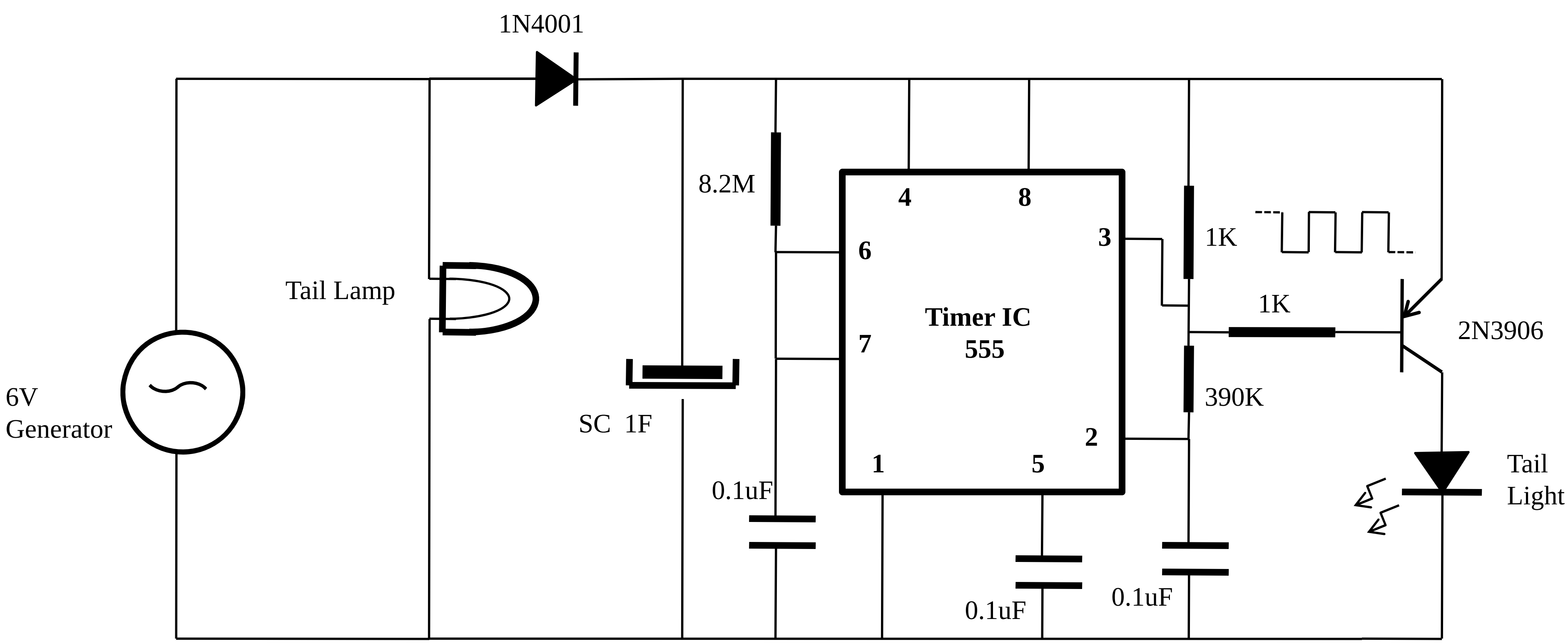
The emergency lamp

Photograph of CAG supercapacitor powered fast charging Emergency Light having two 25F/ 2.5 V, 1-3W Power LED, which takes ~90 second for full charging and provide energy to light power LED for 30 min to 90 min, depending on LED lamp power (1-3W). Note this is not pulsed power application.

The circuit for bicycle lighting system



The tail light flashes even while cycle stops

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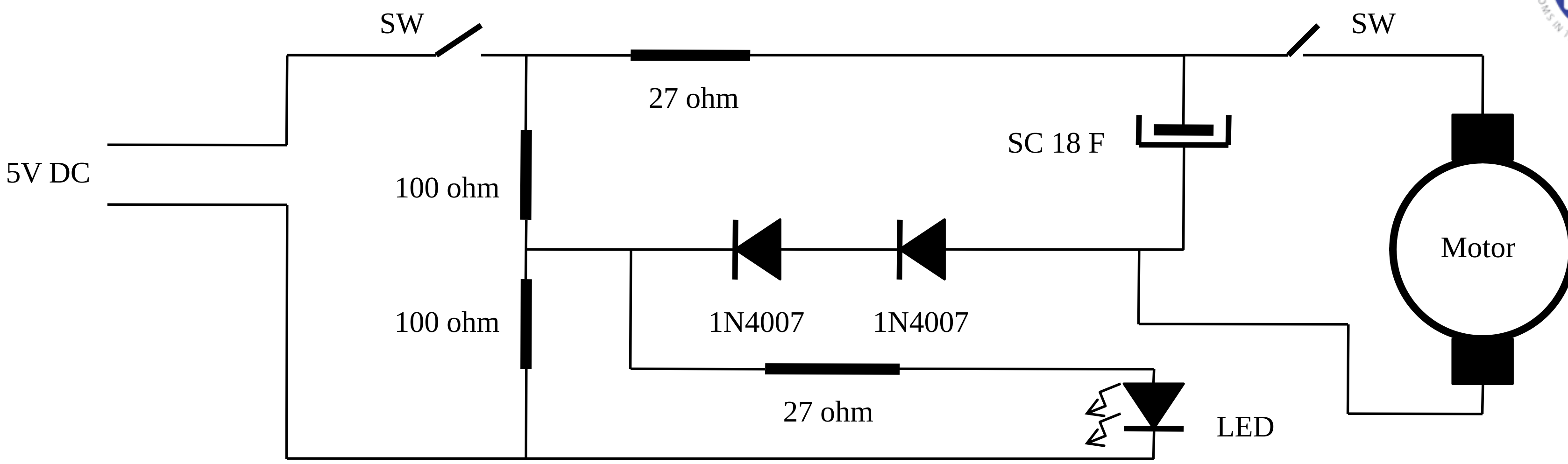
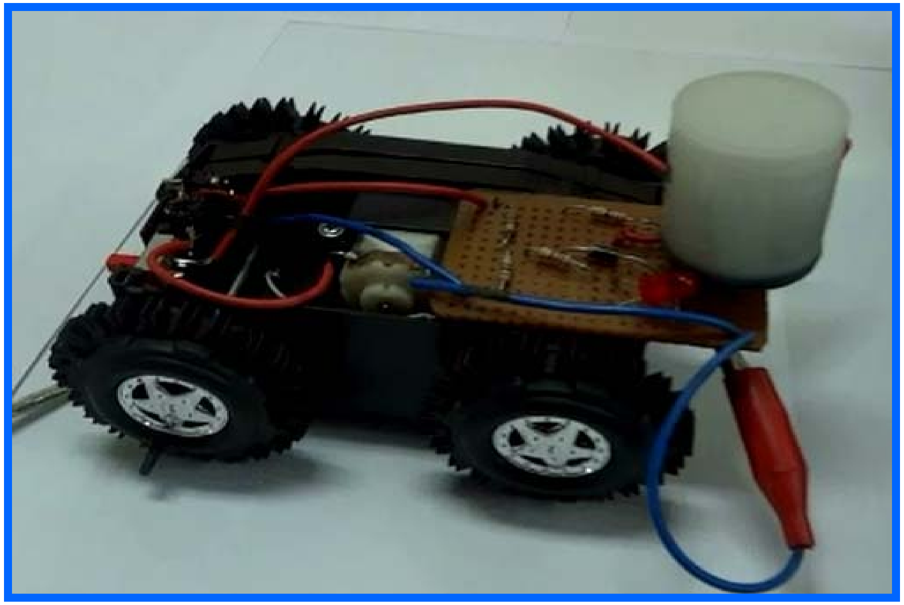
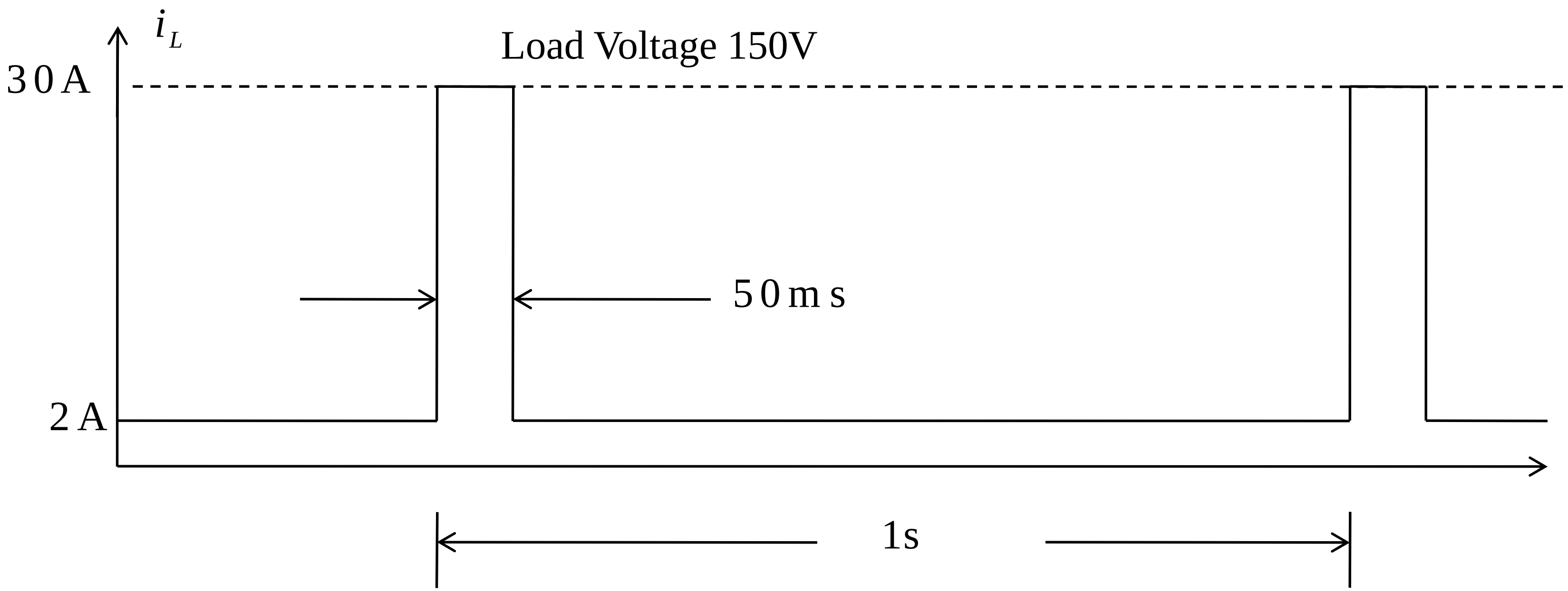
The circuit driving toy car with geared wheel





The pulsed current load and super-capacitor usage

A requirement of pulsed current load



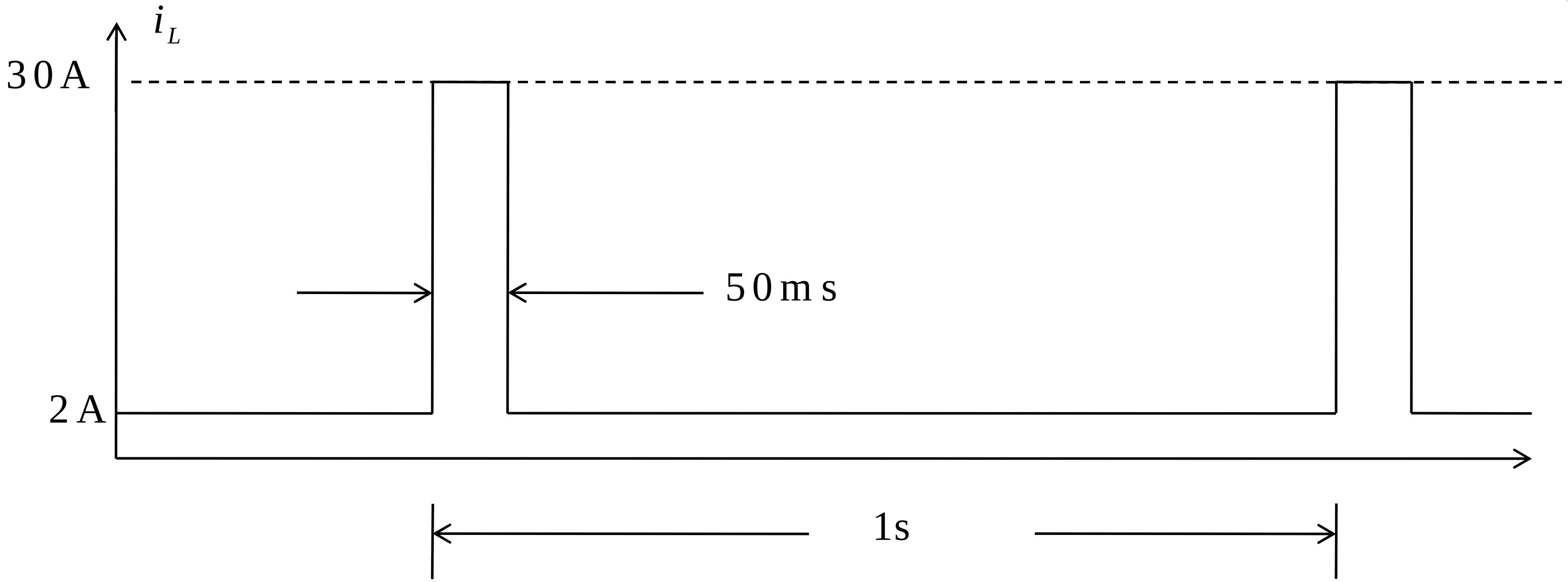
If we size the battery with 30A peak current, at 150V the size would be enormous and also due to slow dynamics of battery ,the battery will unable to provide burst power to the load

We have to therefore use the battery or cell to cater continuous average current/power , and let the super-capacitor deliver the peak current/load

This way battery/cell size gets reduced for continuous average power rather than peak power, the system becomes smaller and lighter. Moreover the super-capacitors provide response times which cannot be matched by battery/cell.

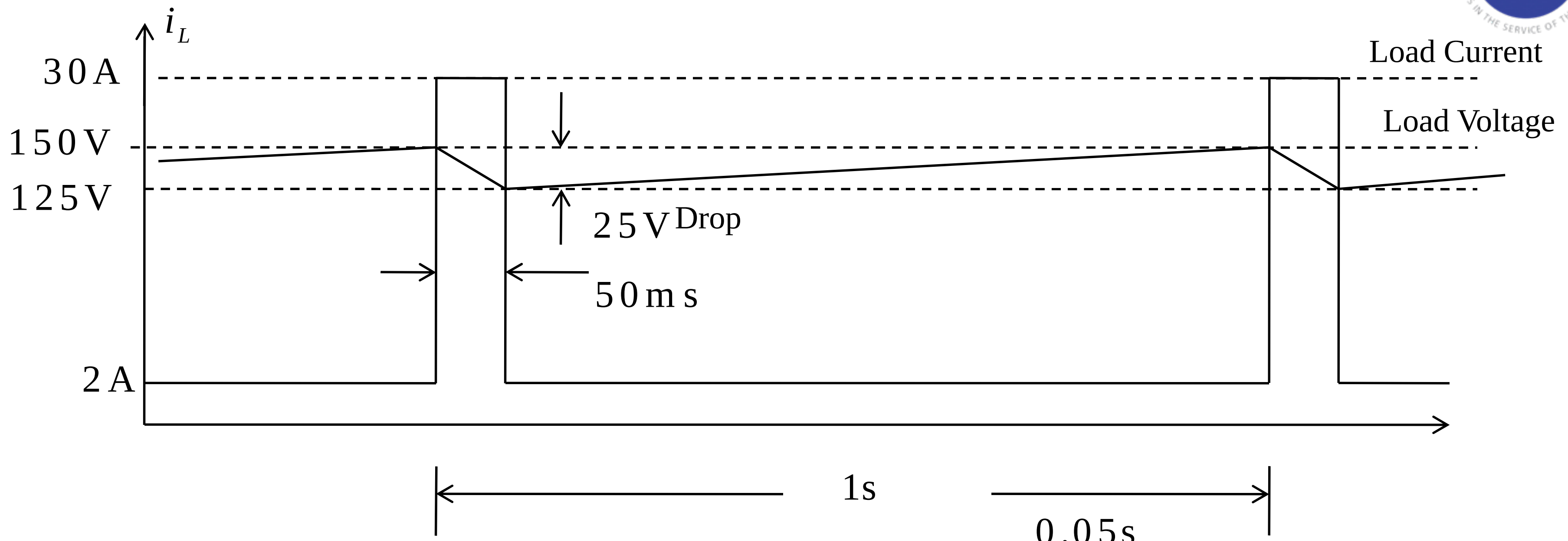
Moreover battery life gets reduced while delivering pulsing load as compared to constant load

A requirement of pulsed current…cont.

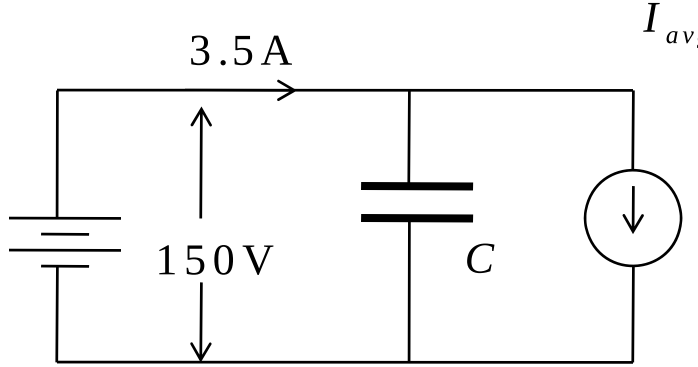


Continuous Average Current: $I_{avg} = 2A + 30A \times \frac{0.05s}{1s} = 3.5A$

A requirement of pulsed current…cont.



$$I_{avg} = 2A + 30A \times \frac{0.05s}{1s} = 3.5A$$



Instead of having battery/cell to be sized for 30 A at 150V we have battery/cell that provides 2A required to support quiescent current as well as added 1.5A to re-charge super-capacitor between the peak pulses

Super-capacitor must be able to deliver 30A for 50ms with voltage drop (d*V*) not greater than 25V

A power pack required from CAG super-cells of 2.5 V each

Say we have CAG cell of 33.3 F of ESR 45 milli-ohm, with three of them in parallel we make 100F cell of ESR 15 milli-ohm of terminal voltage 2.5 V. We are having load voltage of 150V, therefore we require 150 V /2.5V = 60 of such in series stack.

With this series stacking of 60 cells of 100F each of 15 milli-ohm ESR we get the following

Total DC Capacity = 100F / 60 = 1.66 F

Total DC ESR = 15milli-ohm X 60 = 900 milli-ohm=0.9ohm (approx. 1 ohm)

The DC time constant of the power-pack is 1.66 F X 0.9 ohm = 1.5 seconds

AC ESR is 60 % of DC ESR therefore we have AC ESR approximately 0.6 ohm

AC Capacity is 30 % of DC capacity therefore we have AC capacitance as 0.3 X 1.66 approx 0.5F

From the voltage drop and droop expression

$$dV = I\left(R_C + \frac{dt}{C}\right)$$

We obtain

$$dV = 30\,\mathrm{A}\left(0.6\,\mathrm{ohm} + \frac{0.05\,\mathrm{s}}{0.5\,\mathrm{F}}\right)$$

$$= 21\,\mathrm{V}$$

This is well below our required maximum voltage drop of 25 V

The AC ESR and AC capacity of super-capacitor are lower than DC values because the super-capacitor electrode is highly porous, therefore the AC electric fields when applied does not penetrate infinite pores therefore the capacity and the ESR are lower for AC fields.

From time constant calculation

We have a power pack with DC time constant 1.5 second

AC ESR is 0.6 times DC ESR and AC capacity is 0.3 times DC capacity. Therefore the AC time constant is 0.18 times DC time constant

We obtain the AC time constant as 1.5 s X 0.18 = 0.27 s

Thus $R_C C = 0.27\text{s}$

$$dV = I\left(R_C + \frac{dt}{C}\right)$$

$$= I\left(R_C + \frac{dt}{0.27/R_C}\right) = I \times R_C\left(1 + \frac{dt}{0.27}\right)$$

Taking $dV = 20\text{V}, \quad I = 30\text{A}, \quad dt = 0.05\text{s}$

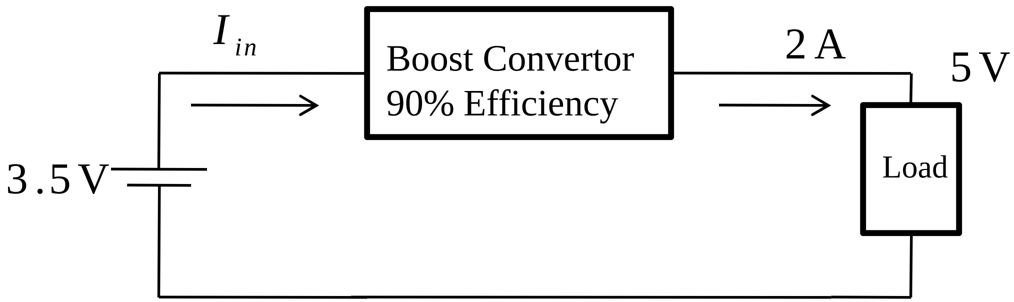
We obtain from above $R_C = 0.56\text{ohm}$ this is AC ESR., therefore DC ESR is 0.94 ohm

From time DC time constant that is 1.5 s we get DC capacity as 1.5 s /0.94 ohm = 1.6 F

Sometimes specifications of super-capacitors are in time-constant which has various ESR values. This way too one can select the capacity and ESR and go ahead with calculations required.

A simpler example

We have a pulsed load of 2A and load voltage required is 4.8 V, and we have a battery of 3.5V, therefore we require to boost the battery voltage

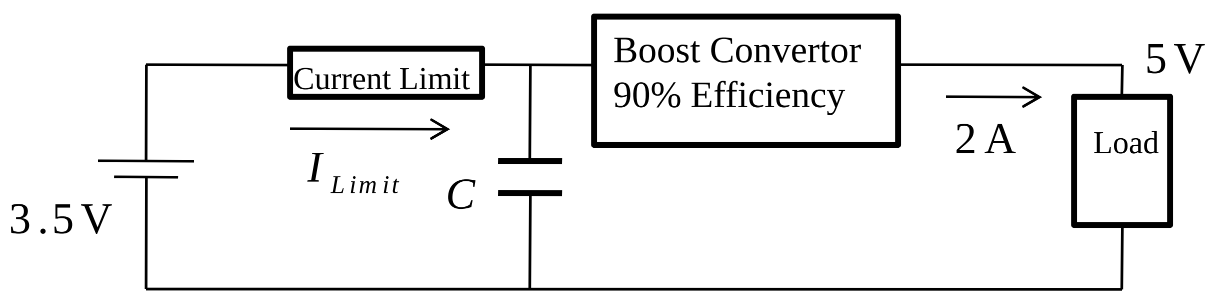


Total load voltage driving the load is 5V i.e. 4.8V+200mV (taking this 200mV as overhead for current control circuit)

$$P_{in} = 3.5\text{V} \times I_{in} \qquad P_{out} = 5\text{V} \times 2\text{A} = 10\text{W}$$

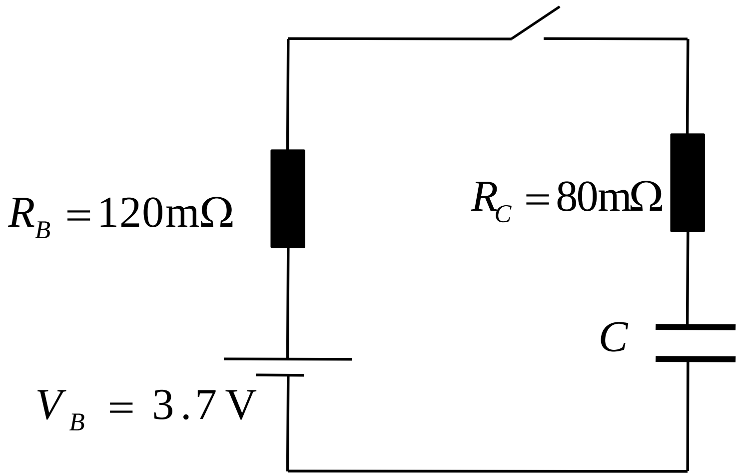
$$P_{in} = \frac{10}{0.9}\text{W} = \frac{100}{9}\text{W} \qquad I_{in} = \frac{100}{9}\text{W} \times \frac{1}{3.5\text{V}} = 3.17\text{A}$$

This 3.17A is too high for this battery-either can cause the battery protection circuit to shut down or cause a low voltage shut down with plenty of energy still remaining in the battery



One solution is to have super-capacitor *C* for pulsed power and a current-limit circuit to limit the in-rush current

Current limit ?



Current limit circuit like AAT4601A manages in-rush current on power-on –while charging the super-capacitor

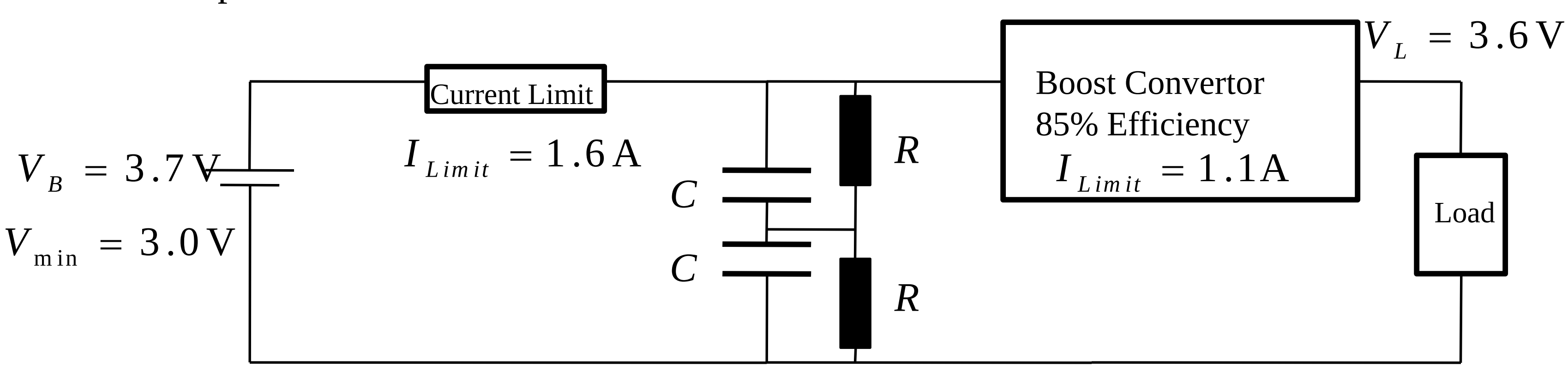
$$I_{in-rush} = \frac{3.7\text{V}}{120\text{m}\Omega + 80\text{m}\Omega} = 18.5\text{A}$$

Say our load is 2A peak, with 50% duty cycle so we have average current of 1A. The other circuit takes 100mA as quiescent current. Then current limit should be set for output of dc-dc convertor for > 1.1A.

Assume minimum battery voltage is 3.0V load voltage is 3.6V and converter efficiency 85% then

$$I_{in-Limit} = \frac{1.1\text{V} \times 3.6\text{V}}{3.0\text{V}} \times \frac{100}{85} = 1.55\text{A}$$

Then we set input current limit at 1.6A



With two R of equal value the scheme is passive charge balance in series C connection

Power Pack out of CAG super-capacitor cells and active charge balancing circuit

Need for charge balancing while stacking super-capacitors

If capacitors $C1$ and $C2$ charging and discharging currents are relatively small 1-2 mA then one connects equal resistances of “charge balancing resistors”. Say we are connecting two capacitors of 10F if the tolerance spread is almost nil and the charging discharging currents are in range 1-2 mA the circuit of figure-1 will do.

Say the tolerance mismatch is 10% and we have thus 11F and 9F in series; then instantly the node voltage of the $C1$ and $C2$ will go to 2.75 V. Thus one of the super capacitor (rated for 2.5 V) will go bad; unless very quickly we make the node voltage to 2.5 V. Now in this unbalance case (figure-2) the charge extra to be pumped or drained (through one of the C = 10F) is $\Delta Q = 0.25\text{V} \times 10\text{F} = 2.5\text{C}$

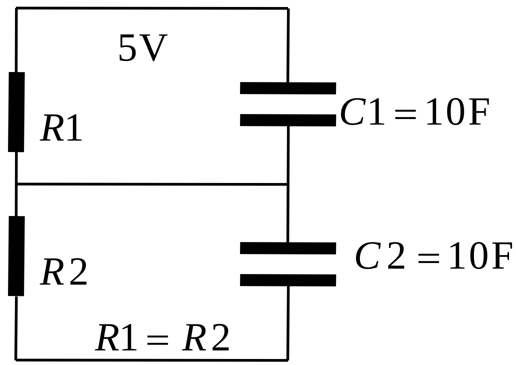


Figure-1

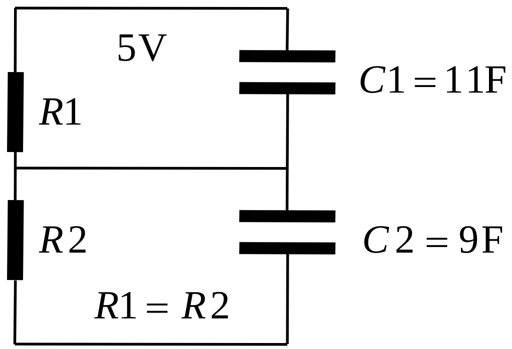


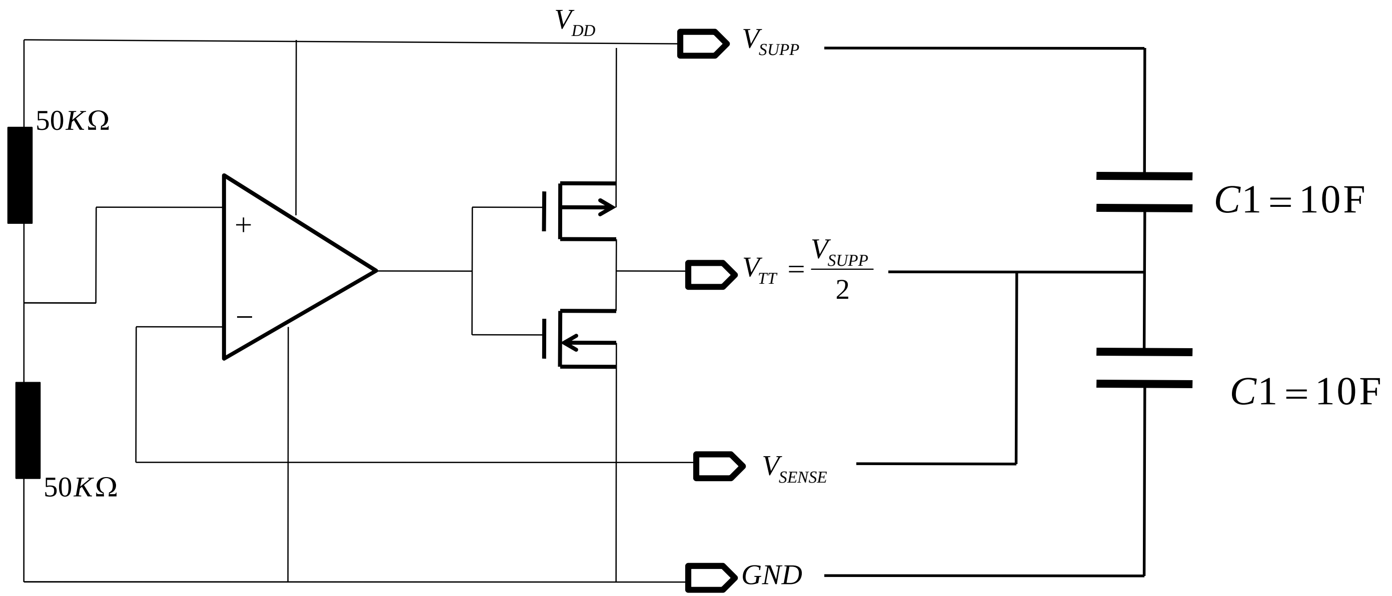
Figure-2

Well had these capacitors be of micro-farads say, 11 and 9 micro-farads, then the charge to be balanced is 2.5 micro-C. This discussion thus points out that well, “efficient approach” is required when the capacitors are in FARAD range.

One active charge balance scheme

Here we shall be taking charging to 5V since each super capacitor is rated for 2.5V and with charging discharging currents of 1A. Say to balance the 2.5 C charge in 1 second requires the sinking of 2.5 A current! Thus we have to maintain regulated voltage by ‘active’ circuit the node should be maintained at V /2 =2.5V , at the C1 and C2 junction

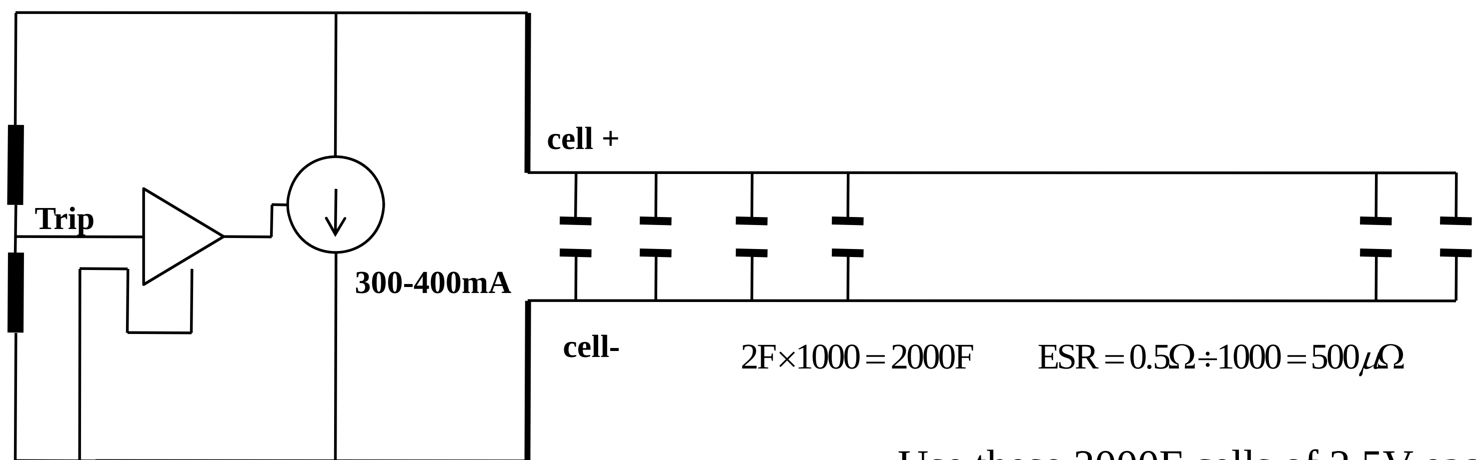
The circuit gives a active regulated circuit to achieve this objective. This circuit shows an operation amplifier which shall be ‘single supply’ operational amplifier. The output stage current 1.5 A, continuous, transient peak 3A.



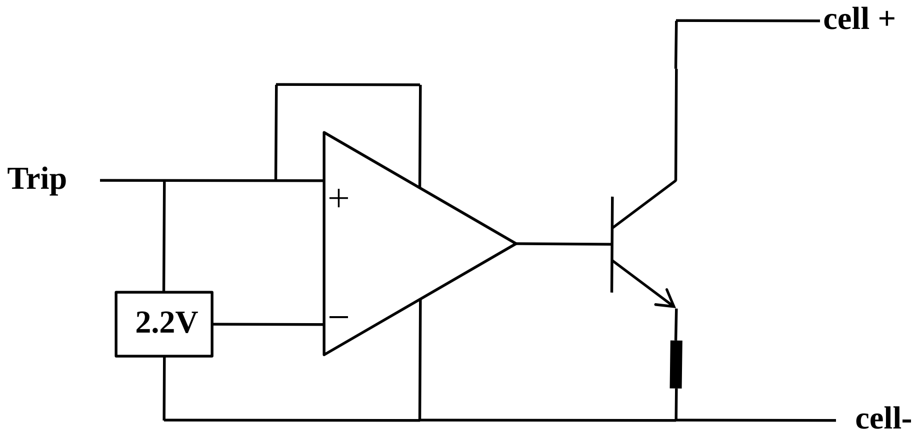
This scheme is fine for pair of capacitors in series

Active charge Balance Circuit-with floating current sink

This floating current sink circuit is across each cell. Here one cell interface is shown to scheme the working logic. That is if any of the stacked series connected cells of these unit cells gets over-voltage (more than or equal to 2.73 V the 300-400 mA current sink turns on till the cell voltage is lessened to safe value.



$2F \times 1000 = 2000F$ $\quad ESR = 0.5\Omega \div 1000 = 500\mu\Omega$



Integrated Reference Comparator and current sink

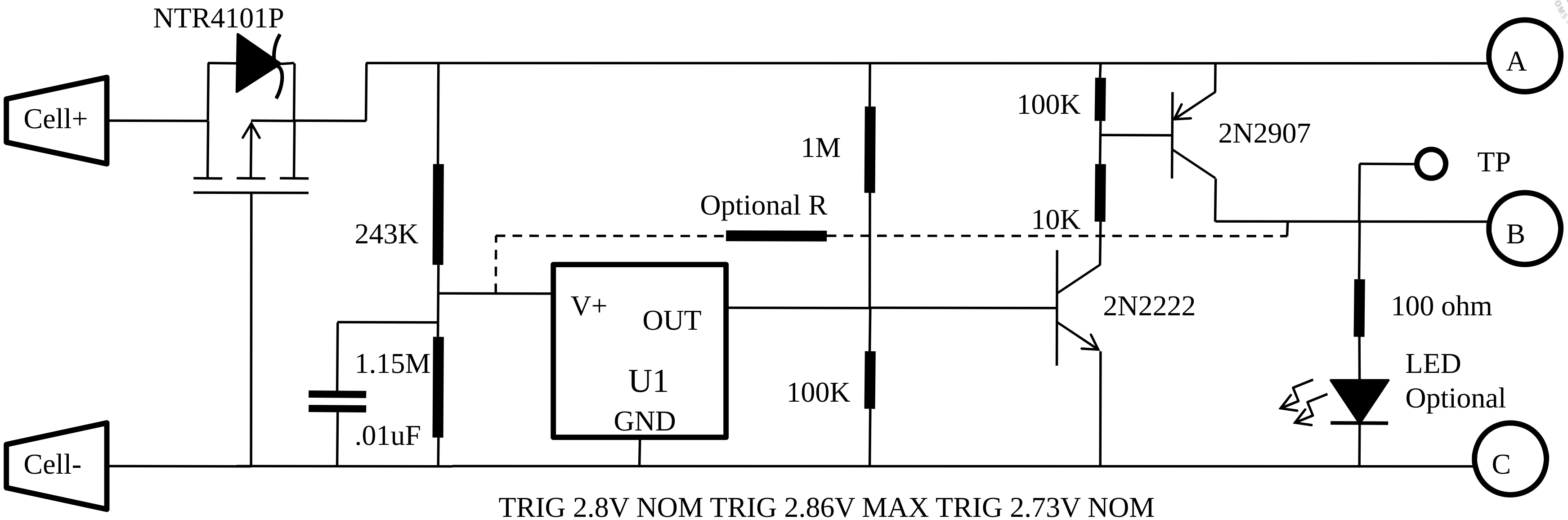
Use these 2000F cells of 2.5V each to get 100F 50V super-capacitor power pack system, by series connection of 20 such cells

Here module to module balancing is eliminated.

Pairing correction not required

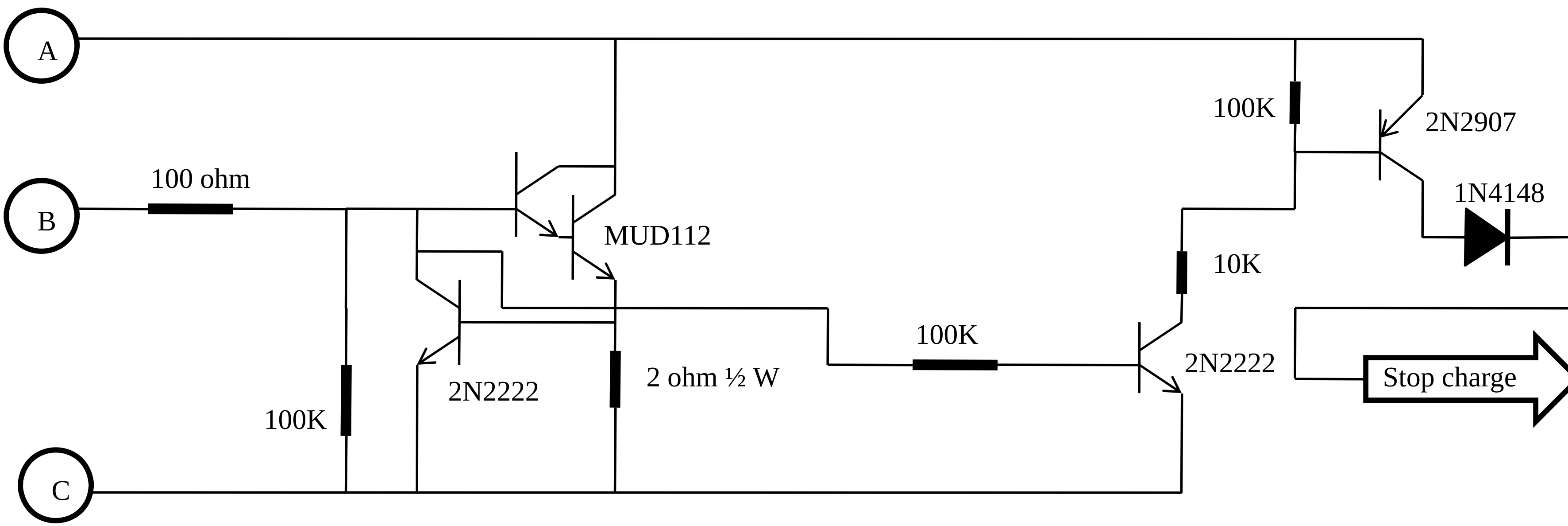
Under development

Detailed circuit active cell balance-floating current sink



TRIG 2.8V NOM TRIG 2.86V MAX TRIG 2.73V NOM

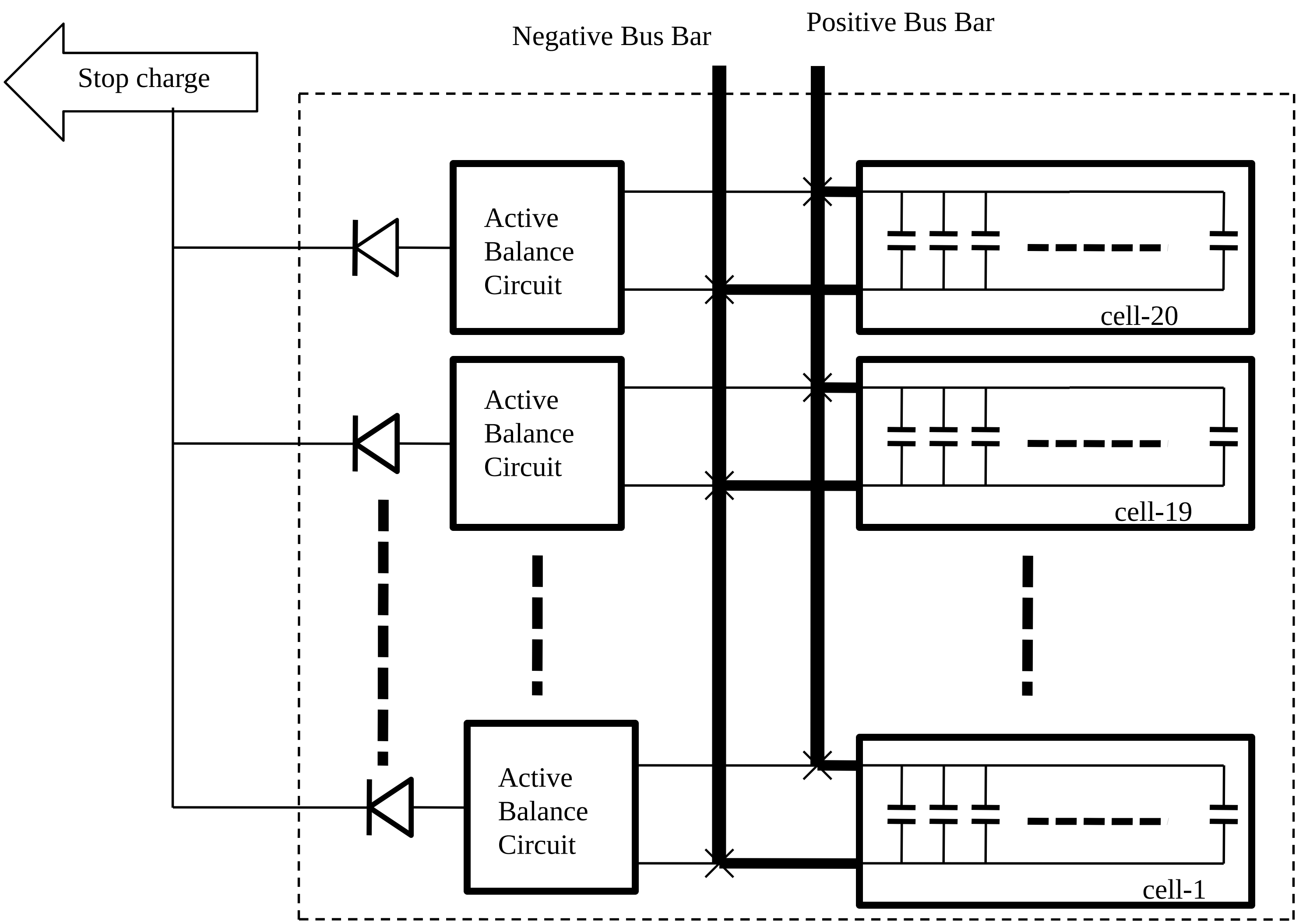
U1: Integrated Reference Comparator TRIG 2.3V, 2% Accuracy, 110mV Hysteresis.
Open Drain output , pulls low until thresh hold 2.2V reached



Ref: US patent

Under development

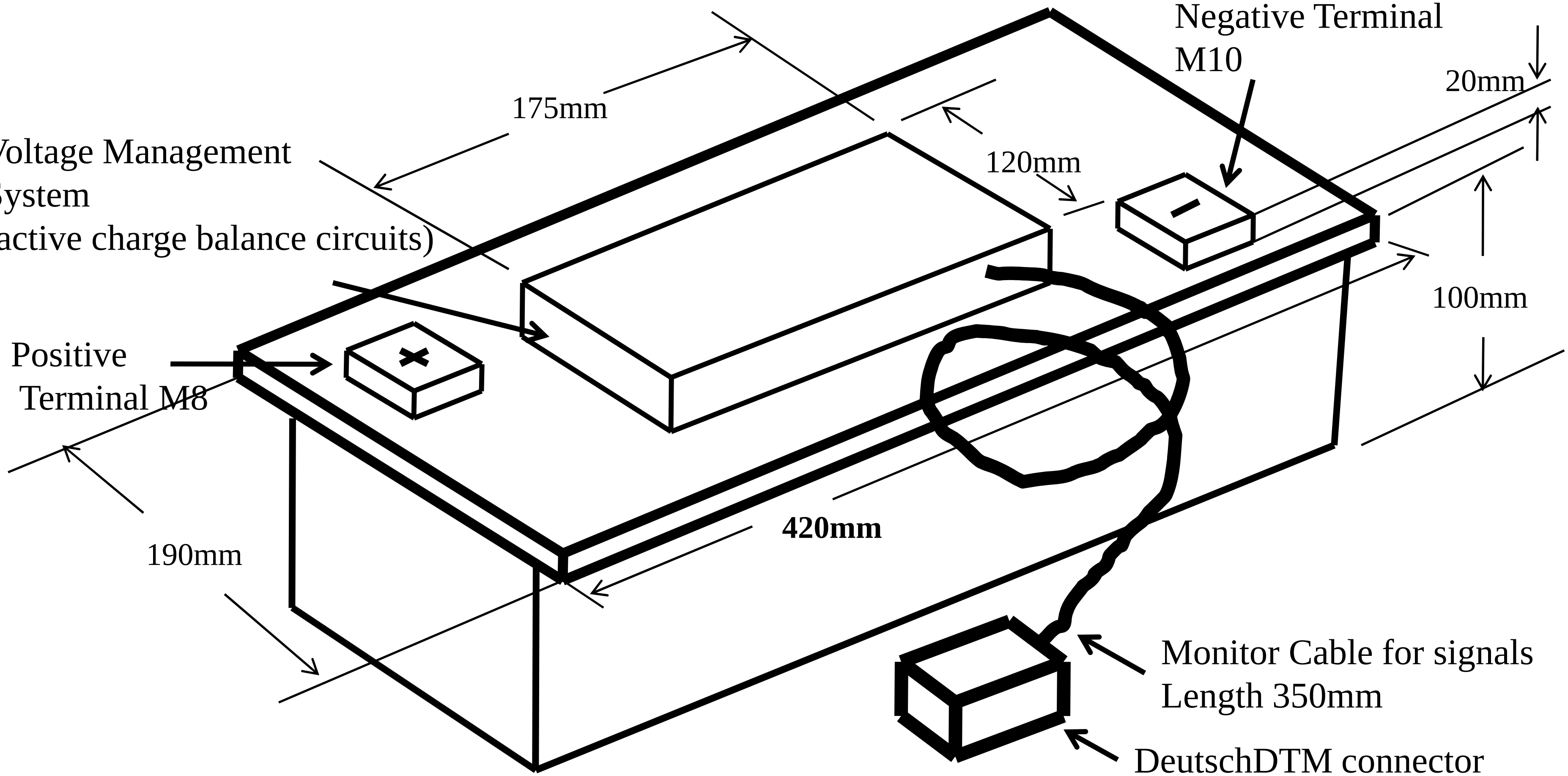
Stacking of cells to get power pack



Power Pack 100F, 50V ESR 10 milli-ohm

Under development

Overall assembled Power Pack 100F, 50V ESR 10 milli-Ohm



Monitor Signals:
Over voltage Alarm, Temperature (Analog)
State of Charge (Analog) & Facility WLL connectivity
Stop Charge Signal (Digital)

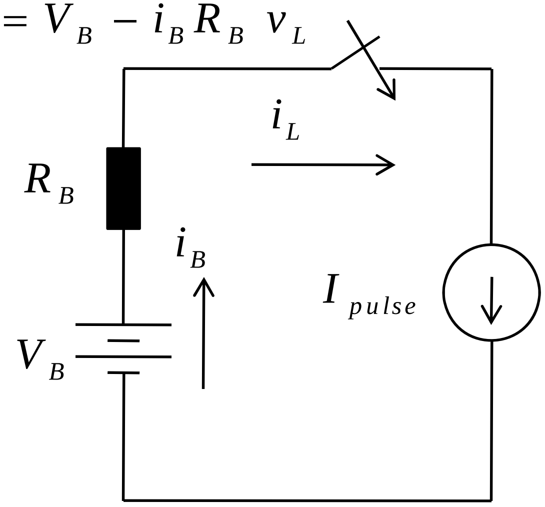
Weight 10kg

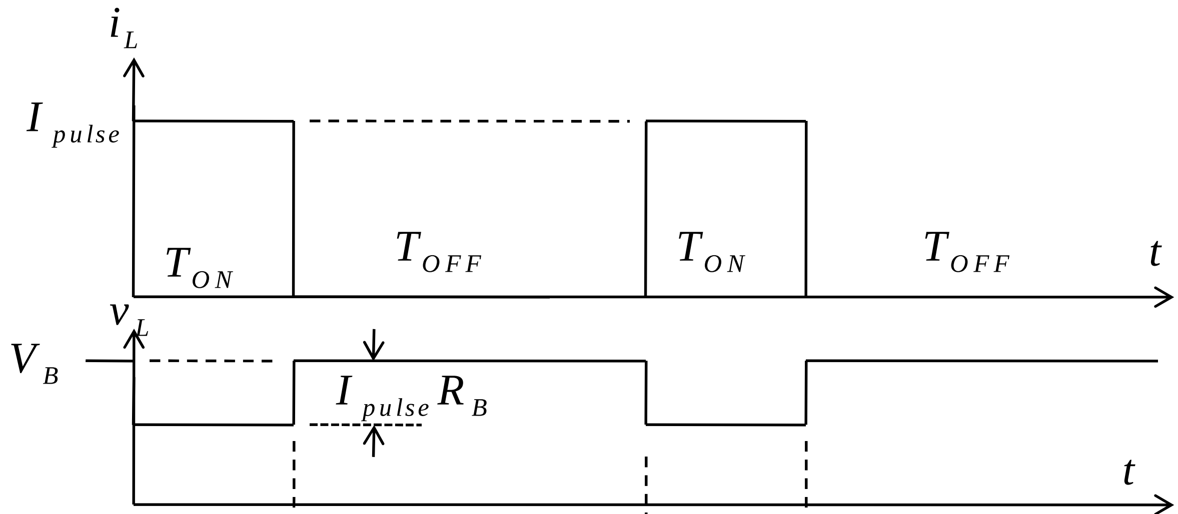
Under development

Detailed calculations and derivation of circuits with battery and super-capacitor for pulsed current load

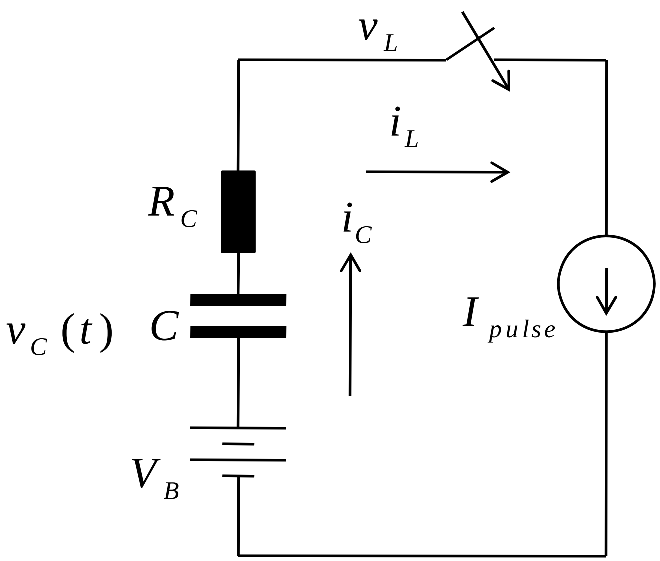
Pulse load current supplied by battery & charged super-capacitor separately

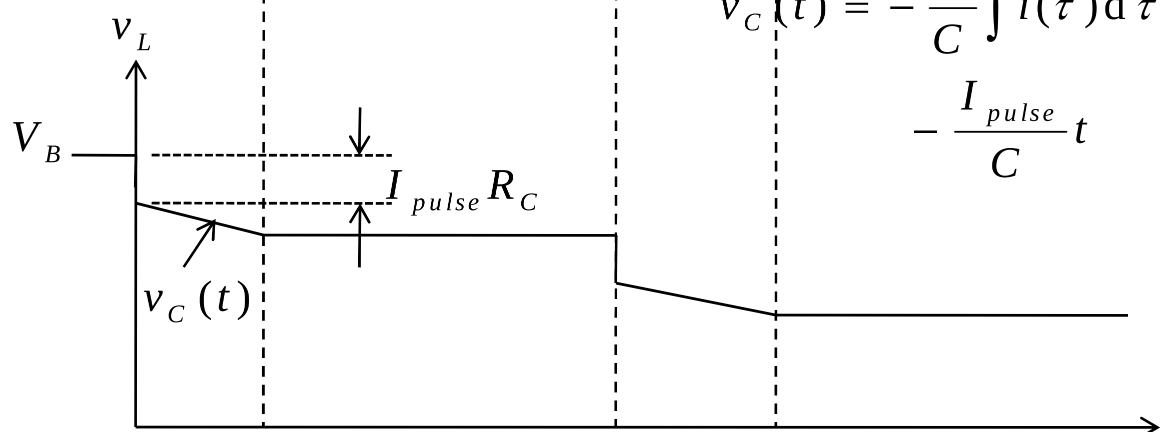
$$v_L = V_B - i_B R_B$$





$$v_L = V_B + v_C(t) - i_C R_C \qquad i_C = I_{pulse}$$



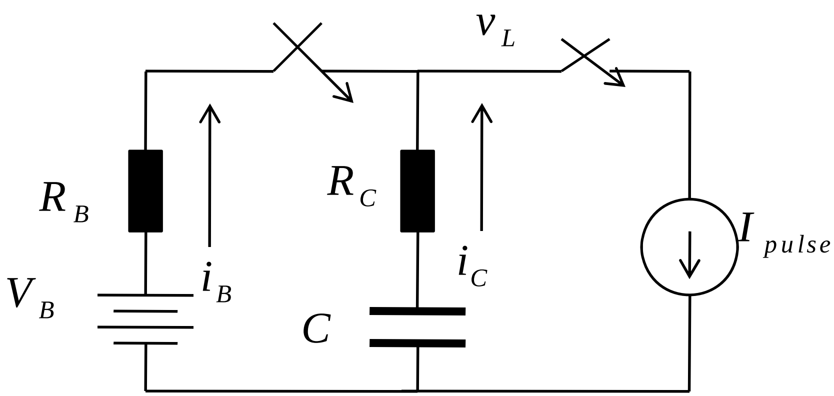


$$v_C(t) = -\frac{1}{C}\int i(\tau)\,d\tau$$

$$-\frac{I_{pulse}}{C}t$$

$$v_L(t) = V_B - \frac{I_{pulse}}{C}t - I_{pulse} R_C$$

If we charge the super-capacitor during OFF time then

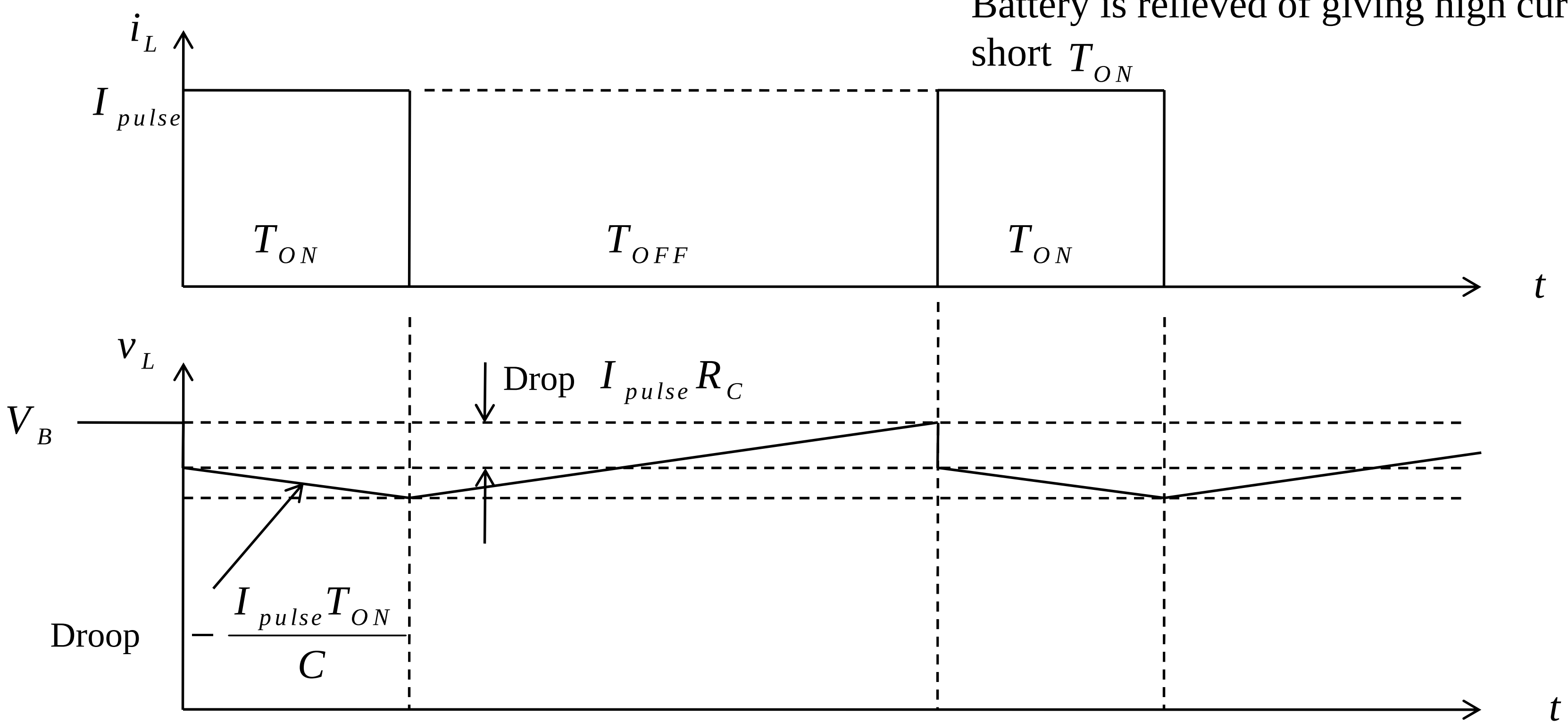


We get almost smooth load voltage

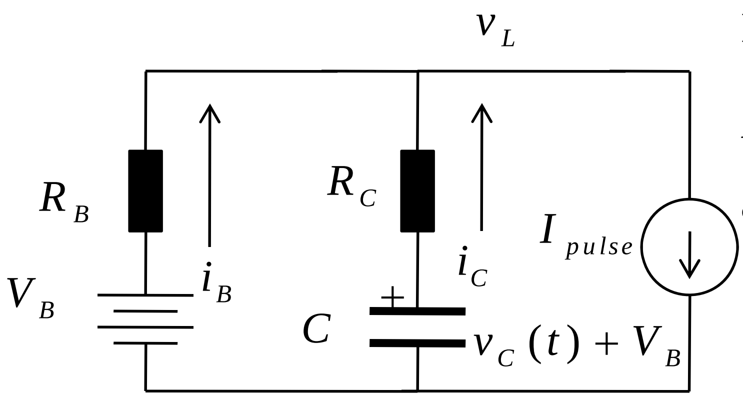
Voltage Drop is little $R_C << R_B$

For C large Voltage Droop is small

Battery is relieved of giving high current for short T_{ON}



Calculations



Initially assume that capacitor is charged to V_B

At time $t = 0$ the current sink starts to draw constant I_{pulse} and then after we have $I_{pulse} = i_C(t) + i_B(t)$

The capacitor current at $t > 0$ is

$$i_C(t) = i_C(0) + \frac{[v_C(t) + V_B] - V_B}{R_B + R_C}$$

This is superposition of two currents, one due to I_{pulse} and second due to voltage on the capacitor that is $v_C(t) + V_B$ minus the battery voltage V_B divided by resistance of the circuit comprising of battery and capacitor that is $R_B + R_C$. The current through C due to I_{pulse} is $i_C(0)$ which is constant and is

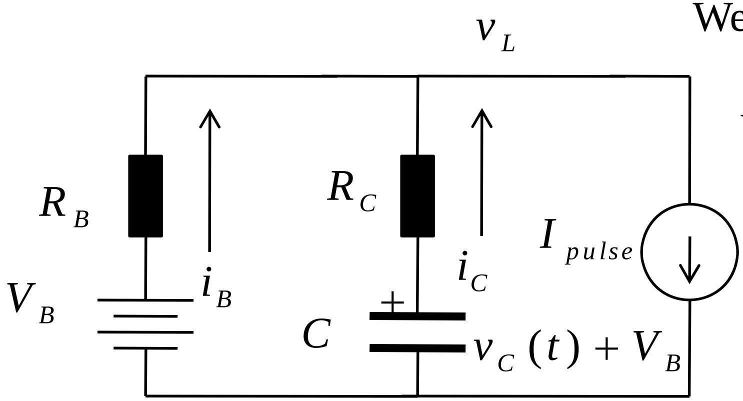
$$i_C(0) = I_{pulse} \times \frac{R_B}{R_B + R_C}$$

Therefore $$i_C(t) = I_{pulse}\frac{R_B}{R_B + R_C} + \frac{v_C(t)}{R_B + R_C}$$

The capacitor voltage $v_C(t)$ is $v_C(t) + V_B = -\frac{1}{C}\int_0^t i_C(t)\,\mathrm{d}t$ or $i_C(t) = -C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$

With these we write $$-C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t} = I_{pulse}\frac{R_B}{R_B + R_C} + \frac{v_C(t)}{R_B + R_C}$$

Calculations…contd.



We have $-C\dfrac{\mathrm{d}v_C(t)}{\mathrm{d}t}=I_{pulse}\dfrac{R_B}{R_B+R_C}+\dfrac{v_C(t)}{R_B+R_C}$

Writing $\tau=(R_B+R_C)C$ we get

$$\tau\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}+v_C(t)=-I_{pulse}R_B$$

We do Laplace transform and write

$$\tau s v_C(s)+v_C(s)=-\frac{I_{pulse}}{s}R_B$$

Giving us $v_C(s)=-\dfrac{I_{pulse}R_B}{s(\tau s+1)}$; which after doing partial fractions we obtain

$$v_C(s)=-\frac{I_{pulse}R_B}{s}+\frac{I_{pulse}R_B\tau}{\tau s+1}=-\frac{I_{pulse}R_B}{s}+\frac{I_{pulse}R_B}{s+\frac{1}{\tau}}$$

We now take inverse Laplace transform of above to get

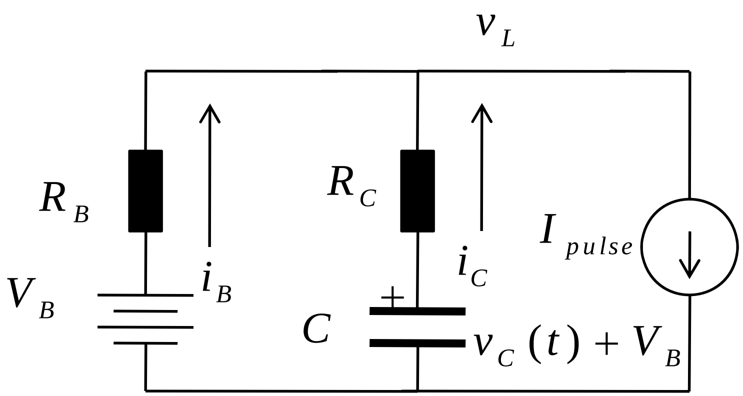
$$v_C(t)=-I_{pulse}R_B+I_{pulse}R_Be^{-t/\tau}=I_{pulse}R_B\left(e^{-t/\tau}-1\right)$$

We now calculate $i_C(t)$ from above by using $i_C(t)=-C\dfrac{\mathrm{d}v_C(t)}{\mathrm{d}t}$ and get

$$i_C(t)=-C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}=-C\times I_{pulse}R_B\times\left(-\tfrac{1}{\tau}\right)e^{-t/\tau}\qquad\tau=C(R_B+R_C)$$

$$=\frac{I_{pulse}R_B}{R_B+R_C}e^{-t/\tau}$$

Calculations…contd.



We have $i_C(t) = \dfrac{I_{pulse} R_B}{R_B + R_C} e^{-t/\tau}$

Also we have $i_C(t) + i_B(t) = I_{pulse}$

Therefore

$$i_B(t) = I_{pulse} - \frac{I_{pulse} R_B}{R_B + R_C} e^{-t/\tau}$$

$$= \frac{I_{pulse} R_C + I_{pulse} R_B \left(1 - e^{-t/\tau}\right)}{R_B + R_C}$$

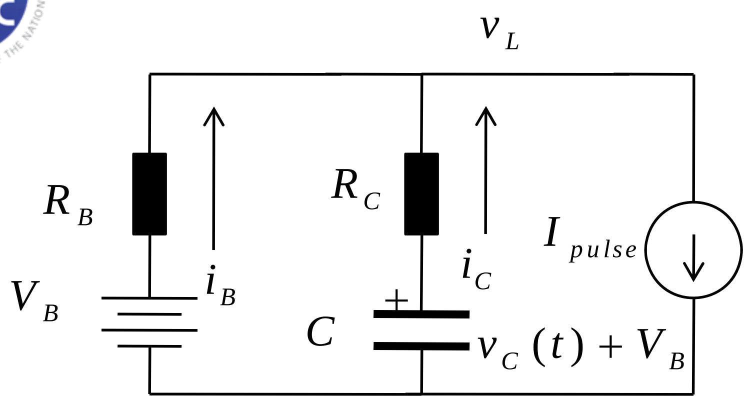
We note here that $i_B(0) = I_{pulse} \dfrac{R_C}{R_B + R_C}$ and $i_B(\infty) = I_{pulse}$ and also

$i_C(0) = I_{pulse} \dfrac{R_B}{R_B + R_C}$ and $i_C(\infty) = 0$

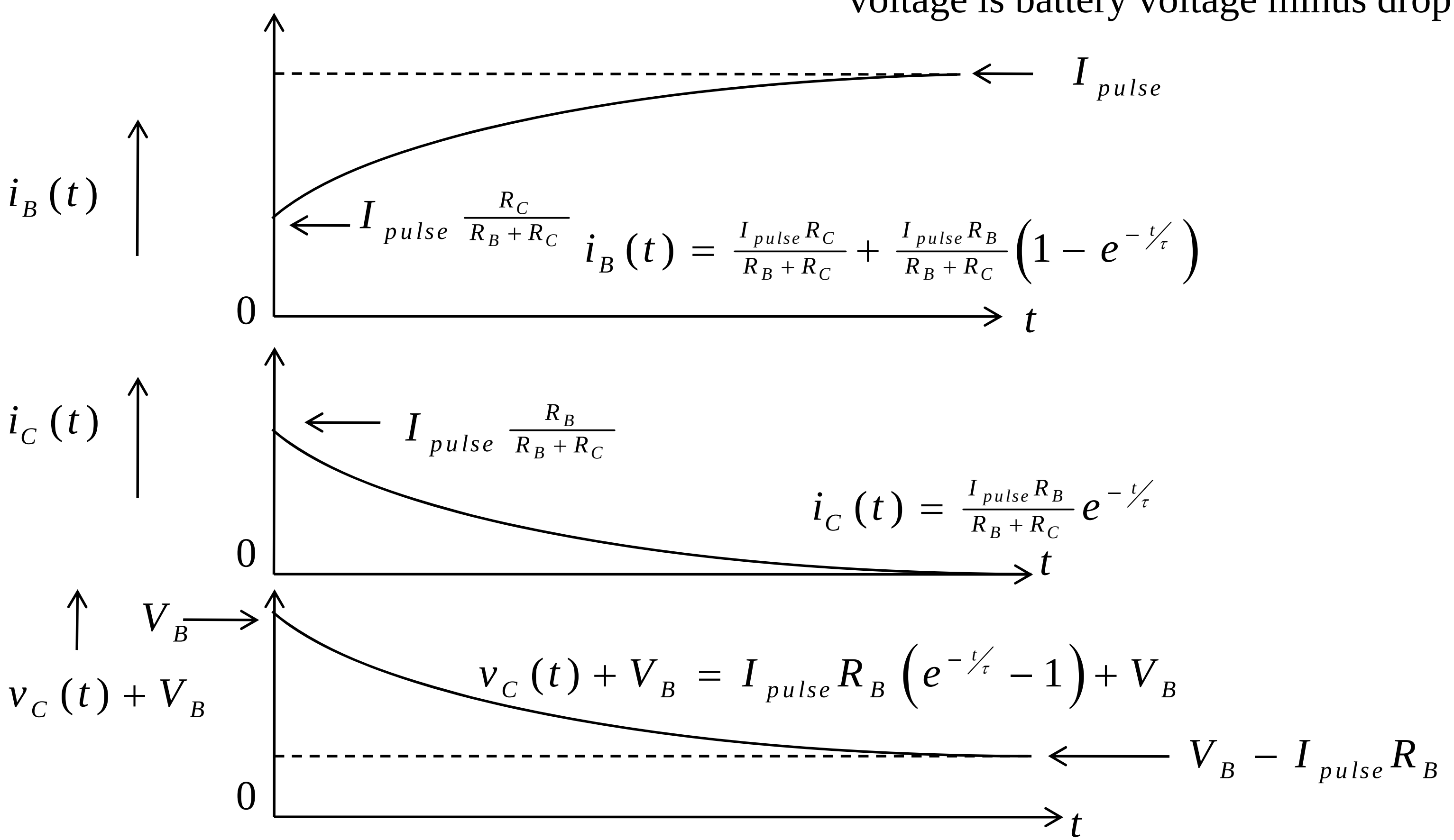
We observe that assuming $R_C << R_B$, that initially all the load current is given by the capacitor and at large times all the current comes from battery, if we extend the pulse width to infinity.

Therefore from a small duty cycle pulsed current, the capacitor will be supplying the pulsed current

Calculations…contd.

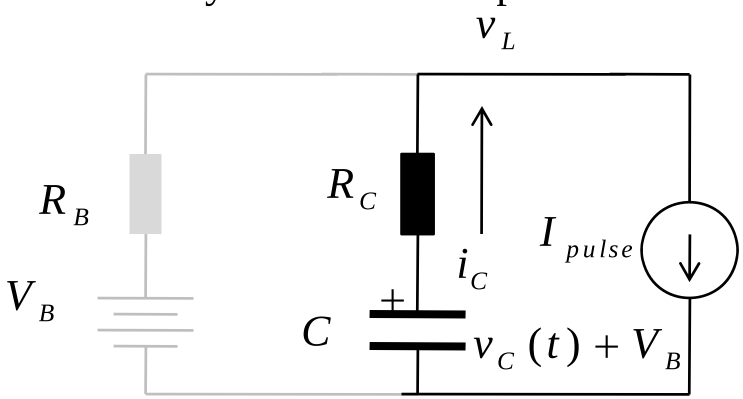


On application of a pulse current load to a circuit with super-capacitor and a battery, earlier kept at steady-state i.e. the super-capacitor charged to battery volts, the super-capacitor takes most of the load current pulse and deliver the same, and as time goes, this super-capacitor current decrease and the battery current increase-and at large time, the entire load is given by battery. The load voltage starts at battery volts decrease as time goes on and at large times the load voltage is battery voltage minus drop at battery.



Load voltage calculations

This is most likely condition. The pulse current load is supplied by the super-capacitor $R_C << R_B$



$$v_L(t) = V_B + v_C(t) - i_C(t) R_C$$

$$= V_B + I_{pulse} R_B \left(e^{-t/\tau} - 1\right) - \left(\frac{I_{pulse} R_B}{R_B + R_C} e^{-t/\tau}\right) R_C$$

$$= V_B + I_{pulse} R_B e^{-t/\tau} - I_{pulse} R_B - \frac{I_{pulse} R_B R_C}{R_B + R_C} e^{-t/\tau}$$

with $R_C << R_B$

$$v_L(t) \approx V_B + I_{pulse} R_B e^{-t/\tau} - I_{pulse} R_B - I_{pulse} R_C e^{-t/\tau}$$

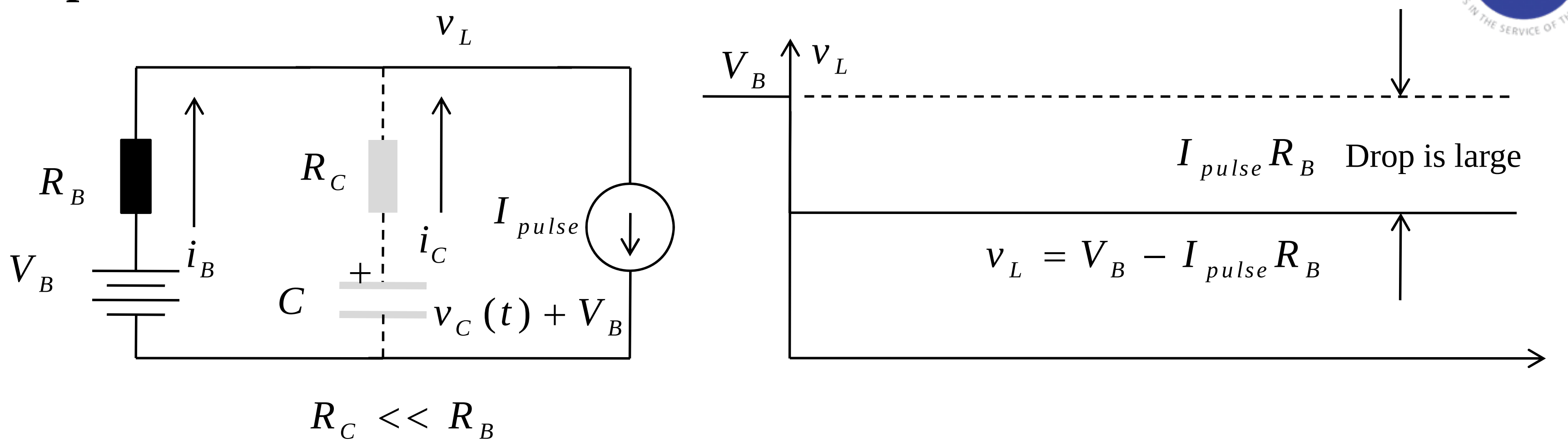
$$= (V_B - I_{pulse} R_B) + I_{pulse}(R_B - R_C) e^{-t/\tau}$$

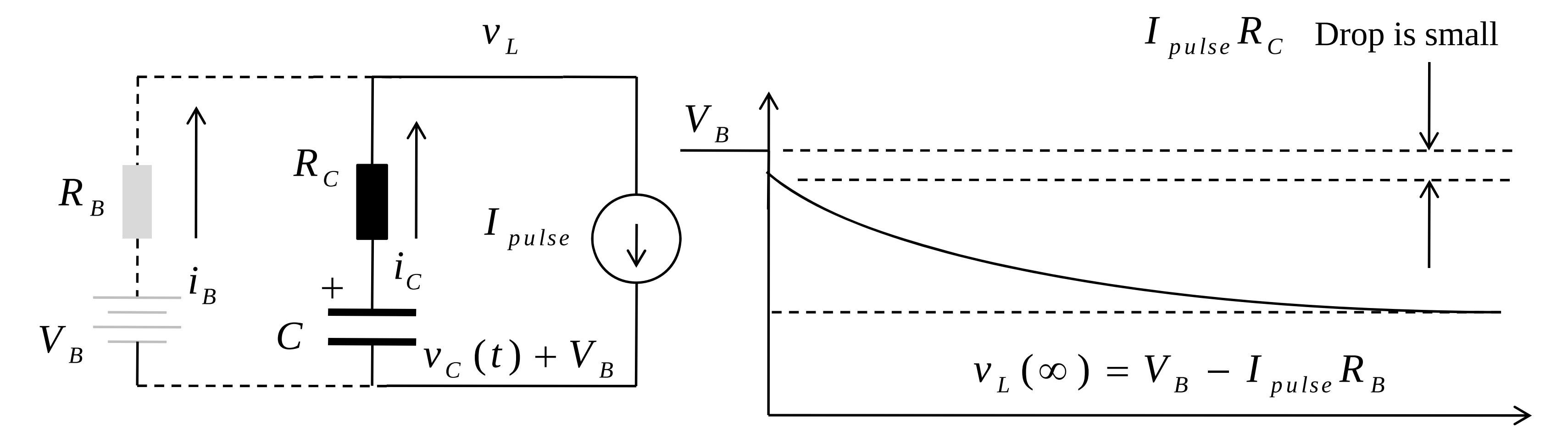
From above obtained expression we get

$$v_L(0) = V_B - I_{pulse} R_C$$

$$v_L(\infty) = V_B - I_{pulse} R_B$$

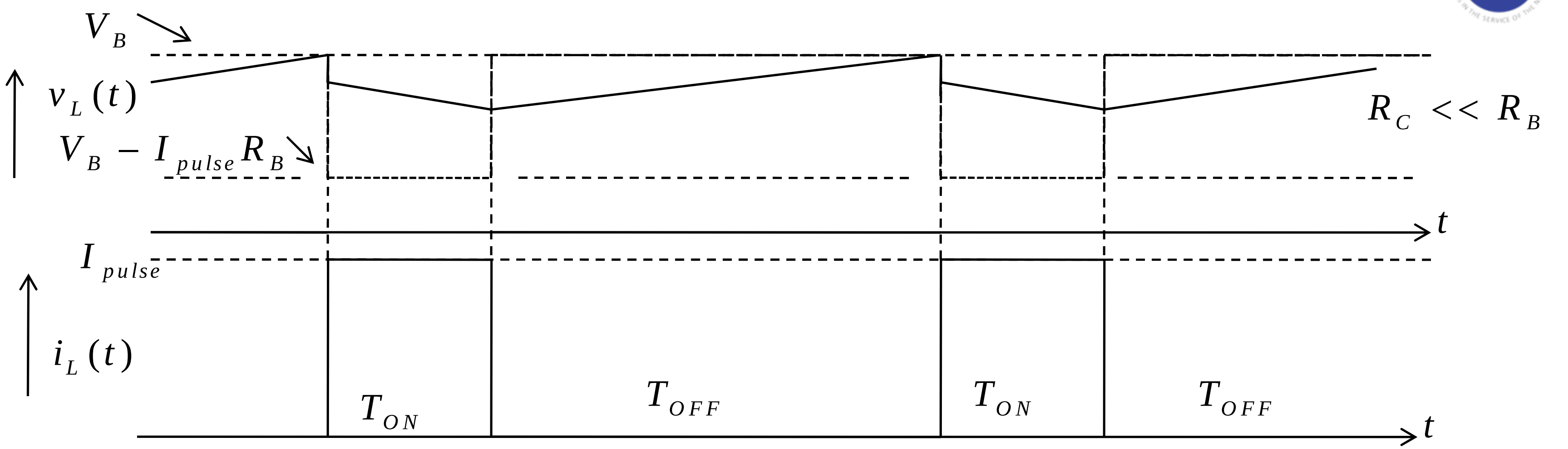
Comparison of load voltage of battery power and super-capacitor powered pulse





We have for $R_C << R_B$ and large C smaller initial drop and less droop

Pulsing load current and load voltage with super-capacitor-some points



We are getting a smoother load voltage v_L as filtering action is due to large C and low ESR R_C

Average v_L increases compared to a case without C. Therefore power to the load also rises.

With duty cycle less than 50% voltage droop/drop can be significantly reduced with C parallel with power source (battery).

Say all of I_{pulse} during ON time is given by C with large C and low ESR $R_C << R_B$ then

$$V_{drop} = I_{pulse} \times R_C + I_{pulse} \times \frac{T_{ON}}{C}$$

Small Example of Voltage drop during pulse ON time & comparison

$C = \frac{1}{2}\,\mathrm{F} \quad R_C = 20\,\mathrm{m}\Omega \quad I_{pulse} = 2\,\mathrm{A} \quad T_{ON} = 577\,\mu\mathrm{s}$ and $R_B = 100\,\mathrm{m}\Omega$

Load current being given by C alone, we get the voltage drop as

$$V_{\mathrm{drop}} = 2\,\mathrm{A} \times 20\,\mathrm{m}\Omega + 2\,\mathrm{A} \times \frac{577 \times 10^{-6}\,\mathrm{s}}{\frac{1}{2}\,\mathrm{F}} = 40\,\mathrm{mV} + 2.31\,\mathrm{mV} = 42.31\,\mathrm{mV}$$

Without C we have voltage drop as

$$V_{\mathrm{drop}} = I_{pulse} \times R_B = 2\mathrm{A} \times 100\mathrm{m}\Omega = 200\mathrm{mV}$$

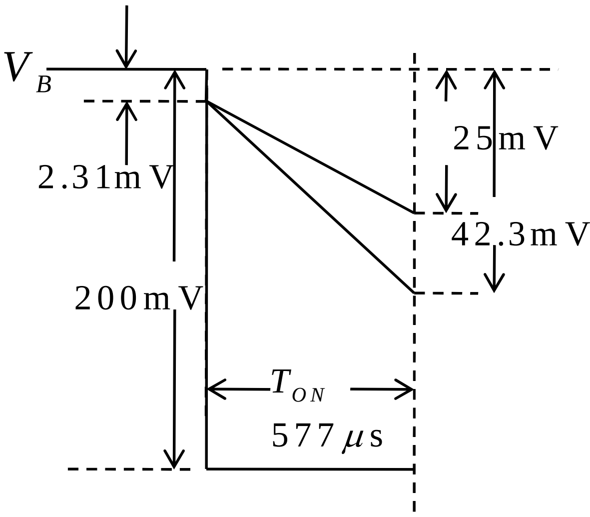
Say we have ON time as 577 mico-seconds in a frequency of 217 Hz , that is 4.16 milli-second. This gives duty cycle $D = 12.5\%$ of load current.

This gives average current as 2A X 12.5 / 100 = 0.25 A= $i_L(avg)$; with this we can calculate average load voltage as $v_L(avg) = V_B - i_L(avg) \times R_B$

$$V_{\mathrm{drop}}(avg) = i_L(avg) \times R_B = 0.25\mathrm{A} \times 100\mathrm{m}\Omega = 25\mathrm{mV}$$ less than V_B

Summary of voltage drop during ON time

Circuit configuration	Voltage Drop
With only battery	200 m V
With only C	42.3 m V
With battery and C both	25 m V



Power delivered to pulsed load

The power delivered to the load with battery alone

$$p_L(t) = v_L(t) \times i_L(t) = \left[V_B - i_L(t) R_B\right] \times i_L(t)$$
$$= \left[V_B - I_{pulse} R_B\right] \times I_{pulse}$$

The power delivered to the load with battery in parallel with super-capacitor

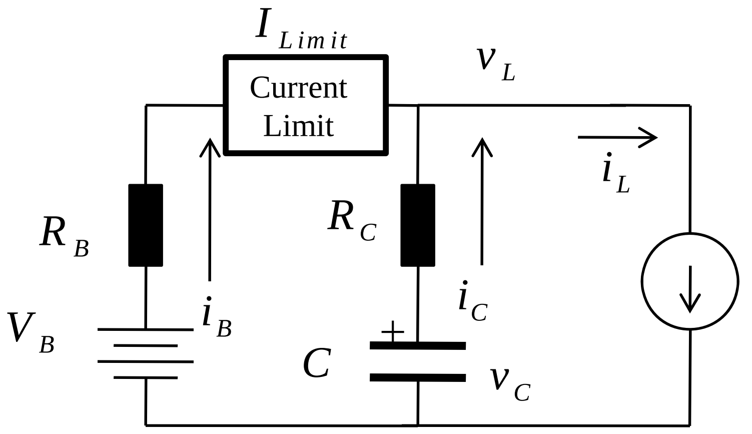
$$p_L(t) = v_L(avg) \times i_L(t) = v_L(avg) \times I_{pulse}$$
$$= \left[V_B - i_L(avg) \times R_B\right] \times I_{pulse}$$
$$= \left[V_B - I_{pulse} D \times R_B\right] = \left[V_B - R_B I_{pulse} D\right] \times I_{pulse} \qquad D = \frac{T_{ON}}{T_{ON} + T_{OFF}} \qquad 0 < D < 1$$

This is approximation when we have low ESR and high C. For high ESR and low C the ripple is large and this approximation breaks down

The approximate difference between power and energy delivered to the load during the pulse without super-capacitor and with supper-capacitor (assuming voltage ripple is small i.e. for high C and low ESR)

$$p_{difference} \cong R_B I^2_{pulse} - R_B I^2_{pulse} D = R_B I^2_{pulse} (1 - D)$$
$$e_{difference} \cong R_B I^2_{pulse} (1 - D) T_{ON}$$

Expressions for circuit simulation of the circuit response to a given load



$$i_C = i_L \frac{R_B}{R_B + R_C} - \frac{V_B - v_C}{R_B + R_C}$$

$$= \frac{v_C - V_B + i_L R_B}{R_B + R_C}$$

$$i_B = i_L - i_C$$

$$= i_L - \frac{v_C - V_B + i_L R_B}{R_B + R_C}$$

$$= \frac{V_B - v_C + i_L R_C}{R_B + R_C}$$

$$\Delta v_C = -\frac{i_C \Delta t}{C}$$

v_C at the next step is

$$v_C(n+1) = v_C(n) + \Delta v_C$$

If a current limit is specified and if $i_B > I_{Limit}$ then $i_B = I_{Limit} = \frac{V_B - v_C + i_L R_C}{R_B + R_C}$

here we will also thus write $v_C = V_B + i_L R_C - I_{Limit}(R_B + R_C)$

These are simulation equations for constant current pulsed load

Useful expressions in summary

The following equations define the instantaneous values of load voltage, the power delivered to the load, the power loss in ESR of battery and power loss in ESR of super-capacitor, having found i_B and i_C

$$v_L = V_B - i_B R_B$$

$$p_L = v_L i_L$$

$$p_{R_B} = i_B^2 R_B$$

$$p_{R_C} = i_C^2 R_C$$

The energy expended in time Δt are

$$e_L = p_L \Delta t$$

$$e_{R_B} = p_{R_B} \Delta t$$

$$e_{R_C} = p_{R_C} \Delta t$$

At the time the first pulse of a repeated pulse load is applied, the super-capacitor's internal voltage is the same as the voltage on the battery, V_B; provided the C has sufficient time to charge.

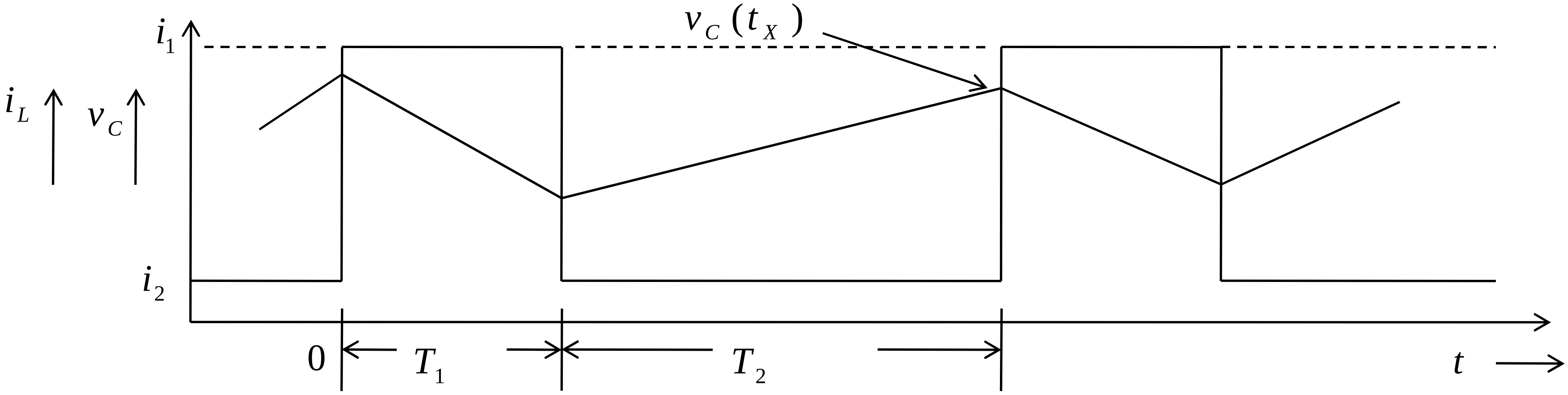
With each applied pulse, the voltage on the C drops slightly, with its average voltage in steady state reaches the average value given by

$$v_L(avg) = v_C(avg) = V_B - R_B \times i_L(avg) < V_B$$

At steady state a train of pulses after C reaches steady state, looks as in next slide

The steady state expression

At steady state a train of pulses after C reaches steady state, looks as

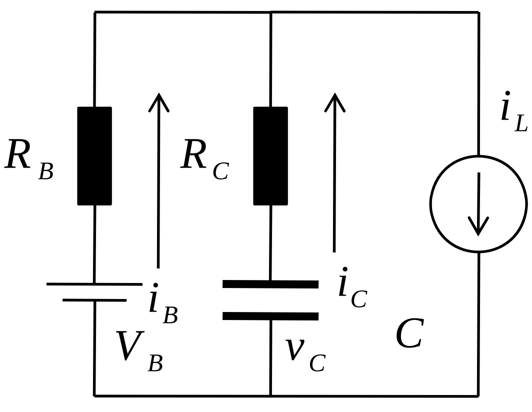


At $t = 0$ to $t = T_1$, $i_L = i_1$. At $t = T_1$ to $t = T_2$, $i_L = i_2$

In the time T_1 to T_2 the v_C follows charging equation (curve)

$$v_C(t) = v_C(t_X)\left[1 - e^{-t/\tau}\right]; \quad \tau = C(R_B + R_C)$$

Battery is always charging the C



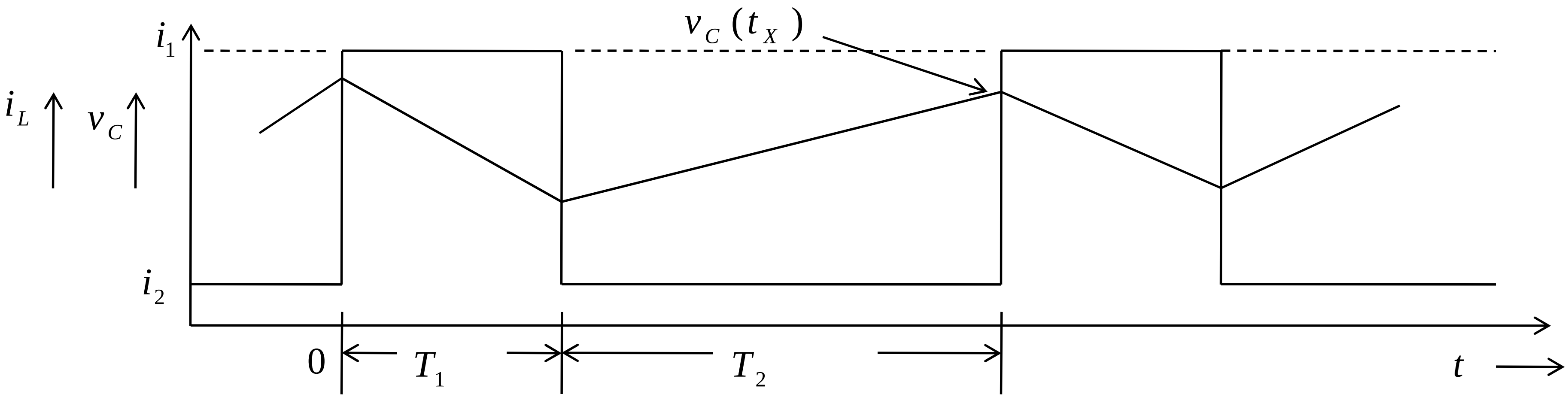
Where $v_C(t_X)$ is the steady-state charged voltage at $t = T_1 + T_2$

At steady state there is no current i_C and we have

$$v_C(avg) = v_L(avg) = V_B - i_L(avg) \times R_B$$

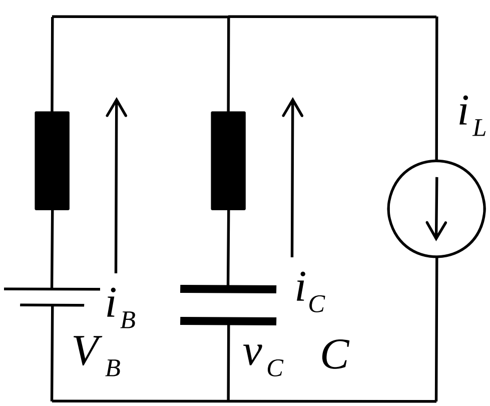
Implying net charge rate and discharge rate at steady-state are equal for C.

The steady state expression…contd.



At the end of $t = T_1 + T_2$, we thus have steady state governed by various superposition's of individual charging and discharging (decay) by several individual components of load currents as:

$$\text{Charge on } C\,;\, v_C = (\text{Charge due to } i_2 \text{ alone at end of } t = T_2)$$
$$- (\text{Charge due to decay of } v_C \text{ due to } i_1 \text{ in } t = T_1 + T_2)$$
$$- (\text{Charge due to decay of } v_C \text{ due to } i_1 - i_2 \text{ in } t = T_2)$$



$$\text{Charge on } C\,;\, v_C\,;\text{ at } t = T_1 + T_2 \text{ is} : v_c(t_X)\left(1 - e^{-\frac{T_1+T_2}{\tau}}\right)$$

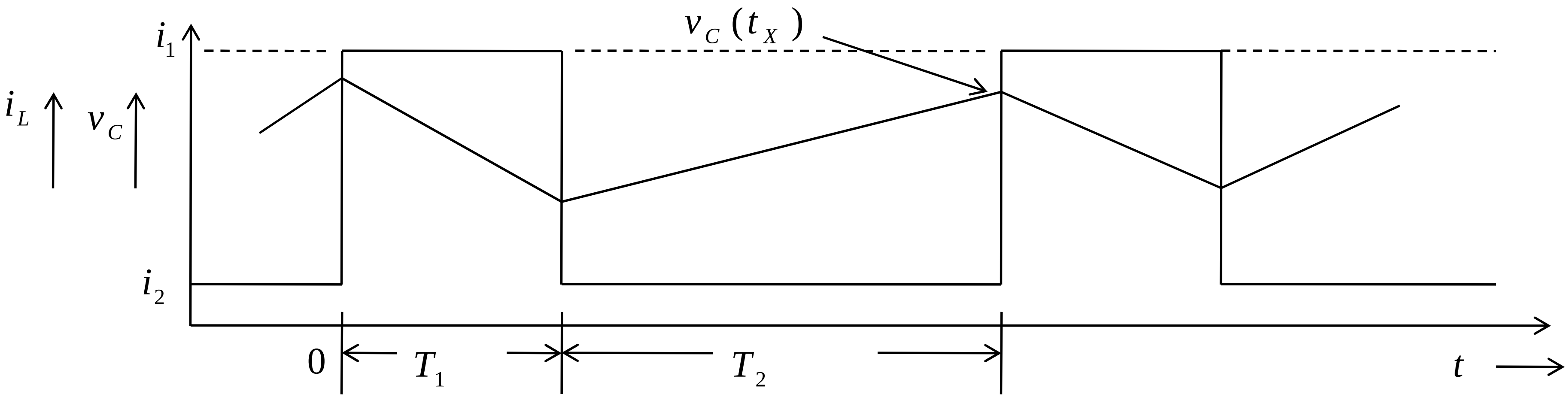
$$\text{Charge due to } i_2 \text{ alone at end of } t = T_2 \text{ is} : V_B - i_2 R_B$$

$$\text{Charge due to decay of } v_C \text{ due to } i_1 \text{ in } t = T_1 + T_2 \text{ is} : \left(V_B - i_1 R_B\right) e^{-\frac{T_1+T_2}{\tau}}$$

$$\text{Charge due to decay of } v_C \text{ due to } i_1 - i_2 \text{ in } t = T_2 \text{ is} : R_B (i_1 - i_2) e^{-\frac{T_2}{\tau}}$$

(Here V_B gets cancelled)

The steady state expression…contd.



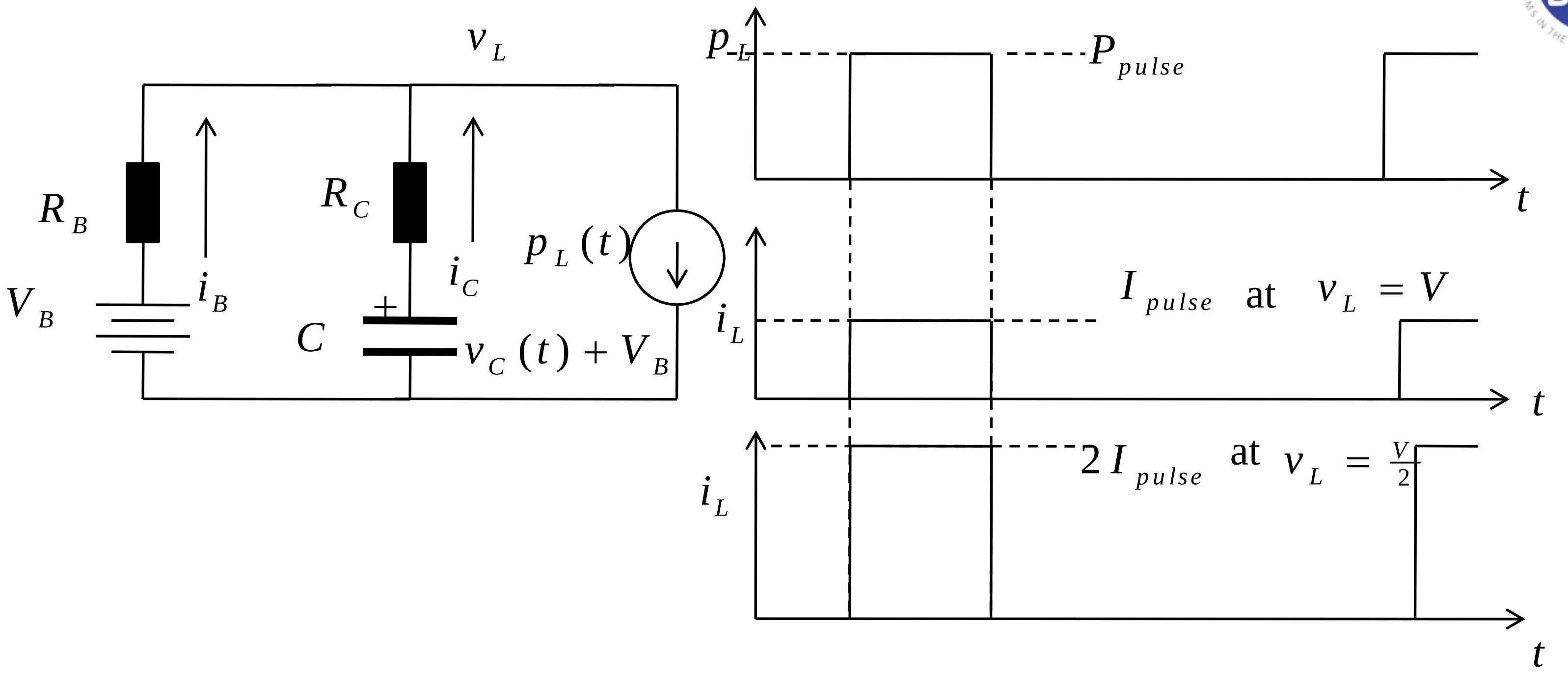
We write the balance equation as

$$v_C(t_X)\left(1-e^{-\frac{T_1+T_2}{\tau}}\right)=(V_B-i_2R_B)-(V_B-i_1R_B)e^{-\frac{T_1+T_2}{\tau}}-R_B(i_1-i_2)e^{-\frac{T_2}{\tau}}$$

$$v_C(t_X)=\frac{V_B-i_2R_B+(i_1R_B-V_B)e^{-\frac{T_1+T_2}{C(R_B+R_C)}}+R_B(i_2-i_1)e^{-\frac{T_2}{C(R_B+R_C)}}}{1-e^{-\frac{T_1+T_2}{C(R_B+R_C)}}}$$

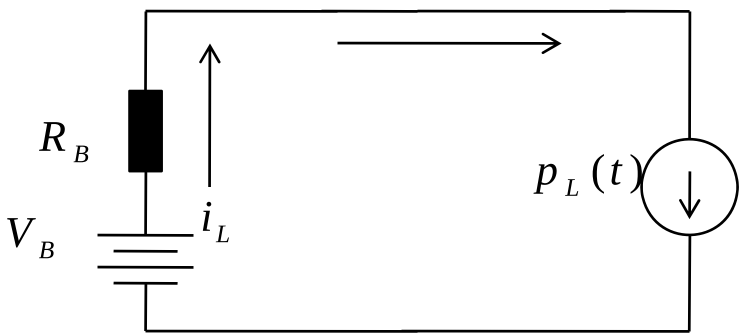
About fixed power pulsed load

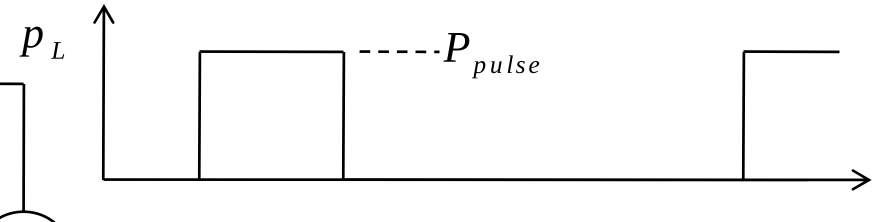
Fixed power load



Here the current drawn by load depends upon the load voltage but the power remains fixed

Fixed power load-calculations





The power to the load is approximately $p_L = V_B \times i_L$ neglecting R_B

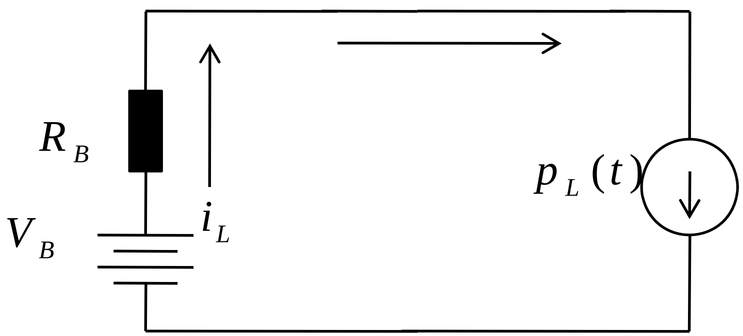
The approximate current drawn is $i_L \cong \frac{p_L}{v_L} = \frac{p_L}{\left[V_B - \left(\frac{p_L}{V_B}\right)R_B\right]}$

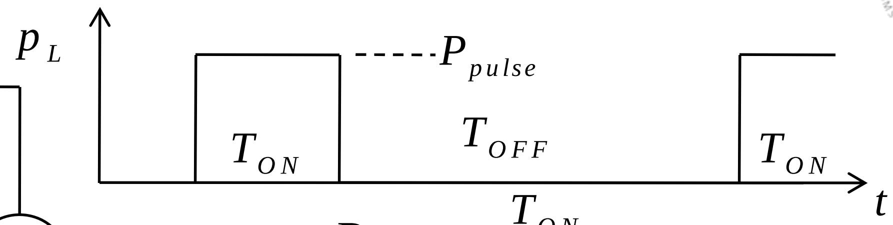
In above expression is power delivered to the load is reduced by the voltage drop across the internal resistance. This in turn means load has to draw more current , in order to receive the required power, hence increasing the current demand over that of an ideal system in which the $R_B = 0$

As output voltage v_L drops, the current drawn by load increases. The power-equation is

$$V_B i_L - i_L^2 R_B = p_{pulse}$$

Fixed power load-calculations…contd.





$$D = \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

This is energy store in *VI* minus loss in impedance equals load energy in unit time.

$$V_B i_L - i_L^2 R_B = p_L$$

$$R_B i_L^2(t) - V_B i_L(t) + p_L(t) = 0$$

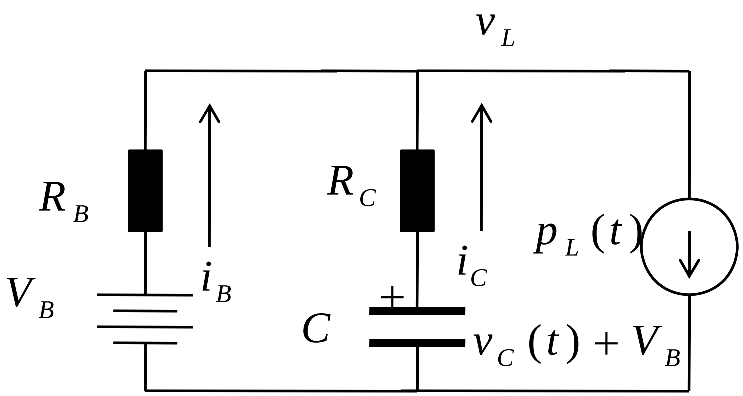
$$i_L(t) = \frac{V_B \pm \sqrt{V_B^2 - 4\,p_L(t) \times R_B}}{2R_B} = i_B(t)$$

This is basic expression-without C in parallel. However, if C is in parallel, then voltage drop here $i_L R_B$ gets reduced as discussed earlier. This has effect of reducing current from battery for pulse current. With C in parallel the output voltage under load may be approximated by average voltage, and this is determined by average power.

For a load pulse of a given power, i.e. P_{pulse} with duty cycle D we write

$$p_L(avg) = P_{pulse} \times D = P_{pulse} \times \frac{T_{ON}}{T_{ON} + T_{OFF}}$$

Fixed power load-with C –calculations…contd.



For high C and low ESR, the voltage ripple at the load will be smaller. Then the approximate average battery current required to deliver average power with C present

$$i_B(avg) \cong \frac{p_L(avg)}{v_L(avg)} \cong \frac{p_L(avg)}{V_B - \frac{p_L(avg)}{V_B} \times R_B}$$

A more accurate expression is

$$i_B(avg) \cong \frac{V_B \pm \sqrt{V_B^2 - 4\,p_L(avg) \times R_B}}{2R_B}$$

Note that the current in above equation depends on average power rather than the peak power drawn in each pulse, resulting in a significant reduction in peak battery current compared to a case without C

These solutions are good for small D

The average voltage at the output with high C and low ESR is approx. $v_L(avg) \cong V_B - i_B(avg) \times R_B$

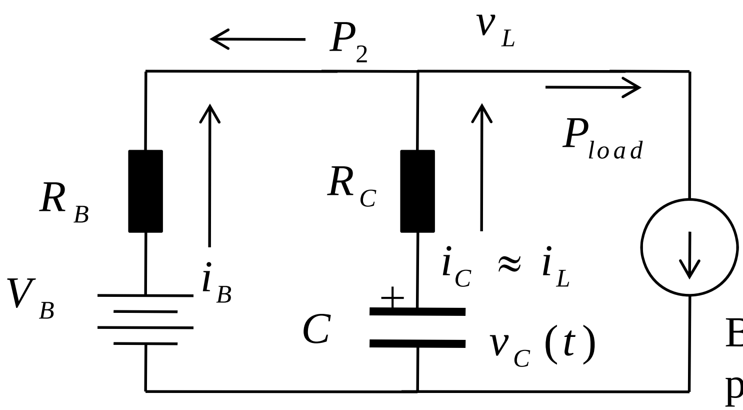
As in case of fixed current load, the voltage drop during the pulse with C is reduced significantly over that of battery alone. With C present , the approx. load current during a fixed power pulse is

$$I_{pulse} \approx \frac{P_{pulse}}{v_L(avg)}$$

The corresponding drop in output voltage under load can again be approximated roughly by voltage drop when only C supplies the full load (neglecting battery contribution) $V_{\text{drop}} \approx I_{pulse} \times R_C - I_{pulse} \times \frac{T_{ON}}{C}$

If battery is connected this drop will be lesser than obtained by this above expression

Fixed power load-with C –calculations…contd.



For all the load current is by capacitor we write

$$v_C i_C - i_C^2 R_C - i_C^2 Z_C = P_{load} \qquad Z_C \approx \frac{\Delta t}{2C}$$

Note that loss in ESR of capacitor due to $i_C \approx i_L$ is $i_C^2 R_C$

and loss in capacitor impedance is $i_C^2 Z_C$

But here we have capacitor apart from delivering out power delivers power to the battery branch, i.e.

$$P_2 = v_C \times i_B = v_C \left(\frac{v_C - V_B}{R_B}\right)$$

Due to the branch comprising of battery there will be extra *VI* terms and extra losses too.

We note the extra loss as some fraction of $i_L \approx i_C$ flows into R_B, and that extra loss is

$$\left[\left(\frac{R_C}{R_B + R_C}\right) i_C\right]^2 \times R_B \cong \frac{R_C^2}{R_B} i_C^2$$ here we take $R_C << R_B$

The other *VI* products are

$$1: \quad v_C \times i_C \left(\frac{R_C}{R_B + R_C}\right) \approx v_C \left(\frac{R_C}{R_B}\right) i_C$$

$$2: \quad (v_C - V_B) \times i_C \left(\frac{R_C}{R_B + R_C}\right) \approx (v_C - V_B)\left(\frac{R_C}{R_B}\right) i_C$$

Use power balance equation $\sum v_m i_m - \sum i_m{}^2 Z_n = \sum P_n$ and we obtain

$$\left[v_C i_C + v_C \left(\frac{R_C}{R_B}\right) i_C + (v_C - V_B)\left(\frac{R_C}{R_B}\right) i_C\right] - \left[R_C i_C^2 + \left(\frac{R_C^2}{R_B}\right) i_C^2 + \left(\frac{\Delta t}{2C}\right) i_C^2\right] = P_{load} + v_C \left(\frac{v_C - V_B}{R_B}\right)$$

$$\left[v_C + \left(\frac{R_C}{R_B}\right)(2v_C - V_B)\right] i_C - \left[R_C\left(1 + \frac{R_C}{R_B}\right) + \left(\frac{\Delta t}{2C}\right)\right] i_C^2 = \left[P_{load} + v_C \left(\frac{v_C - V_B}{R_B}\right)\right]$$

Summary of constant power pulsed load

The super capacitor current may be thus found directly that it satisfies the following equation involving constant load power

$$a_C i_C^2 + b_C i_C + c_C = 0$$

$$a_C = R_C\left(1 + \frac{R_C}{R_B}\right) + \frac{\Delta t}{2C} \qquad b_C = \frac{R_C}{R_B}(V_B - 2v_C) - v_C \qquad c_C = v_C\left(\frac{v_C - V_B}{R_B}\right) + P_{load}$$

$$i_C = \frac{-b_C \pm \sqrt{b_C^2 - 4a_C c_C}}{2a_C}$$

The change in super-capacitor voltage by the end of time interval Δt is then

$$\Delta v_C = -\frac{i_C}{C}\Delta t$$

The load voltage and battery current may be determined from the two following

$$v_L = v_C - i_C R_C \qquad i_B \approx \frac{(V_B - v_C) + i_C R_C}{R_B}$$

These are simulation equations for fixed power pulsed load.

Conclusions

This effort is in tune with ‘make-in-India’ program of Government of India .

There are lots of scientific and engineering challenges in circuits & systems with super capacitors especially in Indian context.

The power store-delivery circuits by super-capacitor if applied to rural power-systems based on concept of ‘micro-grid’ will be boon to our Nation-apart from using these in automobile EV/HEV or strategic electronics and commercial power electronics like UPS systems.

There are lot of new science in electro-chemical and electrical science fields to be explained developed by use of fractional calculus concept.

The concept of EDLC seen in 1875 still is nascent and real explanation of charge arrangement yet to be convincing.

Recently in 2015 ‘Nature’ brought out article about possibility of arero-gel super-capacitors; where as we are having a device out of the same today, with possible applications!

Our sincere prayer to Government , academicians and to reputed national laboratories is to have some modest funding and encouragement to these in house developed science

Thank you all

We have lot to do and will do and try to do

Require institutional support