Epsilon-Mu Zero Meta-material with Zero Refractive Index
An Electromagnetic Nihility

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Key words

Resonance, zero effective permittivity, zero permeability, zero refractive index, nihility, Meta-material, cylindrical harmonics, Hankel's function

Abstract

The interesting property of meta-material having simultaneous zero permittivity and zero permeability is noted in the note. Here it is demonstrated that if a material has near zero refractive index by realizing the real part of permittivity and permeability tuned at electric and magnetic plasma frequency, then within a narrow pass band, we get electrostatic electric field and magneto static magnetic field inside the 'nihility' cylinder, and with usual cylindrical travelling waves outside the cylinder. This exotic property if experimented will give immense potential applications in Electromagnetic devises and circuit's applications. May be extended in Left Handed Maxwell (LHM) project in future for experimentation, along with the present plasmonic meta-material prisms for realizing simultaneous negative permittivity and permeability, to demonstrate negative refractive index systems.

1. Introduction

The Meta-materials having near zero refractive index can offer exciting potential applications in electromagnetic devises. This phenomenon is called nihility in optics. We demonstrate that Electromagnetic fields in a matched zero-index medium take on static character in space yet remain dynamic in time; in such a manner that underlying physics remains associated with propagating (travelling waves) fields. This phenomenon is realized via artificial structural inclusions to get near zero effective permittivity and near zero effective permeability. We have presented here volumetric meta-materials that exhibit near zero index medium properties. These zero index electromagnetic structures studies include working in a pass band, by introducing a source into a zero index meta-material with an excitation frequency that is within the pass band, interesting field distributions are generated, giving electro static electric field and magneto static magnetic field. All these zero index medium if made can have exotic properties, and can cater to specialized narrow band novel electromagnetic circuits and devices. In this short noter we assume the solutions of Maxwell's equation as wave functions in cylindrical coordinates, derived in several text books. The standard solutions of travelling wave in cylindrical harmonics is via Hankel's function ($H_\upsilon^{(1)}$, and $H_\upsilon^{(2)}$) of first and second kind, appearing in several electrodynamics text books. These Hankel's functions are cylindrical harmonics of order υ with $\upsilon \in \Re$. They are linear superposition of plane waves. We are used to Bessel's function in cylindrical geometry as solutions for cylindrical wave-guides. Bessel functions are class of Hankel's functions with real arguments, giving standing wave solutions; whereas the Hankel's functions are of complex arguments denoting harmonic travelling wave solutions. Bessel's and Hankel's are related to each other. The derivation of cylindrical harmonics via Hankel's function we are not noting here, as these are taken from text books. We take a (infinite) line current source of intensity I_0 directed towards z direction, in cylindrical geometry given by coordinate ρ, φ and z a region, of radius $\rho = a$ enclosing region with $\mathrm{Re}\,\varepsilon \cong 0$, and $\mathrm{Re}\,\mu \cong 0$, that is having zero index of refraction (for $\rho \le a$). We study the travelling waves inside the nihility cylinder and outside the nihility cylinder, by looking at distributions of $\mathbf{E}$ and $\mathbf{H}$ fields.

2. Derivation of Travelling Wave inside zero indexed Meta-Material

EM fields in a matched zero-index medium take on static character in space yet remain dynamic in time; in such a manner that underlying physics remains associated with propagating fields. This we demonstrate in this section. Consider the figure-2 a representing an infinite cylinder excited by a line current source. The travelling wave representation in cylindrical harmonics for a Helmholtz's equation $(\nabla^2 + k^2)\psi = 0$, is given by Hankel's function as ($\mathrm{i} = \sqrt{-1}$)

$$\psi(\rho,\varphi) = \begin{Bmatrix} H_\upsilon^{(1)}(\rho) \\ H_\upsilon^{(2)}(\rho) \end{Bmatrix} e^{\mathrm{i}\upsilon\varphi} e^{\mathrm{i}k_z z} e^{-\mathrm{i}\omega t} = H_\upsilon(kr) e^{\mathrm{i}\upsilon\varphi} e^{\mathrm{i}k_z z} e^{-\mathrm{i}\omega t} \qquad \omega^2 = c^2 k^2 + c^2 k_z^2$$

The Hankel's functions are defined as follows

$$\left.\begin{matrix} H_\upsilon^{(1)} \\ H_\upsilon^{(2)} \end{matrix}\right\} = \frac{e^{\mathrm{i}\upsilon\pi/2}}{\pi} \int_{\alpha_1-\varphi}^{\alpha_2-\varphi} e^{\mathrm{i}\rho\cos\alpha} e^{\mathrm{i}\upsilon\alpha} \mathrm{d}\alpha$$

$$H_\upsilon^{(1)}(\rho) = \frac{e^{-\mathrm{i}\upsilon\pi/2}}{\pi} \int_{C_1} e^{\mathrm{i}\rho\cos\alpha + \mathrm{i}\upsilon\alpha} \mathrm{d}\alpha \qquad H_\upsilon^{(2)}(\rho) = \frac{e^{-\mathrm{i}\upsilon\pi/2}}{\pi} \int_{C_2} e^{\mathrm{i}\rho\cos\alpha + \mathrm{i}\upsilon\alpha} \mathrm{d}\alpha$$

The integral representation of the Hankel's function has standard contour of integration in α complex plane with $\mathrm{Im}\,\alpha_1 = +\infty$ and $\mathrm{Im}\,\alpha_2 = -\infty$. The, C_1 and C_2 are integration contours. However, in this note we are not going to discuss properties of Hankel's function.

Consider Maxwell's equations

$$\nabla \times \mathbf{E} = -\mathrm{i}\omega\mu\mathbf{H} \qquad (1)$$

$$\nabla \bullet (\varepsilon\mathbf{E}) = \rho \qquad (2)$$

$$\nabla \times \mathbf{H} = \mathrm{i}\omega\varepsilon\mathbf{E} + \mathbf{J} \qquad (3)$$

$$\nabla \bullet (\mu \mathrm{H}) = 0 \qquad (5)$$

When $\mathrm{Re}\,\varepsilon \cong 0$ and $\mathrm{Re}\,\mu \cong 0$, the Maxwell equation is reduced to

$$\nabla \times \mathbf{E} = 0 \qquad (6)$$

$$\nabla \bullet (\varepsilon\mathbf{E}) = 0 \qquad (7)$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad (8)$$

$$\nabla \bullet (\mu\mathbf{H}) = 0 \qquad (9)$$

The equations (7) (8) are automatically satisfied in the zero-index medium for finite fields. Thus we see the static like equations within medium of zero indexes. Let is consider a cylinder made with zero index medium surrounded by free space, the solutions for infinite line currents (figure-2 a); the line current density is thus;

$$\mathbf{J}(\rho,\varphi,z) = \frac{I_0}{2\pi a}\delta(\rho)\delta(\varphi)\hat{\mathbf{z}} \tag{10}$$

For $\rho \leq a$, we have, from solutions of cylindrical harmonics, via Hankel's function as

$$\mathbf{E}(\rho,\varphi,z) = -Z_0 \frac{I_0}{2\pi a}\frac{\mathrm{i}H_0^{(2)}(k_0 a)}{H_1^{(2)}(k_0 a)}\hat{\mathbf{z}} \tag{11}$$

$$\mathbf{H}(\rho,\varphi,z) = \frac{I_0}{2\pi\rho}\hat{\boldsymbol{\varphi}} \tag{12}$$

The solutions show that the electric field is constant inside the cylinder, and the magnetic field decays as radial distance increases as in magneto static case.

For $\rho > a$, we have, from solutions of cylindrical harmonics, via Hankel's function

$$\mathbf{E}(\rho,\varphi,z) = -Z_0 \frac{I_0}{2\pi a}\frac{\mathrm{i}H_0^{(2)}(k_0 \rho)}{H_1^{(2)}(k_0 a)}\hat{\mathbf{z}} \tag{13}$$

$$\mathbf{H}(\rho,\varphi,z) = \frac{I_0}{2\pi a}\frac{H_1^{(2)}(k_0 \rho)}{H_1^{(2)}(k_0 a)}\hat{\boldsymbol{\varphi}} \tag{14}$$

where $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$, $Z_0 = \sqrt{\varepsilon_0 / \mu_0}$ is free space impedance. This show the outside the nihility cylinder, the both fields are harmonic spatially, and decaying with radial distance $\sim H_{0,1}^{(2)}(k_0\rho)$.

Thus inside the cylinder we have spatially constant electrostatic electric field and magneto static magnetic field, (11) (12) where the field outside the cylinder fields propagates as cylindrical waves away from the source. Nonetheless there is power flowing outwards from the source in both regions that is time averaged Poynting's vector that is

$$\langle \mathbf{S} \rangle = \frac{Z_0 I_0^2}{(2\pi a)^2}\frac{1}{\left|H_1^{(2)}(k_0 a)\right|^2}\frac{2}{\pi k_0 \rho}\hat{\boldsymbol{\rho}} \tag{15}$$

If time variation is taken into account and driving source is localized as line-source then one can approximately write the following, inside the zero index cylinder;

$$\mathbf{H}(\rho,\varphi,z,t)=\frac{I_0}{2\pi\rho}f(t)\hat{\boldsymbol{\varphi}} \tag{16}$$

$$\mathbf{E}(\rho,\varphi,z,t)\cong -Z_0\frac{I}{2\pi a}f(t)\hat{\mathbf{z}} \tag{18}$$

The driving alternating current (AC) source could be standard oscillating one that is $\sim e^{-i\omega t}$.

3. The Field Patterns

We make approximate field pattern in the cross section of cylinder having zero index meta-material. The material is made via choosing the magnetic structures via Split Ring resonators (SRR) having characteristics of resonance in figure-1 b, and the resonating electrical structures realized by wire array medium giving resonating structures of figure 1 a. The plasma frequencies of electric plasma and magnetic plasma are chosen to be overlapped as $\omega_{ep}=\omega_{mp}=\omega_0$, where we get $\mathrm{Re}\,\varepsilon\cong\mathrm{Re}\,\mu\cong 0$. The design of these is reported in several references as listed. When the both values of $\mathrm{Re}\,\varepsilon$ and $\mathrm{Re}\,\mu$ are negative (DNG region) or both values are positive (DPS region) we get travelling solution to Maxwell's equation, and that is the pass band at $\omega<\omega_0$; discussed in several references as listed. We choose a frequency of excitation in this pass band region very close to ω_0 but slightly lesser or greater (to allow travelling wave solutions else the mismatch will give evanescent solution). This is the frequency of $f(t)$ in (16) and (17).

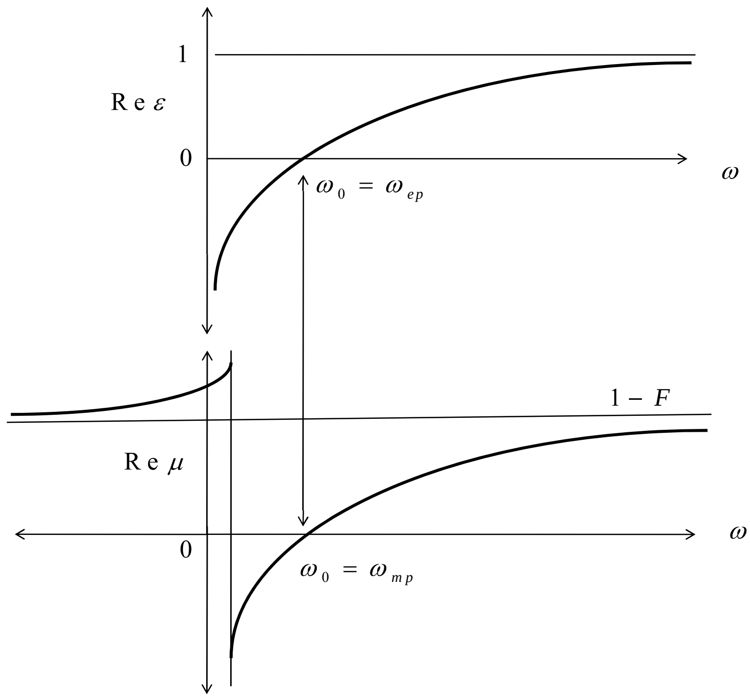

Figure-1: Resonating structures giving simultaneous epsilon and mu zero, chose operating frequency near magnetic and electric plasma frequency to get DNG or DPS region

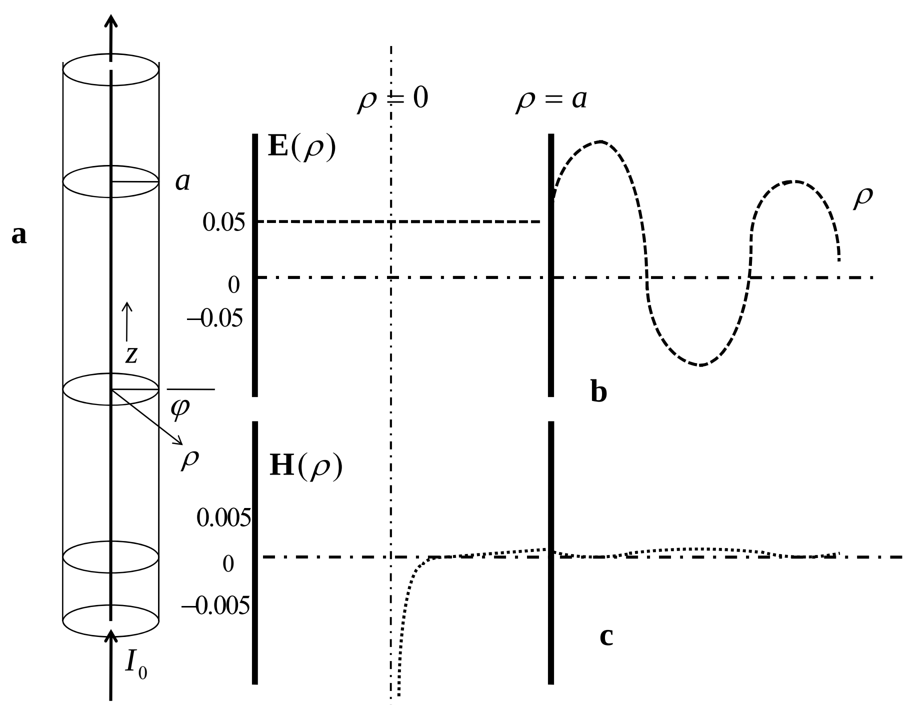

Figure-2: Cylinder with zero refractive index excited by a line current source, and E and H field

The, $\mathbf{E}$ and $\mathbf{H}$ fields are plotted in figure-2 b and c, (side section view of cylinder) for a particular time freeze. The Electric field inside the nihility region ($\rho \leq a$) is static character, and magnetic field in side nihility cylinder show magneto-static character. Outside the nihility cylinder, the Electric and Magnetic fields show travelling wave character, with spatial crests and troughs and their amplitude decaying as the radial distance increases. Due to symmetry, the figure-2 b and c we are showing one side only. For a different time freeze we will have similar picture with different constant amplitude of fields. With time the vertical line showing the electric field inside the nihility cylinder will be moving up and down from maximum $\mathbf{E}$ to zero and to minimum $\mathbf{E}$ depending on $f(t) \sim e^{i\omega t}$; similarly the spike of magnetic field will be varying with time-showing up-down movement.

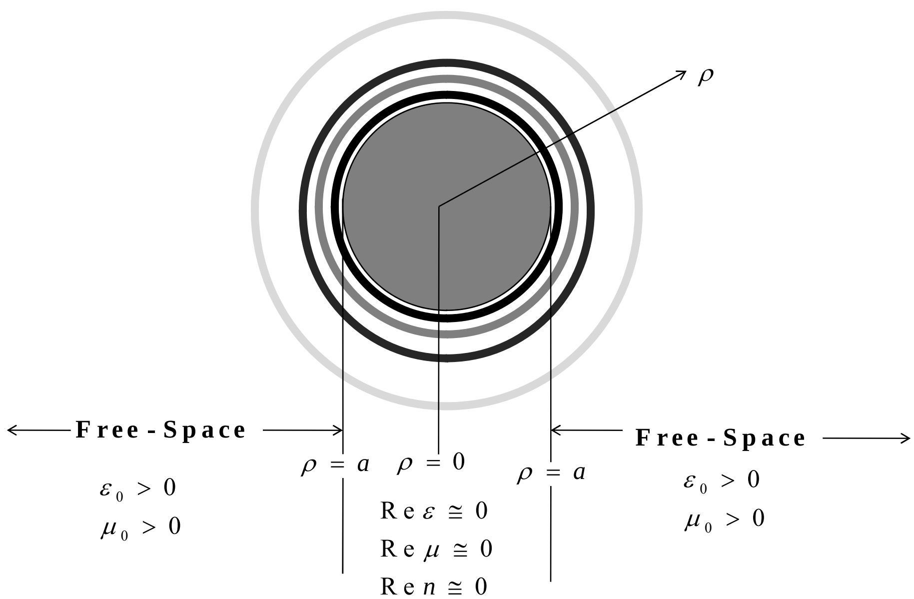

Figure-3 Electric Waves inside the zero index cylinder and outside the cylinder excited from a line current source at the centre of the cylinder

The figure-3 shows the same with top view. The figure only shows the electric field, at a particular time freeze. Inside the cylinder electric field show a constant nature, but outside the cylinder the cylindrical wave patterns in radial direction as for travelling cylindrical waves, amplitude decaying with radial distance.

4. Conclusion

We have discussed very exotic phenomena of travelling electromagnetic wave getting 'static like' behavior (electrostatic and magneto static) while inside a meta-material having both permittivity and permeability as near zero, giving an electromagnetic system with nihility. This nihility discussed has potential applications in the science of electromagnetism for probable usage to make advanced novel electromagnetic devices and circuits. On these phenomena of nihility, electromagnetic experiments are yet to be done. Perhaps we may extend our Left Handed Maxell Project for this novel experiment too, along with what is done to demonstrate plasmonic structures with negative permittivity and negative permeability for negative refractive index (presently at Ka-Band). This experiment on zero refractive index is to be carried out; not reported via any experimental result.

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