Left Handed Maxwell Systems

PART-8

Application of Magneto-Inductive Waves

SAMEER

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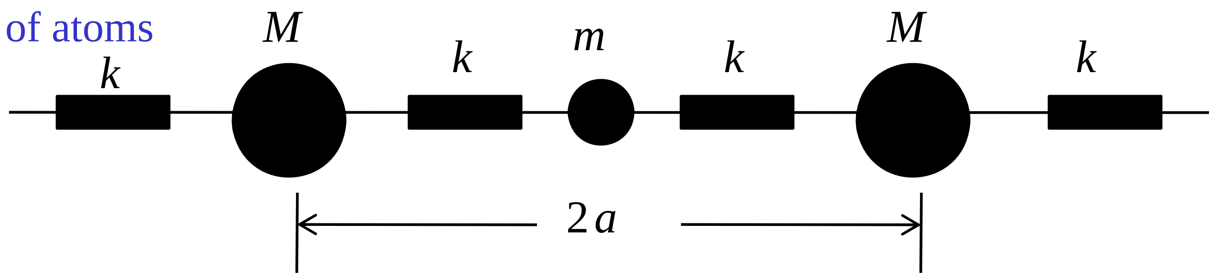
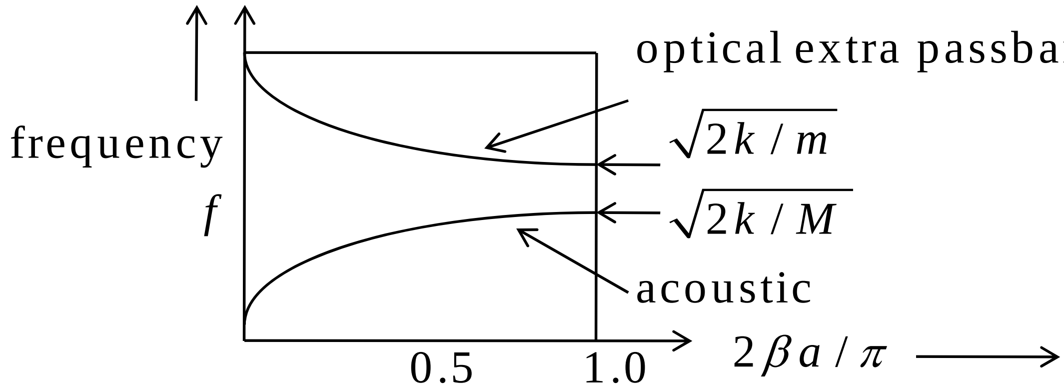
Tailoring the dispersion characteristics of MI waves

In the theory of MI waves we have observed that for a small coupling $\kappa = 2M / L = 0.1$, the pass band is small that is from $\omega = 0.95\omega_0$ to $1.05\omega_0$. A narrow pass band for the MI waves is similar to the acoustic waves.

Consider a di-atomic chain of atoms

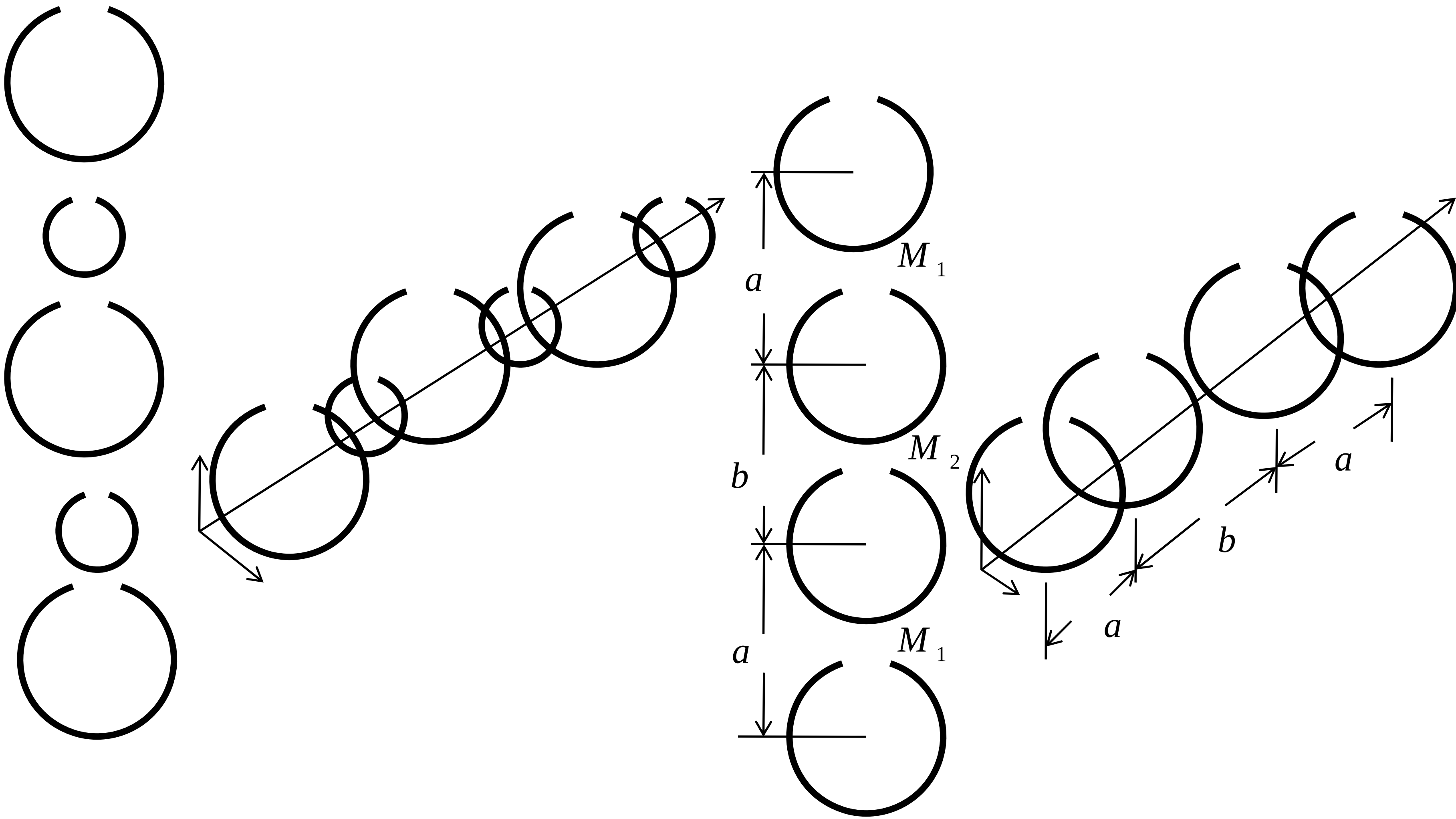




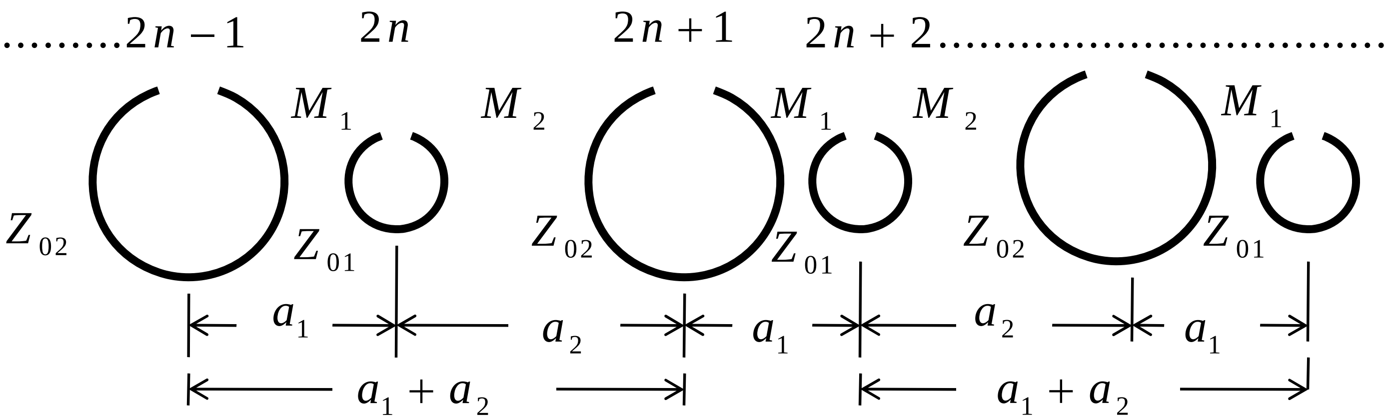
Here phonon dispersion curve appears with two pass bands. The consequence of having a material of Different mass alternate, is appearance of a new pass band known as optical branch in dispersion curve

Possibilities to tailor the MI dispersion

1. To change the parameter of elements (*L* or *C* resulting in change of resonant frequency).
2. To vary the distance between the elements which means there will be two different mutual inductance.



Bi periodic line



$$Z_{01}I_{2n} + j\omega M_1 I_{2n-1} + j\omega M_2 I_{2n+1} = 0 \ldots\ldots\ldots\ldots(1)$$

$$Z_{02}I_{2n+1} + j\omega M_2 I_{2n} + j\omega M_1 I_{2n+2} = 0 \ldots\ldots\ldots\ldots(2)$$

Assume propagating solution on both even and odd elements

$$I_{2n} = A_2 e^{-jk2n(a_1+a_2)} \ldots\ldots(3) \qquad I_{2n+1} = A_1 e^{-jk(2n+1)(a_1+a_2)} \ldots\ldots(4)$$

Substituting (3) – and (4) – in (1) – and (2)

$$\cos\left(\frac{k(a_1+a_2)}{2}\right) = \frac{1}{2}\frac{\sqrt{-\frac{Z_{01}Z_{02}}{\omega^2} - (M_1 - M_2)^2}}{\sqrt{M_1 M_2}} \ldots\ldots\ldots(5)$$

when $\quad Z_{01} = Z_{02} = Z_0 \qquad M_1 = M_2 = M$

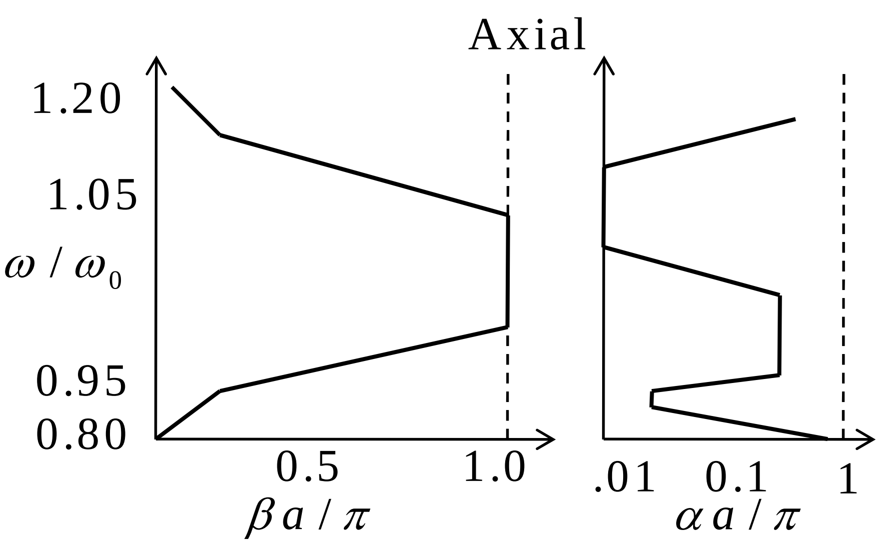
$$\omega = \omega_0 / \sqrt{1 + (2M / L)\cos ka}$$

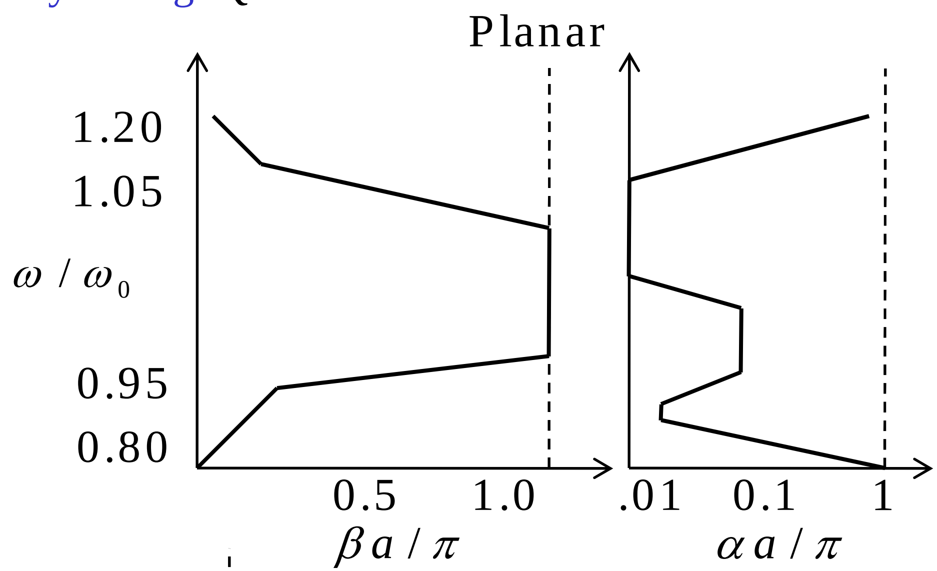
Pass band tailoring broadening!

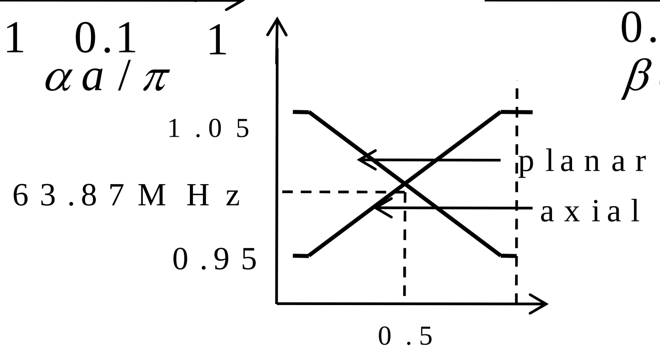
For SRR take $r_0 = 10\text{mm}$, wire diameter $d_w = 2\text{mm}$, the $L = 33\text{nH}$ load with $C_1 = 208\text{pF}$ and $C_2 = 177\text{pF}$ .This gives $\omega_{01} = (LC_1)^{-1/2} = 0.95\omega_0$ and $\omega_{02} = (LC)^{-1/2} = 1.05\omega_0$ with $(\omega_0 / 2\pi) = 63.87\text{MHz}$

This 63.87 MHz. has reason, because it corresponds to Magnetic Resonant frequency of proton for a magnetic field of 1.5 T, one of possible choice to use SRR in MRI (Magnetic Resonant Imaging).

For axial choose $a = 10\text{mm}$, resulting $M / L = 0.149$. For planer have $a = 20.5\text{mm}$, resulting $M / L = -0.104$. Losses are taken into account by setting $Q = 150$







Observations and comments regarding the bi-periodic SRR arrangements

The major distinction between the axial and planar cases in uniform SRR arrangements, is that one gives forward and other give backward MI wave. Well this distinction is not there the bi-periodic configuration In both the cases of bi-periodic configuration, the lower branch is forward wave and upper branch is backward MI wave. Also the observation is that for bi-periodic MI structure, the pass band too has broadened.

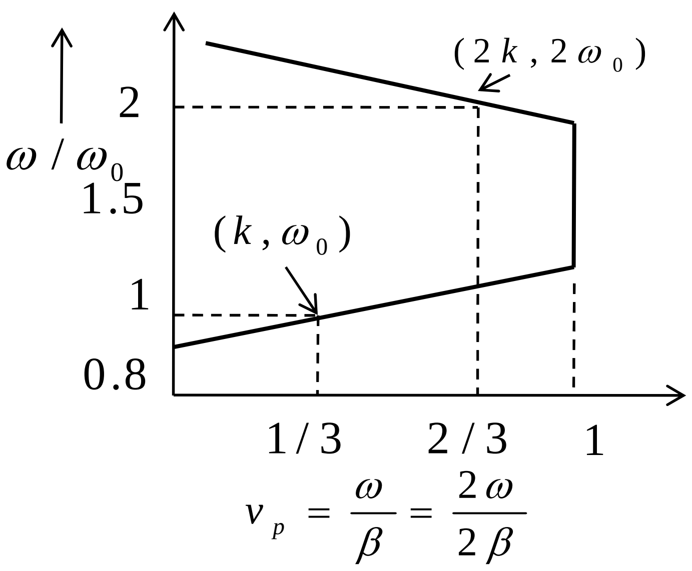
This has one practical utility. In Parametric amplification and problem of synchronization between Signal wave and pump wave can be achieved. How? Chose signal frequency of $\omega_0 / 2\pi = 63.87\,\mathrm{MHz}$ The propagation constant of signal may be chosen (for bi-periodic line) $2\beta a = \pi / 3$. The propagation constant of pump may be chosen as $2\beta a = 2\pi / 3$ at frequency $2\omega_0 / 2\pi = 127.74\,\mathrm{MHz}$ This structure for bi-periodic is with

$$a = 5 r_0 \qquad L = 33\,nH$$

$$M = 0.336\,L$$

$$C_1 = 164\,\mathrm{pF}$$

$$C_2 = 56\,\mathrm{pF}$$



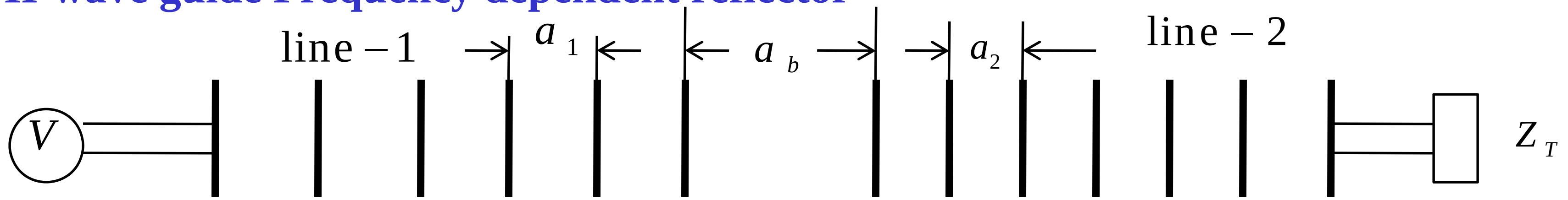
Termination of bi periodic line

$$Z_T(1,2) = \frac{M_{1,2} Z_{01,02}}{M_{1,2} + M_{2,1} e^{jk(a_1+a_2)}}$$

At last element

At second last element

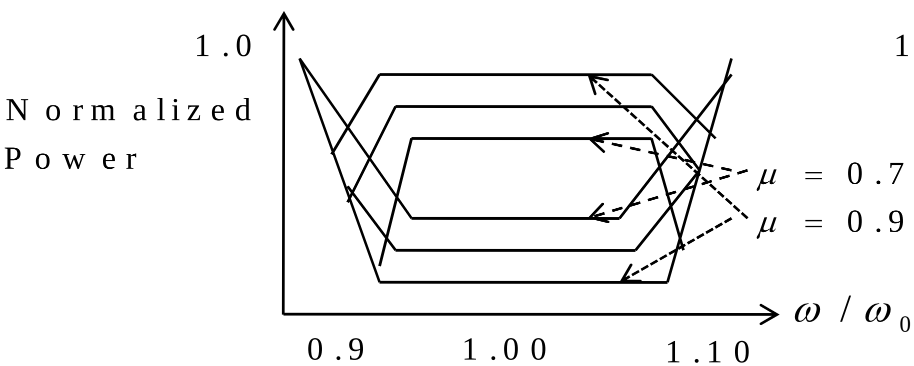
MI-wave guide Frequency dependent reflector

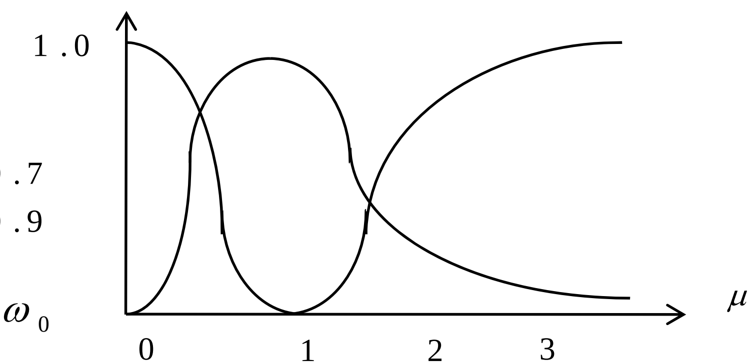


Two one-D lines may be seen to be coupled to each other by M_b. Let us take both the lines identical, so that only new parameter is M_b.

$$R^2 = \frac{(\mu^2 - 1)^2}{D} \qquad T^2 = \frac{4\mu^2 \sin^2(ka)}{D}$$

$$\text{where} \quad D = 1 + \mu^4 - 2\mu^2 \cos(ka) \qquad \mu = M_b / M$$





MI wave guide mirror!

a. Variation of $|R|^2$ and $|T|^2$ with ω / ω_0 for $\kappa = 0.2$ and different μ

b. Variation of $|R|^2$ and $|T|^2$ with μ for $\omega = \omega_0$

Interpretation for MI wave guide frequency dependent reflector

a. Curve are slowly varying across the band. Towards the band edges when $ka = 0$ and $ka = \pi$ the $R \to 1$, the discontinuity reflects everything. Near the centre of the band the T is high but steadily reduces with μ. In this region at the 'Plato' near the centre $\omega = \omega_0$ the approximation is

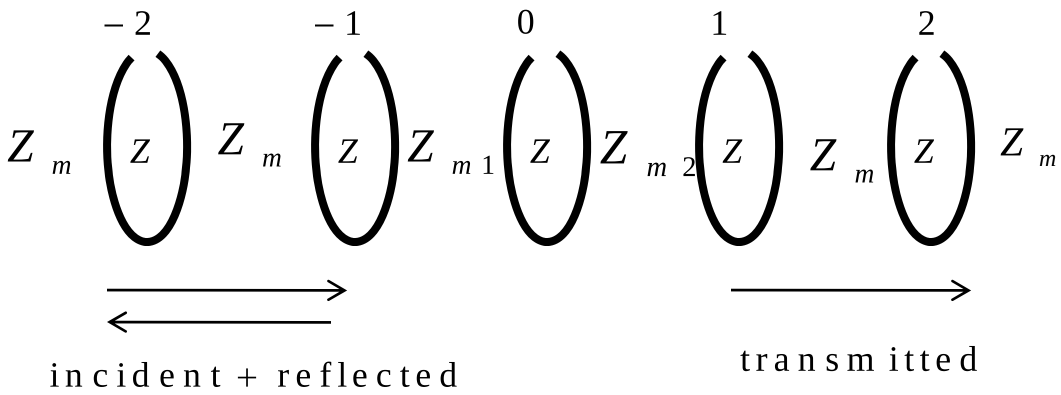
$$R \cong \frac{\mu^2 - 1}{\mu^2 + 1} \qquad T \cong \frac{2\mu}{\mu^2 + 1}$$

b. This curve gives variation of R^2 and T^2 with μ obtained from above approximation, at around the resonances of SRR.

These results are independent of ka and show that reflector with reasonable broad-band performance may be constructed by slight variation in the spacing at junction between the two lines

MI wave guide-Fabry-Perot resonator

As known in optics, two collinear reflectors make Fabry-Perot resonators and that applies to MI waves too.



The current in the uniform line to the left of element -1 may be assumed in the form of and incident and a reflected MI wave, and as a traveling wave to the right of element 1. The basic KVL of the system is thus:

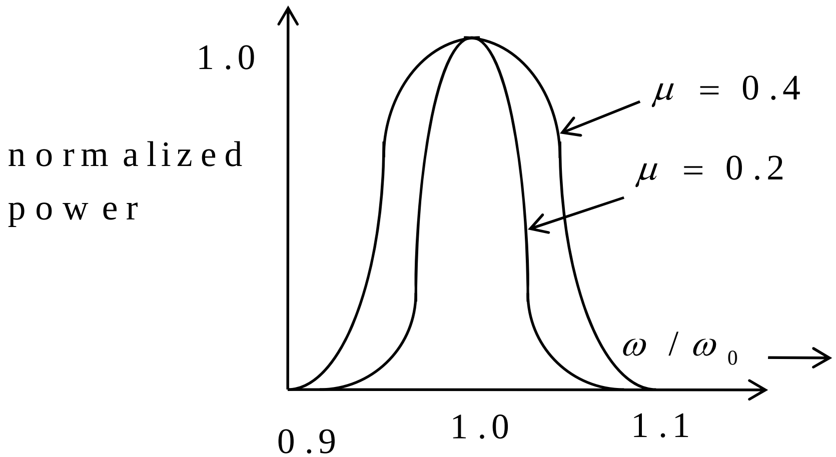
$$Z_0 I_{-1} + j\omega M_1 I_0 + j\omega M I_{-2} = 0$$

$$Z_0 I_0 + j\omega M_2 I_1 + j\omega M_1 I_{-1} = 0$$

$$Z_0 I_1 + j\omega M I_2 + j\omega M_2 I_0 = 0$$

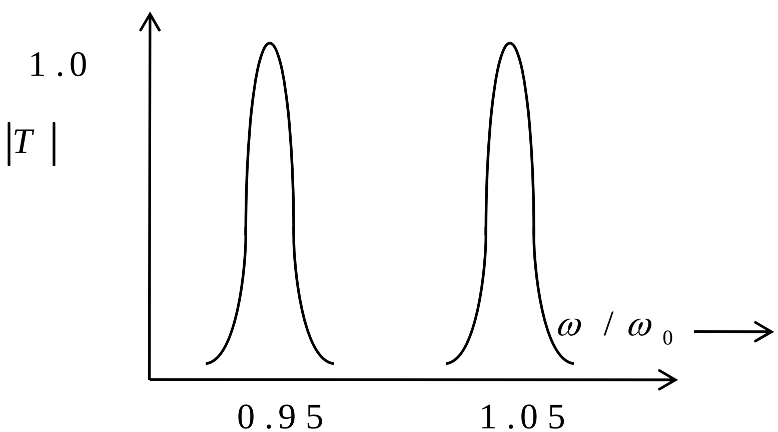
where $Z_m = j\omega M \qquad Z_{m1} = j\omega M_1 \qquad Z_{m2} = j\omega M_2$

Interpretation of MI wave guide Fabry-Perot resonator



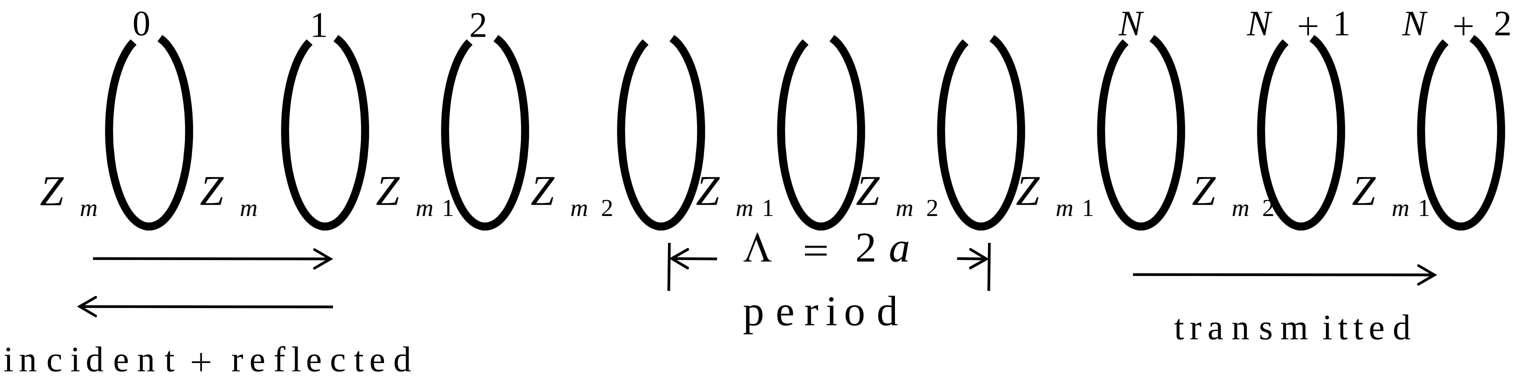
for $\kappa = 0.2$ $\mu_1 = M_1 / M = \mu_2 = M_2 / M = 0.2, 0.4$

MI wave guide Fabry-Perot resonator, variation of $|T|^2$ with ω / ω_0 show clearly there is T peak around resonance ω_0. Now if additional resonant loop is added between element 0 and 1 then we get additional T peaks. The following is figure with two loop cavity having $\kappa = 0.2$ and $\mu = 0.2$

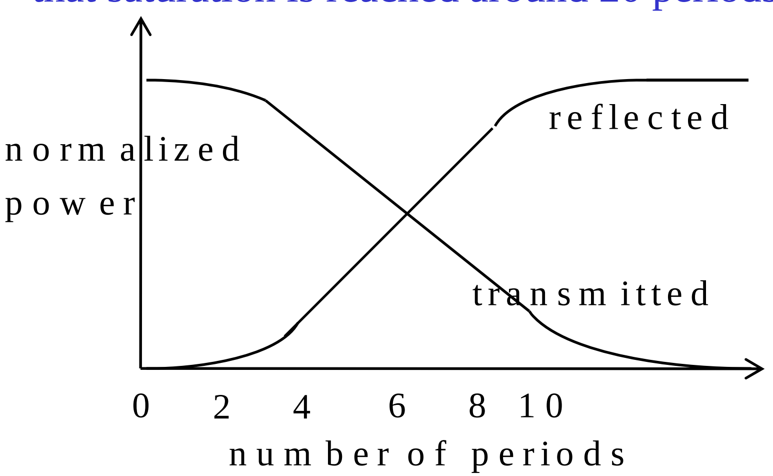


MI waveguide Bragg's grating

Following the optical analogy we should be able to have large reflection in a certain band of frequency-if a large number of small reflectors add coherently-if the mutual inductances undergo a small but periodic variation along the line-then we can achieve this Bragg's reflector.

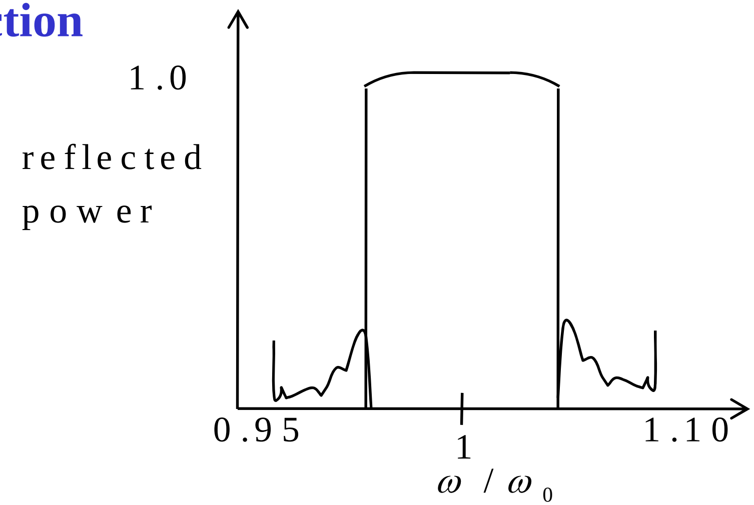


The period is $\Lambda = 2a$. How many periods do we require for 'large' reflection? It may be observed that saturation is reached around 20 periods for $\mu_1 = 1.1$ and $\mu_2 = 0.9$



This show variation of R and T at $\omega = \omega_0$ with number of period

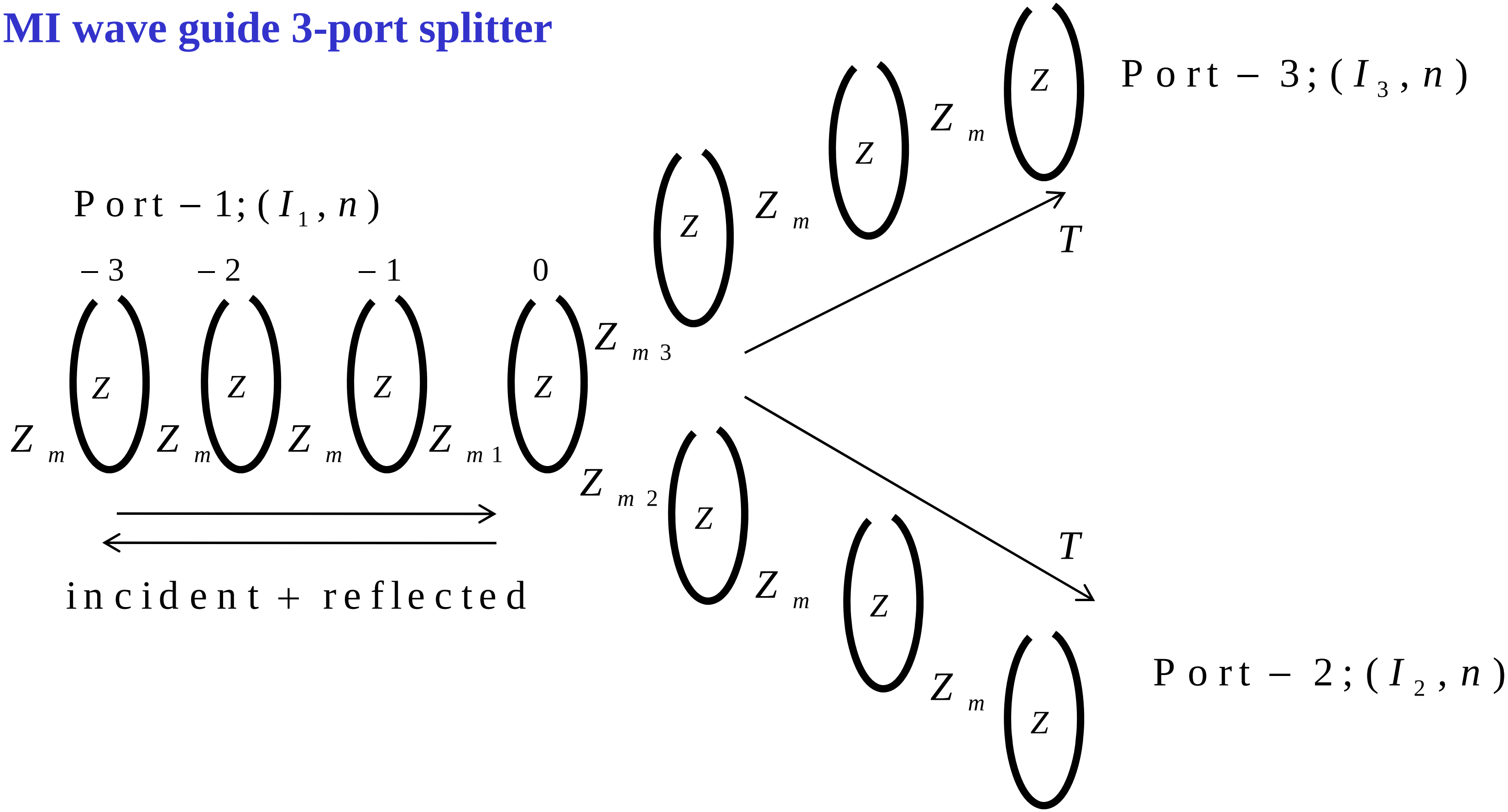
Large Bragg's reflection



Variation of R with ω / ω_0 for similar grating of 20 periods. For 20 periods a close to total reflection occurs within a narrow band centered on the resonant frequency

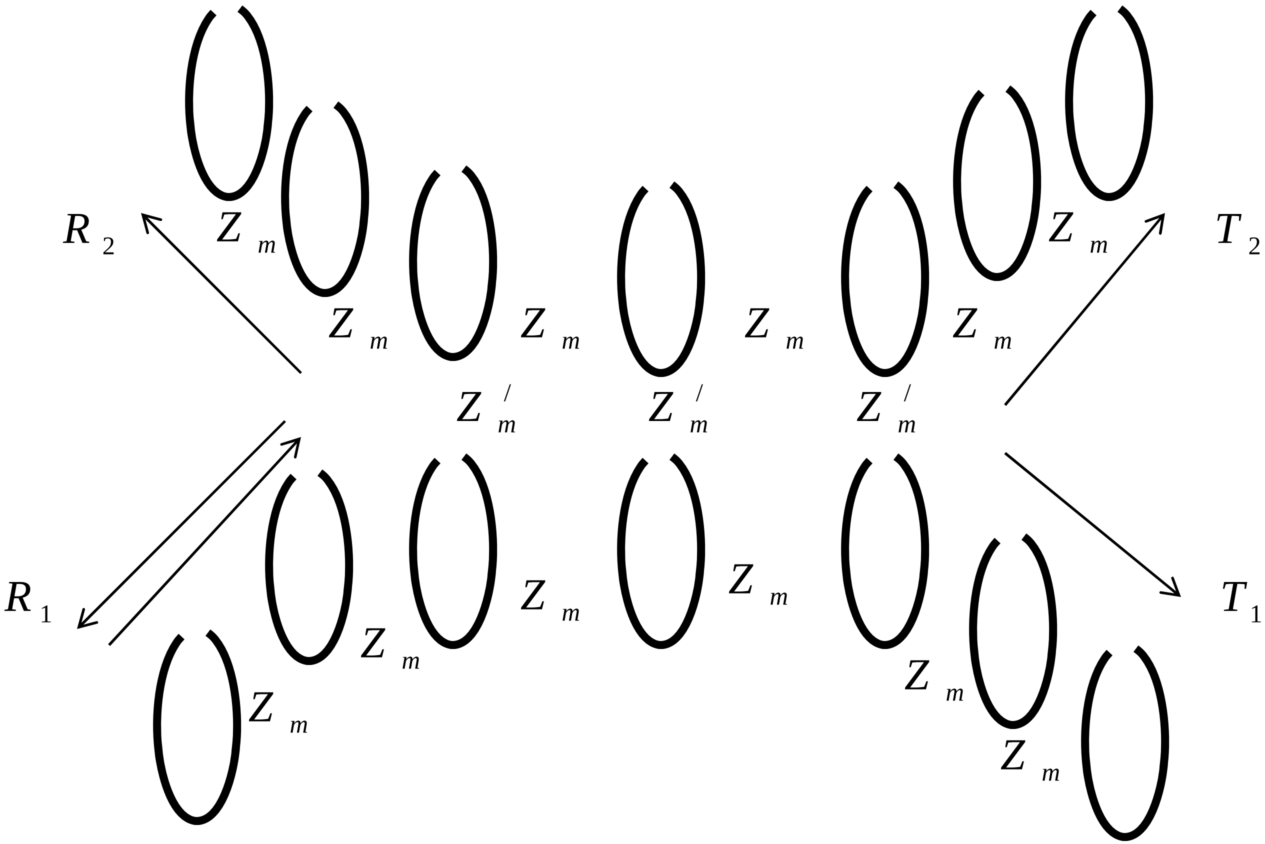
Have we seen a similar thing? Well, it is like bi-periodic line. There we concluded that (assuming infinite line) for bi-periodic line, there is stop-band in the middle of the dispersion curve. Here we are having a finite bi-periodic line, but conclusion is same. Instead of saying stop band, we say there is now a perfect reflection within a certain band due to Bragg's reflection.

MI wave guide 3-port splitter



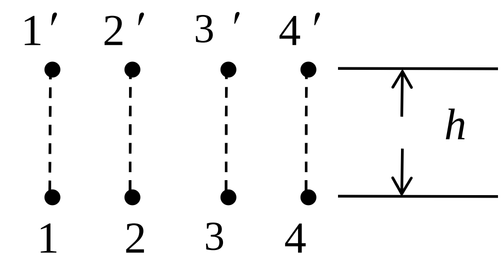
By good choice of M_1, M_2, M_3 any desired power ratio between the 2 outputs can be obtained without *R* in input.

MI wave guide directional coupler

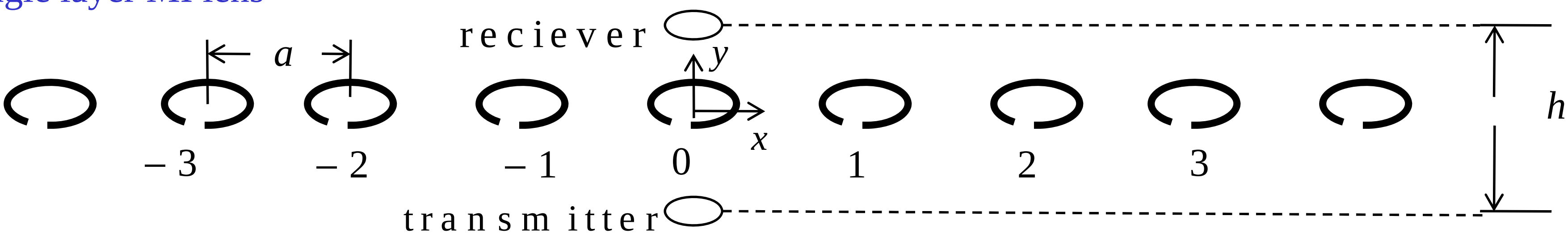


Imaging (MI-lens)

Imaging application is pixel to pixel near field imaging scheme, when all the dimensions are small relative to the wave-length. The object comprising of points 1,2,3,4 is translated to 1’, 2’ 3’ and 4’

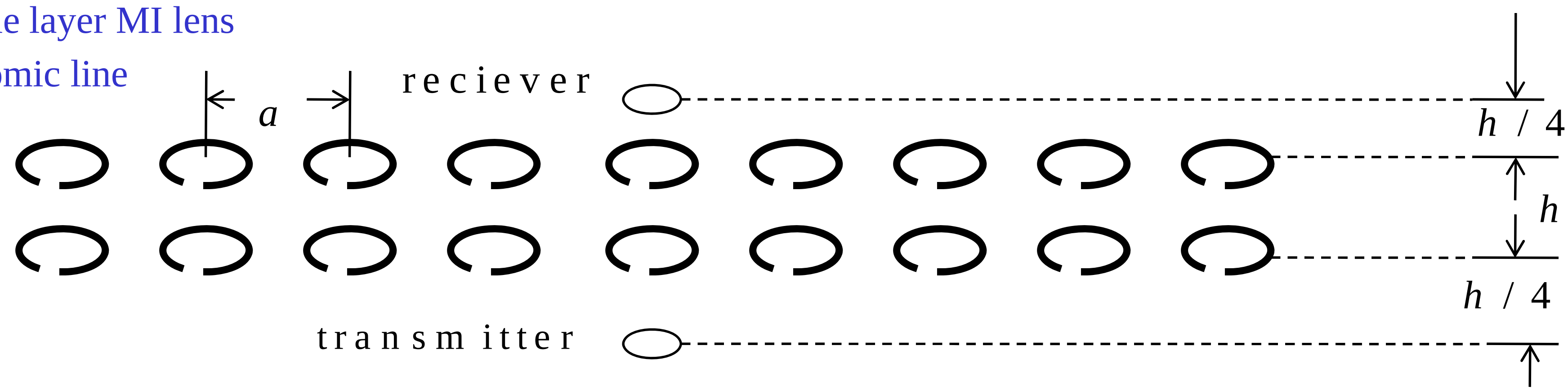


Single layer MI lens



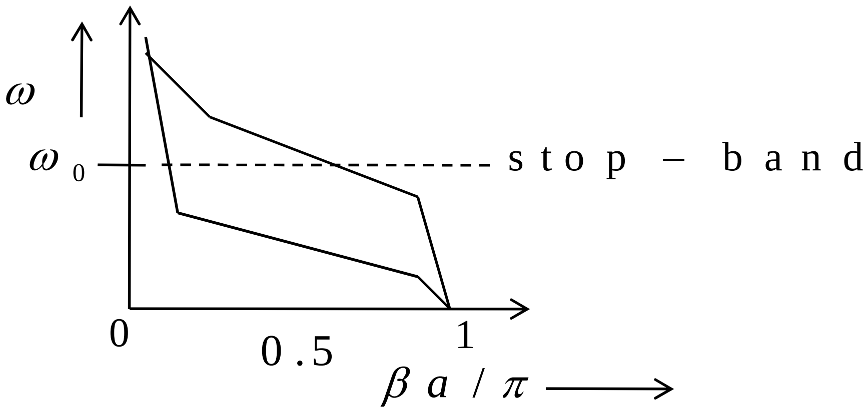
Double layer MI lens

Bi-atomic line



About MI imaging explanation

In order to simplify the problem, we consider a point object and represent it by non resonant transmitter coil at a point $x = 0$ and $y = 0$. The image is 'tested' by moving a small non-resonant receiver coil along line $y = h$. If the received power has a narrow maxima at the vicinity of $x = 0$ and $y = h$, then we regard this as image. However, if we have a small transmitter and receiver coil, then the maxima along the line $y= h$ will be wide, corresponding to field distribution of transmitter coil! How could we make sharpened? Let us insert MI wave guide between transmitter and receiver. We may now claim the field will be higher due to coupling of elements 0 both to transmitter and receiver. This chain need not be correct!! Inserting MI waveguide will not in general make field opposite the transmitter more concentrated because there will be MI propagation in both directions away from the element 0 spreading the power in x direction. The remedy is to have MI wave in y direction but suppress in x direction. This can be done by coupled lines, and elements above each other, then no power is transferred along the coupled lines-provided coupling is high. (Dispersion diagram of bi-atomic structure having stop band around resonance so there will be no power in x-direction)



Detecting rotating dipole by MI wave in Magnetic Resonance MR

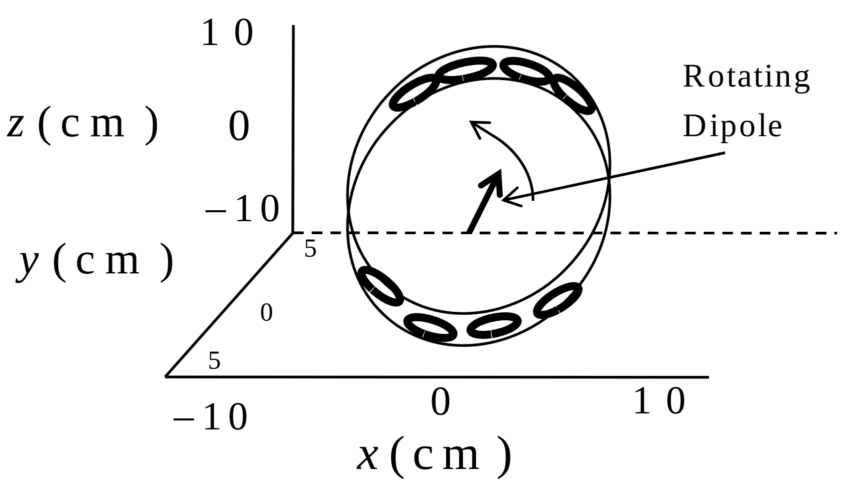
MI waves may propagate along set of metallic nano-particles, loaded electric dipoles and capacitive loaded loops, similar to classical acoustic wave in crystal.

Applications includes Delay Lines, Phase-Shifters and uW lenses.

In MRI the objective is to guide signal and boost its SNR. Classically array of coils have been used for MR signal detection. However it is argued that interaction between elements should be eliminated. For nearest neighbor, this is achieved by overlapping adjacent loops. Other MR detector design do rely on magnetic & electric interactions, and are realized by combination of lumped and distributed elements. Examples are low and high pass bird cages and dome resonators used for imaging human head. It should be noted that in each case physical basis of dipole waves, MI waves and the waves exploited in MRI detectors is interaction between the resonant circuits.

Detection of NMR-interaction with resonant circuits

Idea is of ring resonator in which MI waves travel round the circular path consisting of magnetically coupled, capacitive loaded loops. The frequency range is not significant as long as all the dimensions are small relative to E.M. wavelength. Frequencies of the order of 10 MHz . Here we make maximum use of the mutual inductance to achieve a travelling MI wave resonance, which we shall call 'rotational resonance'. We couple the resonant MI wave to centrally located rotating magnetic field and then the process is similar to magnetron principle where power far excess of that available from isolated element may be delivered from a suitably terminating (modifying) element of the ring.



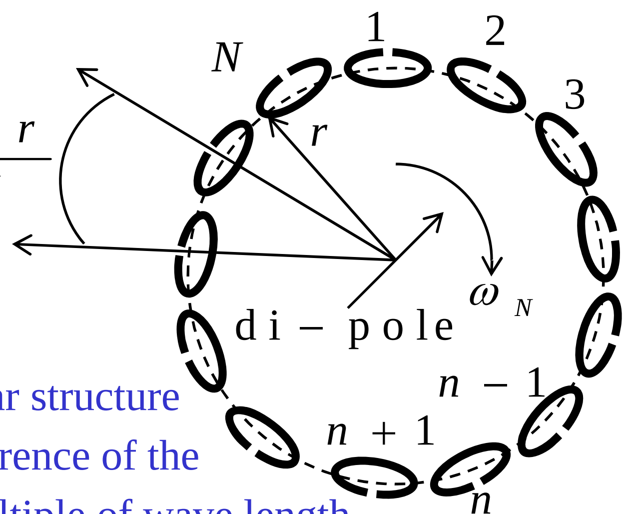
This is travelling of MI waves round the ring structure of resonant elements, call The resulting resonance a rotational resonance

Such structures is also called in uW as 'Strip-Line" Ring Resonator

It operates on principle that total phase shift that MI wave accumulates round the closed path should be $n \times 2\pi$. Alternatively is that the resonance occurs when the circumference of the circle is equal to an $n \times \lambda$ integral multiple of wavelength of MI wave.

Rotational Resonance

$$a = \frac{2\pi r}{N}$$



$$Z_0 I_n + Z_m (I_{n+1} + I_{n-1}) = 0$$
$$Z_0 = R + j(\omega L - 1/\omega C)$$
$$I_n = I_0 e^{-jnka}; \kappa = 2M / L$$
$$\omega_0 = 1/\sqrt{LC}$$
$$\cos ka = -Z_0 / 2Z_m$$
$$k = \beta - j\alpha$$
$$\cos \beta a \cosh \alpha a = \kappa^{-1}[(\omega_0^2 / \omega^2) - 1]$$
$$\sin \beta a \sinh \alpha a = (\kappa Q_0)^{-1}$$
$$\alpha a \cong [Q_0 \kappa \sin \beta a]^{-1}$$

A wave propagating around a circular structure has a ‘resonance’ when the circumference of the circle $2\pi r$ equals an integral multiple of wave length of travelling MI wave.

We couple the resonant MI wave to a centrally located rotating magnetic field and the principle is similar then to magnetron, as the power available will be far in excess than when an isolated element is used, for the detection of rotating magnetic field.

$$2\pi r = s\lambda \qquad s = 1, 2, 3 \ldots$$

Wave length of MI wave is $\lambda = \dfrac{2\pi}{\beta}$ $\quad a = \dfrac{2\pi r}{N}$

Hence $\beta a = \dfrac{2\pi s}{N}$

Hence the frequency of ‘ rotational resonance’ $\cos ka = -\dfrac{Z_0}{2Z_m}$

$$\frac{\omega_r}{\omega_0} = \left[1 + \kappa \cos\left(\frac{2\pi s}{N}\right)\right]^{-1/2}$$

$$r = 85\,\text{mm}; \quad N = 24; \quad \kappa = -0.11; \quad \frac{\omega_r}{\omega_0} = 1.058$$

Coupling a rotating magnetic field

We assume that circular arrangement of resonant loops is excited by nucleus under magnetic resonance. Say a magnetic dipole is rotating with ω_N. If the ring structure is designed and matching $\omega_r = \omega_N$ it will be excited by fundamental resonance. The radial component of this rotating magnetic field will induce a V_n in the n^{th} loop. n^{th} element of the vector $[V]$ will be $V_n = V_0 e^{-j2\pi n/N}$ since a travelling wave is formed, with V_0 is voltage induced by dipole for ring at a distance radius.

$$[V] = [Z][I]$$

$$[Z] = \begin{bmatrix} Z_0 & Z_m & 0 & * & * & 0 & Z_m \\ Z_m & Z_0 & Z_m & * & * & 0 & 0 \\ 0 & Z_m & Z_0 & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * \\ 0 & * & * & * & Z_0 & Z_m & 0 \\ 0 & 0 & 0 & & Z_m & Z_0 & Z_m \\ Z_m & 0 & & & 0 & Z_m & Z_0 \end{bmatrix}$$

Take $n = 2$, then $V_2 = Z_0 I_2 + Z_m (I_1 + I_3) = [Z_0 + 2Z_m \cos ka] I_2$. However ka is now impressed and is real and equal to $2\pi / N$. $\mathrm{Im}[Z_0 + 2Z_m \cos(2\pi/N)] = 0$ Whence we obtain $I_2 = V_2 / R$. This states that current in the element-2 under rotational resonance is same as that obtained for a single uncoupled element excited by the same voltage at its resonant frequency ω_0. Such a simple result can only be obtained at a frequency of rotational resonance. At other frequency current must be found numerically $[I] = [Z]^{-1}[V]$

Frequency variation of the current in rotational resonance

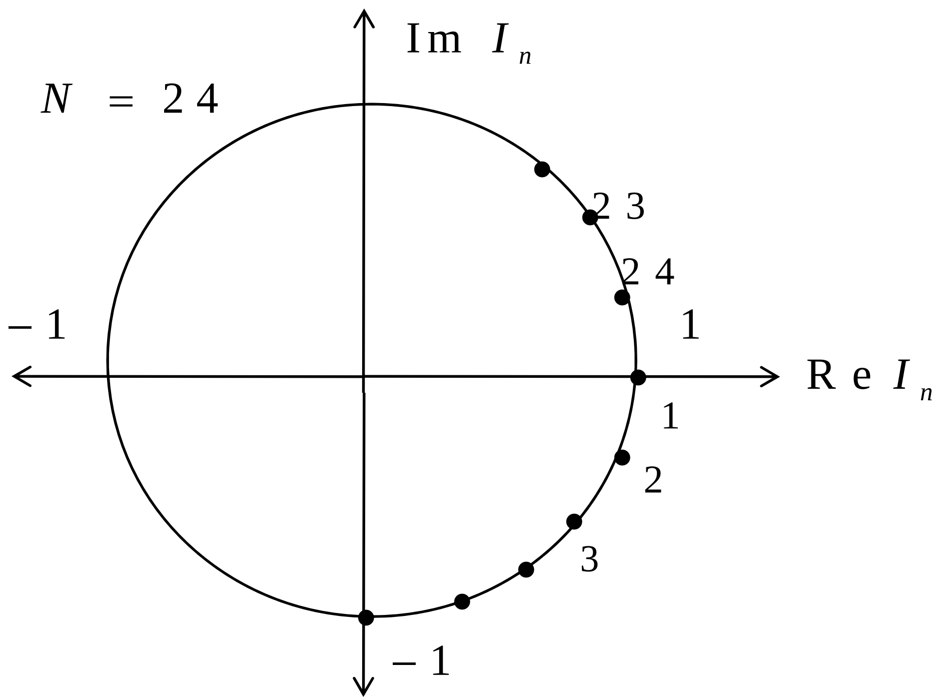
Assume $\omega_N / 2\pi = 63.87\ \mathrm{MHz}$ which is for proton NMR in field of 1.5 T. Also assume $\omega_N = \omega_r$

With SRR loop parameters as $r_0 = 10\ \mathrm{mm}$, $d_w = 2\ \mathrm{mm}$ $L = 33\ \mathrm{nH}$ $C = 187\ \mathrm{pF}$

Gives $\omega_0 / 2\pi = (LC)^{-(1/2)} = 64\ \mathrm{MHz}$ $R = (0.132, 0.0132, 0.00132)$

The values of R is extremely low the second and third cases require cooling and use of superconductor

The current flowing in an element is off course frequency dependent. As frequency departs from ω_r the current is bound to decline



Extraction of power at rotational resonance

So far loops have been excited without extracting any power. Now here we compare the power that may be extracted from an element under rotational resonance with that obtained from a single uncoupled element at the same distance from excitation, the single uncoupled element will have

$$P_{Out,0} = |V_0|^2 Q_0 / 8\omega_0 L; \quad \text{with} \quad |V_0|^2 = (\mu_0 H_r A)^2$$

Where μ_0 is free space permeability, H_r is radial component of the magnetic field at the Centre of the loop due to rotating dipole, and A is area.

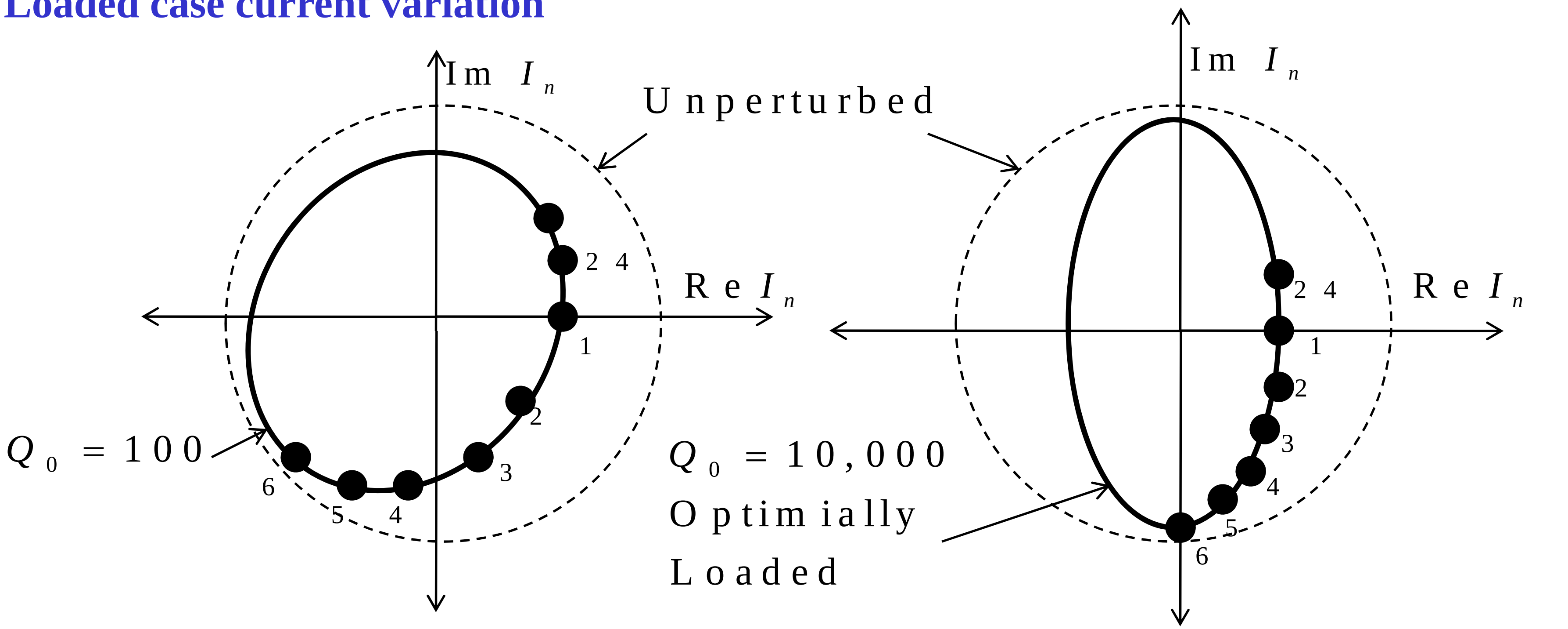
For rotational resonance we choose to extract power from the first element. The maximum power output $P_{Out,m}$, may be obtained by finding optimum impedance $Z_L = R_L + jX_L$ to be inserted into the first loop; therefore the first loop impedance in loaded case is $Z_0 + Z_L$, the other elements remain same at Z_0.

Loading the element for power extraction at rotational resonance

Q_0	$\frac{R_L}{R}$	$\frac{X_L}{R}$	$\xi = \frac{P_{Out,m}}{P_{Out,0}}$
100	4.18	−2.73	6.03
1000	11.5	−0.076	11.8
10,000	12.0	−0.0076	11.98

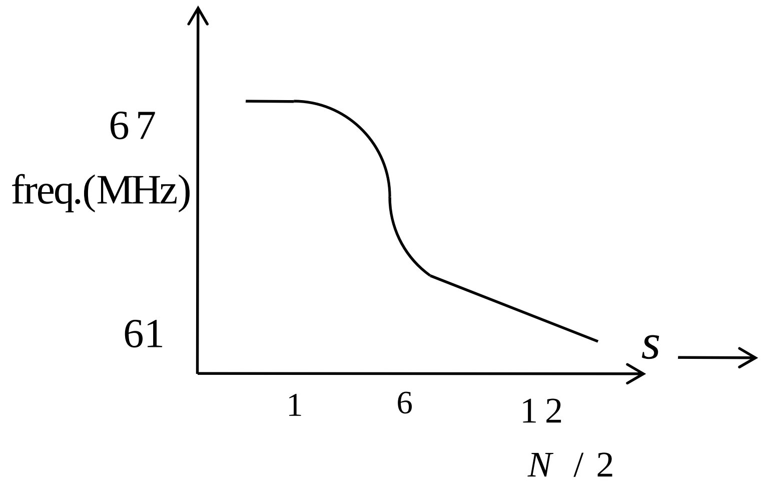
As $Q \to \infty$, the ratio $\xi \to 12 = (N/2)$. The optimum value of $X_L = 0$ and $R_L = R(N/2)$ and optimum power ratio is $N/2$

Loaded case current variation



Observation is the current at rotational resistance obey again same relationship as the currents in simple resonant circuit-the current at the optimum load is just half of that in unloaded case. It may be pointed out that the increase in output power when resonant elements are coupled to each other in a ring structure is analogous to what happens in cavity magnetron; in that case the excitation is provided by electrons passing in front of the coupled resonant cavities, and the joint output is extracted from a single cavity. It is observed that under rotational resonance it is possible to obtain ($N/2$) as much power of a single loaded element as from uncoupled loop. Is there scope for further improvement in SNR? This can be got by inserting further matching impedances in other loops elements at N and at 2.

Higher Order Rotational Resonances

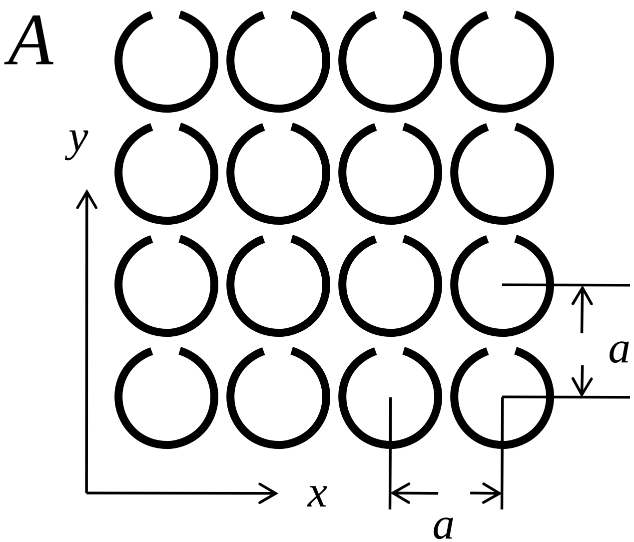


A higher order rotational resonance occurs when, the perimeter of the resonant structure is equal to $s\lambda$. The smaller MI wavelengths means higher value of βa and corresponding lower value of frequency. The highest distinguishable resonance occurs for $s = N/2$ when the amplitude of the wave changes sign from element to element. The variation of the rotational resonance frequency with the order of resonance, as plotted for $N = 24$ is somewhat counterintuitive that is the frequency of resonance should have increases with resonance number! Here the configuration is planar and due to 'backward wave' thus we have resonance frequency falling with increase of resonance number.

MI waves in two dimensions

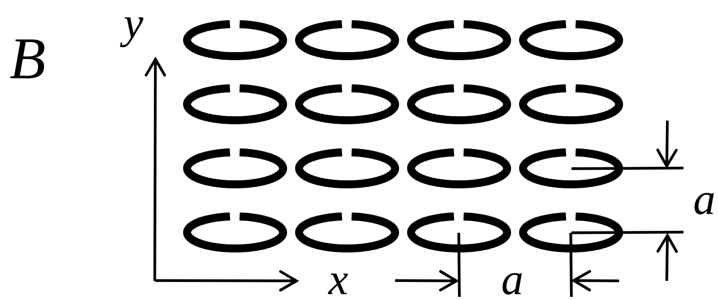
It is rarely necessary to consider interaction between all the particles. For most of the purpose the nearest neighbor interaction is sufficient. A n array of SRR planar or axial may be considered in *x*-*y* plane. KVL for (*n*, *m*) in square lattice is

$$Z_0 I_{n,m} + j\omega M_x (I_{n+1,m} + I_{n-1,m}) + j\omega M_y (I_{n,m+1} + I_{n,m-1}) = 0$$



$M_x = M_y < 0$

Planar – configuration



$M_x < 0; \quad M_y > 0$

Here $M_y << |M_x|$

Planar – Axial – configuration

Assume the periodic current harmonic solution in 2D as

$$I_{n,m} = e^{-j(nk_x a + mk_y a)} \text{ where } \vec{k} = i_x k_x + i_y k_y$$

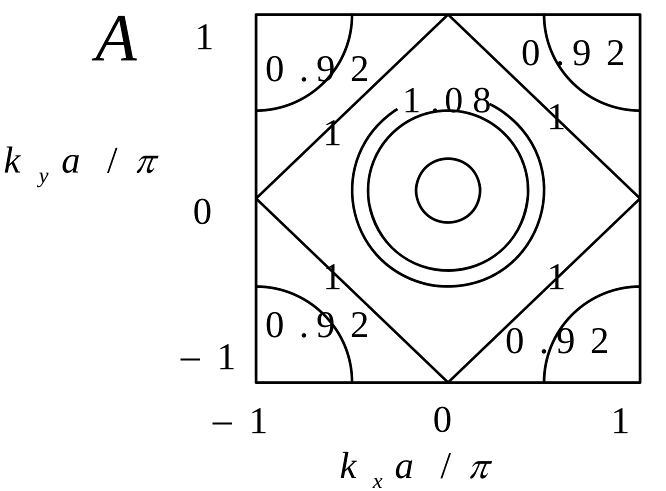
2D-dispersion expression

$$\frac{\omega}{\omega_0} = A^{-1/2}; \quad A = \kappa_x \cos k_x a + \kappa_y \cos k_y a$$

$$\kappa_x = 2M_x / L; \quad \kappa_y = 2M_y / L; \quad \omega_0 = (LC)^{-1/2}$$

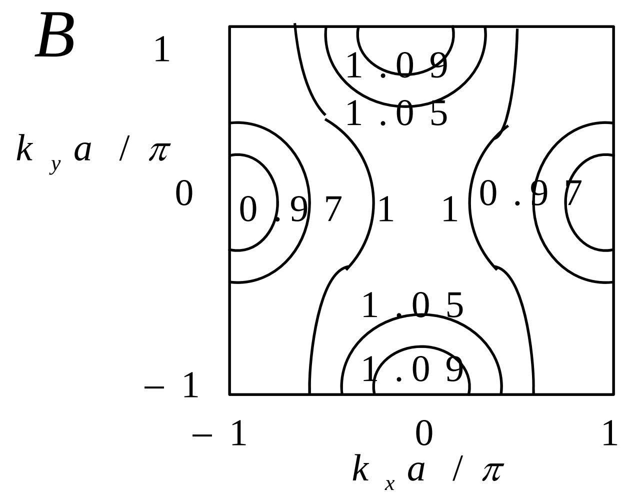
One useful way of representation is plot of $\omega / \omega_0 = \text{constant}$ curves k_x k_y plane.

A



$$\kappa_x = \kappa_y = -0.106$$

B



$$\kappa_x = -0.106; \quad \kappa_y = 0.066$$

Separation of elements is $2.25 r_0$, r_0 is external radius of SRR element. For *A* when $k_x a / \pi$ & $k_y a / \pi << 1$ the iso-frequency curves are circles, where as for *B* for same wave vector range curves are hyperboles

2D case group velocity

$$v_g = (\nabla_k).\omega = i_x \frac{\partial}{\partial k_x}\omega + i_y \frac{\partial}{\partial k_y}\omega$$

$$\omega = \omega_0 A^{-1/2}; \quad A = 1 + \kappa_x \cos k_x a + \kappa_y \cos k_y a \cong 1 + \kappa_x + \kappa_y$$

$$v_g = \frac{a\omega_0}{2} A^{-3/2}[(i_x)\kappa_x \sin(k_x a) + (i_y)\kappa_y \sin(k_y a)]$$

$$\cong \frac{a^2\omega_0}{2\sqrt{(1+\kappa_x+\kappa_y)^3}}[(i_x)\kappa_x k_x + (i_y)\kappa_y k_y]$$

For a planar case with $\kappa_x = \kappa_y = \kappa$ and with arguments of sines small (circles as dispersion diagram), we have

$$v_g \cong \frac{a\omega_0}{2\sqrt{(1+2\kappa)^3}} a\kappa[i_x k_x + i_y k_y], \quad (k_x a) << 1; (k_y a) << 1$$

$$= \frac{a^2\omega_0\kappa \vec{k}}{2\sqrt{(1+2\kappa)^3}}$$

Here the $\kappa < 0$ indicating group velocity is opposite to phase velocity a backward wave

2D-power flow

The direction of power flow is given by group velocity and we may again obtain power density by multiplying the group velocity by stored energy per unit volume (as done in 1D case)

$$S = \frac{1}{2}\vec{v}_g E_s$$

$$E_s = \frac{1}{2a^2}\left[L\left|I_{n,m}\right|^2 + \frac{\left|I_{n,m}\right|^2}{C} + M_x I_{n,m}\left(I^*_{n-1,m} + I^*_{n+1,m}\right) + M_y I_{n,m}\left(I^*_{n,m-1} + I^*_{n,m+1}\right)\right]$$

Assu min g the wave equation for current

$$S = \frac{1}{2}\omega\left|I_0\right|^2\left[(i_x)M_x \sin k_x a + (i_y)M_y \sin k_y a\right]$$

For 1D case derived as:

$$P = W v_g = \frac{\omega_0^2}{\omega^2} L\left|I_n\right|^2 \times \frac{\omega_0 a}{2}\kappa\left(\frac{\omega}{\omega_0}\right)^3 \sin ka$$

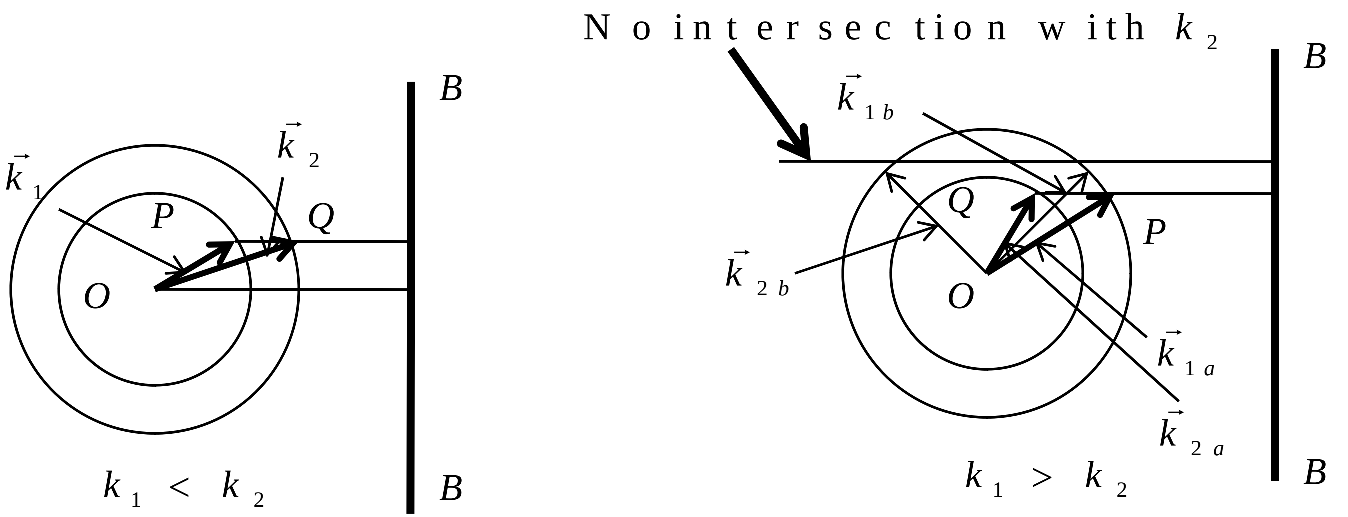
$$= \frac{1}{2}\omega M \left|I_0\right|^2 \sin ka \qquad \text{making} \quad I_n = I_0$$

The power is independent of circuit parameters *L* and *C*, the condition for frequency in pass band off course should be satisfied

Reflection & Refraction at oblique incidence

Revisiting Edward's circle

The loci of the wave-vectors on the two sides of boundary are represented by concentric circles of radius k_1 and k_2 . They show all the possible directions for the wave vectors $\vec{k}_1$ and $\vec{k}_2$

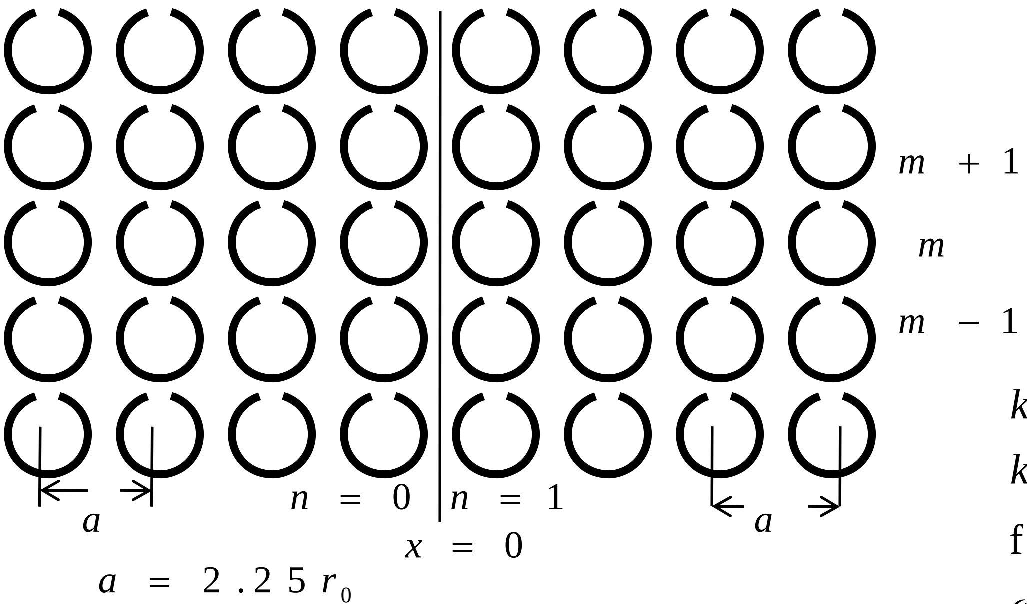


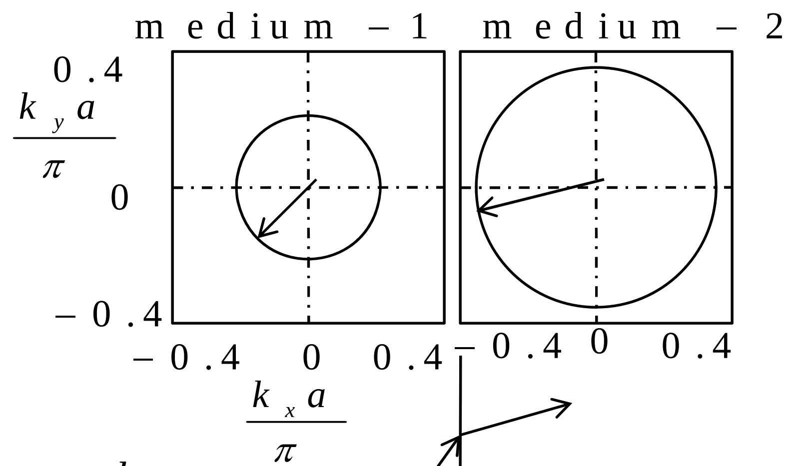
1. Draw a wave vector corresponding to its angle of incident
2. $\vec{k}_1 = OP$
3. Draw perpendicular from *P* to the boundary *BB.*
4. This line intersects the circle of radius k_2 at *Q*. Hence $\vec{k}_2 = OQ$ will be direction of wave in the medium 2, with the direction and angle of refraction or reflection.

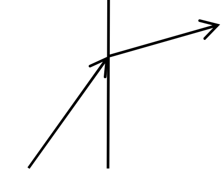
In the case $k_1 > k_2$, the incident wave vector k_{1b} does not intersect the k_2 and this is total internal reflection case, thus manifests as $\vec{k}_{2b}$ in the same media.

Refraction & Transmission for oblique incidence in 2D planar case-positive refraction

Media-2 has slightly different capacitance than media-1; say $\omega_{02} = 1.03\omega_{01}$. Assuming that $k_x a$ & $k_y a$ are very very smaller than unity close to zero, we have 'circles' in 2D dispersion diagram., choosing the constant frequency curve as say $\omega / \omega_0 = 1.11$







$k_{x1} < k_{x2}$

$k_{y1} < k_{y2}$

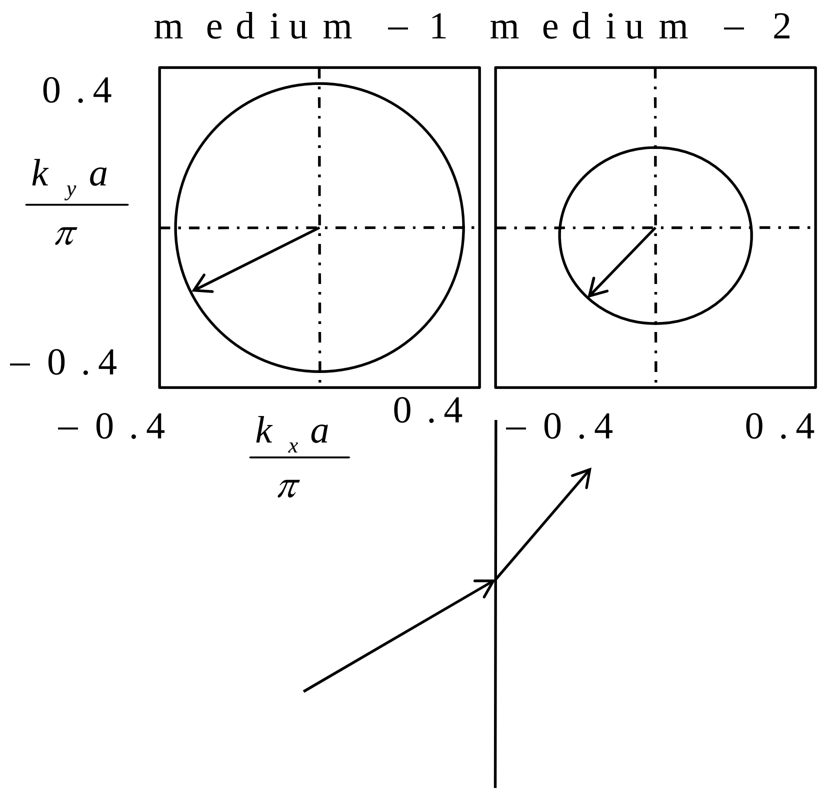
for $\omega / \omega_0 = 1.11$

$\omega_{02} = 1.03\omega_{01}$

Normally we have positive refraction due to forward waves in both the media. In this case both the media support backward wave. It means that if we want a incident wave at positive angle (i.e. group velocity is in first quadrant, we have to choose k_x and k_y in the third quadrant!

At the boundary $v_{px1} = v_{px_2}, \quad v_{py1} = v_{py2}$

Positive refraction with backward waves in both media



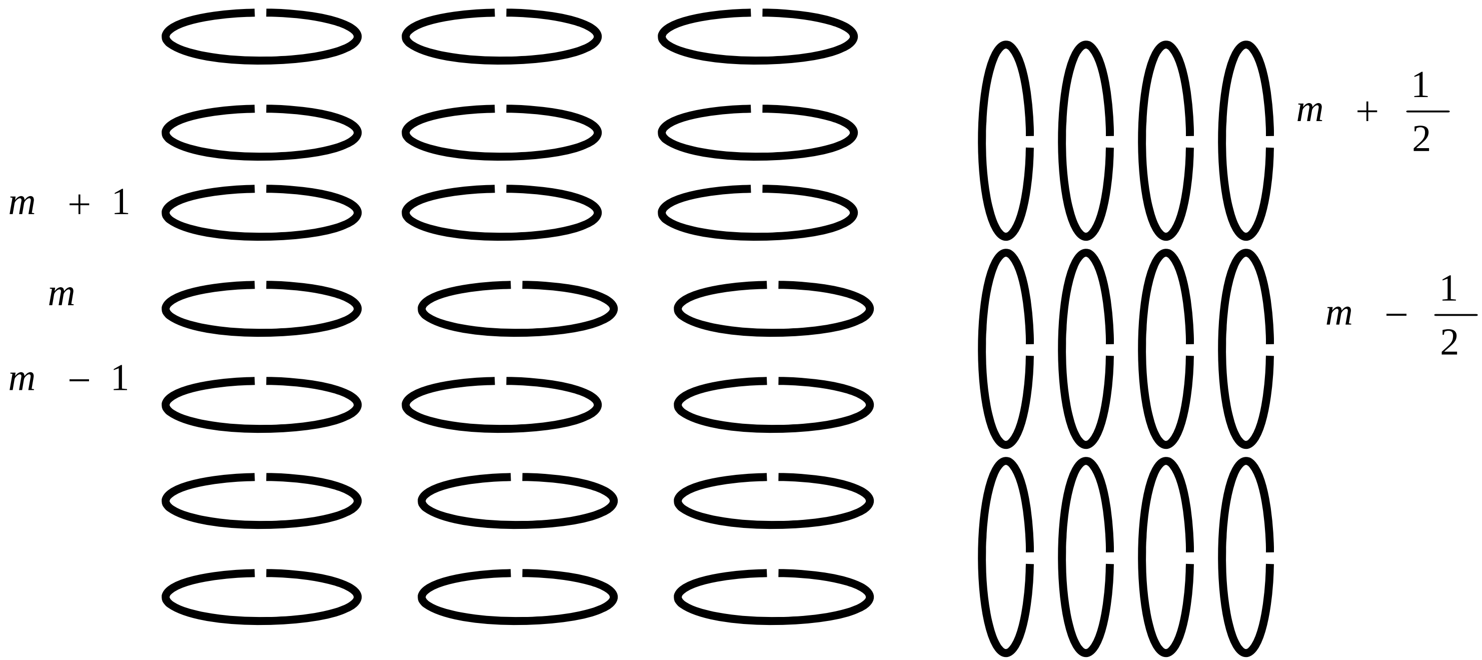
$k_{x1} > k_{x2}$

$k_{y1} > k_{y2}$

for $\omega / \omega_0 = 1.11$

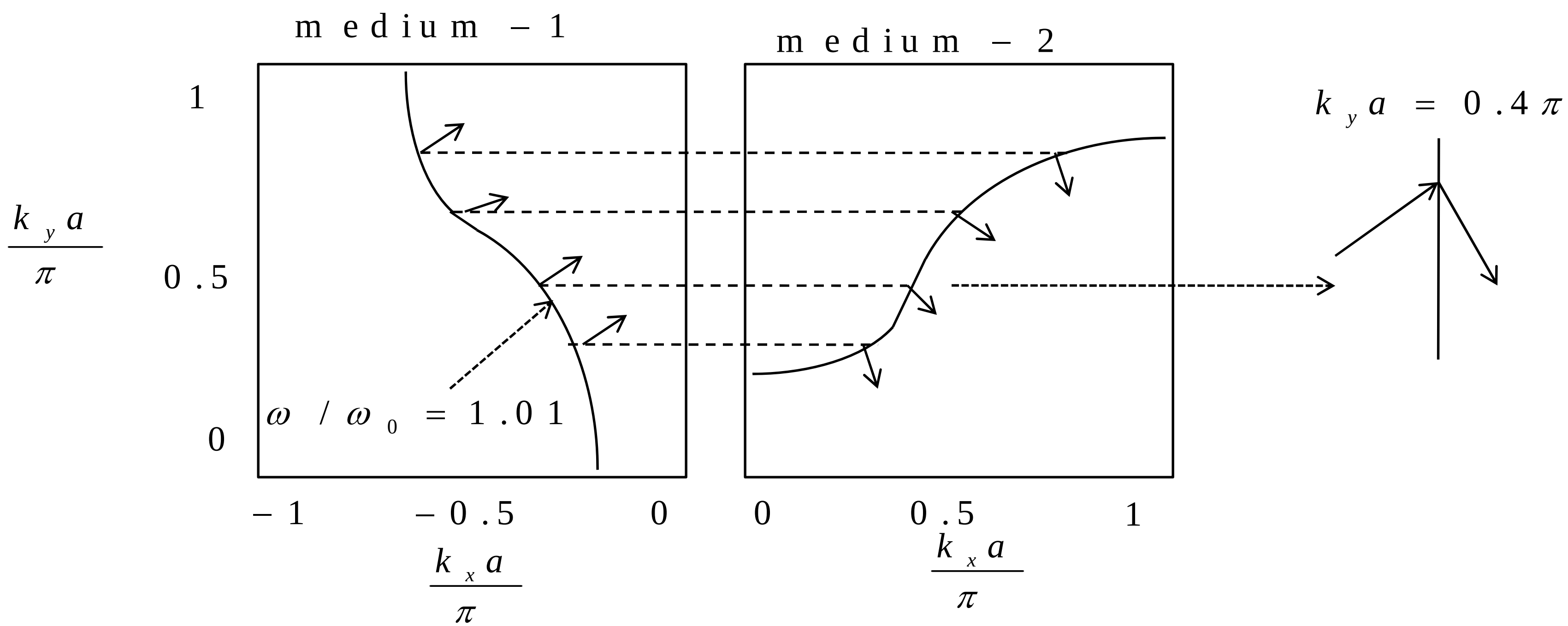
$\omega_{02} = 0.97\,\omega_{01}$

Planar-axial configuration-negative refraction case



We wish to have incident wave with group velocity in first quadrant. Note that group velocity is gradient vector in the ω / ω_0 constant curves of 2D dispersion diagram. Hence 2nd quadrant of 2D dispersion is fit . The medium-2 is rotated 90 degrees, so the second quadrant of the dispersion curve for medium-1 will have to be rotated 90 degrees have dispersion diagram of medium-2.

Dispersion diagram & negative refraction but with forward wave in planar axial configuration



The arrows show the gradient of ω w.r.t. $\vec{k}$, showing the direction of group velocity, for a particular ω / ω_0 curve in the 2nd quadrant. The same curve is rotated 90 degrees for medium-2 It may be seen that there is only a very limited range of incident wave vectors for which negative refraction exists. Say for $k_y a = 0.4\pi$ the refraction is negative, but in both the media the forward wave exists.

Discussing refraction with planer-axial configuration

We could also change the angle of negative refraction in medium-2 by leaving the meia-1 unchanged and ‘rotating’ the orientation in media-2 by less than 90 degree.

In addition we could considerably influence the dispersion curves of MI waves by changing the resonant Frequency of elements and coupling coefficients. In fact leading to four different combinations

1. Both forward waves
2. Both backward waves.
3. Incident wave forward & refracted wave backward
4. Incident wave backward & refracted wave forward

Comments on negative refraction in periodic medium

Most of the effort to find negative refraction have been aimed at EM waves incident from free space upon a periodic medium (WA and SRR Part-1 to 4).

In this MI wave case both the medium are periodic medium.

MI waves quite obviously can exist only in periodic media.

Here in MI waves we have large amount of freedom in choosing the dispersion character in both the media, thereby allowing large variety of refraction angles, positive or negatives to be realized.

Here is a very new topic for EM Research in parlance with MI waves

End of Part-8

We have to go several miles in this LHM research