Left Handed Maxwell Systems

PART-6

Physics of Resonance & Scaling up-the Frequency

SAMEER

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Plasmonics- a new field

$$k.k = \varepsilon \mu \omega^2 / c^2$$

Making real part of permittivity or real part of permeability < 0 makes Electromagnetic radiation evanescent, decay inside them, but also renders the material a rather special status.

For example we know that metals dielectric dispersion or permittivity is dominated by plasma like response, of free electron gas have ENG behavior at UV-optical frequency.

These materials can support host of resonant states localized at their surface (SPP-Surface Plasmon) These surface plasmons can resonantly interact with radiation on structured surfaces, giving rise to a wide variety of novel phenomena, and have given rise to field of Plasmonics.

This generic effect of having ENG and materials with MNG can also be expected to support the analogous ‘surface magnetic plasmons’.

Metals at Optical Frequency

At optical frequency many metals are ENG, when conduction electrons in metals can be assumed to be reasonably free in background of static positive (core) ions, the overall system being charge neutral.

$$m(dv/dt) + m(v/\tau_{col}) = m\ddot{r} + m\gamma\dot{r} = mr(-\omega^2 + j\omega\gamma) = -eE\exp(-j\omega_0 t)$$

The $m\gamma$ is damping viscous force due to inelastic collision. Assume wavelength of light is substantially larger than path length of travel of electron, so that effectively it sees a spatially constant field and the velocities are sufficiently low, so that magnetic field is neglected . This yield 'polarization' per unit volume in the medium as $P = \varepsilon\varepsilon_0 E - \varepsilon_0 E = (\varepsilon - 1)\varepsilon_0 E = -Ner = -(Ne^2E/m)/(\omega^2 + j\omega\gamma)$

From this we get $\varepsilon = 1 - [\omega_p^2/(\omega^2 - j\omega\gamma)]$ $\qquad \omega_p^2 = Ne^2/m\varepsilon_0$

Where N is number density of conduction electrons and each electron contributes to polarization. The $\varepsilon < 0$ up to ω_p , shields the interior of metals from radiation. Above ω_p it is just like positive di-electric material.

The dispersion equation substituting the $\varepsilon(\omega)$ into Maxwell equation we get $k^2c^2 + \omega_p^2 = \omega^2$

Thus the waves $\varepsilon < 0$ corresponds to 'negative-energy' solution are bounded to surface evanescent

Plasmons at Optical Frequency

As discussed in previous parts, rendering ENG makes possible for the surface to support surface resonances, the SPP

$$E = E_0 \exp\left[j(k_x x + k_y y - \omega t) - \kappa_{z+} z \right] \quad \text{for} \quad z > 0$$

$$E = E_0 \exp\left[j(k_x x + k_y y - \omega t) + \kappa_{z-} z \right] \quad \text{for} \quad z < 0$$

$$\text{when} \quad k_x^2 + k_y^2 - \kappa_{z\pm}^2 = \varepsilon_\pm \mu \omega^2 / c^2 = n_\pm^2 \omega^2 / c^2$$

are good solution of Maxwell Equation when $\frac{\kappa_{z+}}{\varepsilon_+} + \frac{\kappa_{z-}}{\varepsilon_-} = 0$

These are 'collective' excitations of electrons with charges displaced parallel to the (real part of the) wave-vector. Thus we have a 'mode' longitudinal component when field decay exponentially into the metal and dielectric. This charge density wave is called SPP (surface wave).

$$k_x = \frac{\omega}{c}\left[\frac{\varepsilon_+ \varepsilon_-}{\varepsilon_+ + \varepsilon_-} \right]^{1/2}$$

For many metals plasma frequency ω_p is UV and damping γ is small compared to ω_p. Thus we have many examples of ENG at optical frequency. However, dissipation in metal is large and we have trouble when we try to extend this behavior to lower frequency when $\omega \approx \gamma$. Then it can be hardly be claimed that electric permittivity is real and negative. The dissipation dominates all phenomena and we do not have good 'plasma'.

Resonating dielectric permittivity-Phonon

It must be pointed out that ENG can be obtained in more ordinary dielectric media with bound charges with in frequency band above resonance frequency ω_0 . Consider a medium in which electrons are bound to positive nuclei and its response to EM radiation. For small displacements of electrons there is restoring force on electron as $-m\omega_0^2 r$.

$$m\ddot{r} + m\gamma\dot{r} - m\omega_0^2 r = -eE\exp(-j\omega t)$$

From here we can have

$$\varepsilon = 1 + \frac{Ne^2/\varepsilon_0 m}{\omega_0^2 - \omega^2 - j\gamma\omega}$$

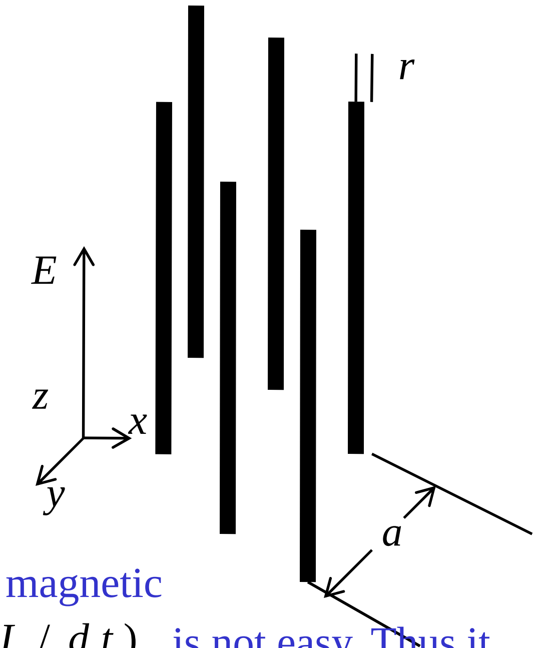
Where *N* is total density of bound electrons. Setting $\omega_0 = 0$ we get the Drude model of dielectric permittivity dispersion obtain for plasmon cases in metals with free electrons. This is longitudinal phonon wave and can get ENG in a small frequency range above resonating frequency .

In Drude model of ours the resonance is at zero frequency!!

Therefore ENG formation is too a resonance case as for MNG!!

Physics of wire mesh (percolated in dielectric background)

$$r << a$$
$$\lambda >> a >> r$$



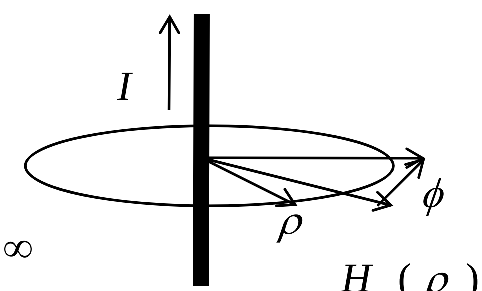
The electrons are confined to move with in wires only, which has effect of 'reducing' the collective free electrons number density, as radiation cannot sense the individual wires structure but only the 'average' charge density. Thus average charge density is

$$N_{eff} = (\pi r^2 / a^2) N$$

The second important effect to be considered is, the thin wires have large magnetic inductance, and thus to have change in current flowing through wires (dI/dt) is not easy. Thus it appears that charge carriers have 'increased mass'.

Consider magnetic field at a distance ρ from a wire. On average we can assume a uniform $D = \varepsilon \varepsilon_0 E$ field with in unit cell. But current density is not uniform leading to non-zero 'non-uniform' magnetic field that is large close to the wire which contributes to most of flux. By symmetry there is point of zero magnetic field in between the wires and can estimate magnetic field along line between two wires.

$$H(\rho) = \frac{\hat{\phi} I}{2\pi}\left(\frac{1}{\rho} - \frac{1}{a-\rho}\right)$$



Note as $\rho \to 0$, the Magnetic field intensifies $H(\rho) \to \infty$

For 'thin' wires this high intense magnetic field (perhaps) helps the wire system to behave as low frequency plasma

The vector potential of this wire system

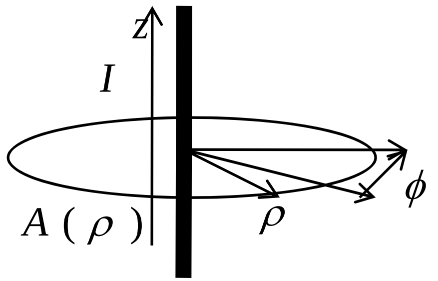
The vector potential associated with the field of a single infinite long current carrying wires is non-unique unless B.C are specified at definite points. In this case we have periodic medium that sets a critical length $(a/2)$. We assume the vector potential associated with single wire is:

$$A(\rho) = \begin{cases} \dfrac{\hat{z} I}{2\pi} \ln\left[\dfrac{a^2}{4\rho(a-\rho)}\right] & ; \rho < \dfrac{a}{2} \\ 0 & ; \rho > \dfrac{a}{2} \end{cases}$$

This choice avoids vector potential of one wire overlapping with other, thus mutual inductance between two adjacent wire is addressed to some extent. Noting that $r << a$ in our case and the current is $I = Nev\pi r^2$ we can write vector potential as

$$A(\rho) \cong \frac{\mu_0 \pi r^2 N e v}{2\pi} \ln\left(\frac{a}{\rho}\right) \hat{z}$$

This is very good approximation in mean field limit. We have considered only two wires, but actually the system has four fold symmetry, however actual one is close to the above approximation.



The momentum of electrons in these wires with vector potential and its effective mass

We note that canonical momentum of an electron in an EM field is $p + eA$. Thus assuming that electrons flow on surface of wires (assuming PEC), we can associate momentum per unit length of wire as:

$$p = \pi r^2 N e A(r) = \frac{\mu_0 \pi^2 r^4 N^2 e^2 v}{2\pi} \ln\left(\frac{a}{r}\right)$$

$$= m_{eff} \pi r^2 N v$$

Therefore effective mass of electron is $m_{eff} = \frac{\mu_0 \pi r^2 N e^2}{2\pi} \ln\left(\frac{a}{r}\right)$

Thus assuming longitudinal plasmonic mode for the system, we have

$$\omega_p^2 = \frac{N_{eff} e^2}{\varepsilon_0 m_{eff}} = \frac{2\pi c^2}{a^2 \ln(a/r)} \quad \text{where} \quad c^2 = (\varepsilon_0 \mu_0)^{-1}$$

For $r = 1\mu\text{m}$ $\quad a = 10\,\text{mm}$ $\quad N = 10^{29}\,\text{m}^{-3}$ $\quad m_{eff} = 2.67 \times 10^{-26}\,\text{kg}$

15 folds increase from that of a proton mass!! This Aluminum conductor gives plasma frequency as 2 GHz well below plasma frequency of pure metal which is near UV.

Well the above plasma frequency formulation is independent of N and v, suggest problem can therefore be casted as in terms of L, C and R !!

Recasting the problem with circuit component

Consider current induced E field along the wires (per unit length)

$$E_z = +j\omega LI = j\omega L\pi r^2 Nev$$

Noting the polarization per volume in homogeneous medium is (note velocity and position are related as by $j\omega$ instead of d/dt)

$$P = -N_{eff}er = N_{eff}e(v/j\omega) = -E_z/\omega^2 a^2 L$$

This is got by substituting (ev) obtained from first expression and putting $N_{eff} = N\pi r^2/a^2$

Calculating magnetic flux per unit length passing through a plane between the wire and point of symmetry between itself and the next wire where field is zero

$$\phi = \mu_0 \int_r^{a/2} H(\rho)\,d\rho = \frac{\mu_0 I}{2\pi} \ln\left[\frac{a^2}{4r(a-r)}\right]$$

Noting that $\phi = LI$ and polarization $P = (\varepsilon - 1)\varepsilon_0 E_z$ where ε is effective permeability we get in limit $r << a$ we get

$$\varepsilon(\omega) = 1 - \frac{2\pi c^2}{\omega^2 a^2 \ln(a/r)} = 1 - \frac{\omega_p^2}{\omega^2}$$

Which is identical to that obtained in plasmonic. We add finite conductivity then,

$$E_z = j\omega LI + \sigma\pi r^2 I$$

From this we obtain:

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega[\omega + j(\varepsilon_0 a^2 \omega_p^2/\pi r^2 \sigma)]}$$

Interpretation

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega[\omega + j(\varepsilon_0 a^2 \omega_p^2 / \pi r^2 \sigma)]}$$

Aluminum has $\sigma = 3.65 \times 10^7 \Omega^{-1} \mathrm{m}^{-1}$, we get damping as $\gamma = 0.1\omega_p$ which is comparable in real metals. Thus this low frequency plasmon is sufficiently stable against absorption to be observable, at 2 G Hz.

The photonic response of superconducting wires of high *Tc* show that system has low frequency cut-off that is much smaller than the bulk super conductor.

The role of plasmon in superconductor as a massive Higg's Boson is today's query in scientific world.

The Basic Concept of Macroscopic Material Response vis-à-vis Microscopic Response

The dielectric constant and magnetic permeability characterizes 'macroscopic' response of a homo- -geneous medium, to the applied E-M fields. These are macroscopic parameters because one usually seeks time-averaged and spatially-averaged responses averaged over sufficiently long times and sufficiently large volumes. All that survives the averaging in macroscopic measurement are the frequency components of the individual (atomic or molecular) oscillators driven by external field.

This idea can now be extended to a higher class of in-homogeneous materials where in-homogeneity are on the length-scales much smaller than the wavelength of EM/Optical radiation, but can be large compared with atomic or molecular length scales.

The radiation then does not resolve these individual meso-structures, but responds to the (atomically) macroscopic resonances of the structure. These are meta-materials, and can be characterized by macroscopic parameters permittivity and permeability, that define their responses to the EM/Optical field much like homogeneous medium.

LHM vis-à-vis Photonic Crystal & Photonic Band Gap

Meta-materials, in some sense, can be strictly be distinguished from other structured photonic material (Photonic Crystal or Photonic Band Gaps).

In photonic crystal or the band-gap materials the stop-band arise as a result of multiple Bragg’s scattering in a periodic array of dielectric scatterers.

In fact the periodicity of the structure here is of the order of wavelengths, and hence homogenization in the sense cannot be carried out.

In meta-materials the periodicity is by comparison far less important and all the properties mainly depend on single scatter resonances.

Causal materials and LHM

All the causal materials are dispersive i.e. the permittivity and permeability are in general, complex function of frequency. This is because the polarization is such media do not respond instantaneously to the applied fields but depends on the history of the applied field! This particularly becomes visible when applied fields have frequencies close to the resonant frequencies of the material oscillator. In fact the real and imaginary parts of permittivity and permeability are related to each other. Noting that the imaginary part of permittivity and imaginary part of permeability relate directly to absorption (losses) of EM radiation in material, we realize that the dispersion and dissipation in thermodynamic medium always accompany each other. As with any resonance, the response follows the applied field below the resonance frequency, and above the resonance frequency the response in anti-phased, with respect to the applied field. If now the resonance can be made sharp enough it will be possible to drive the real parts of effective permittivity and effective permeability negative. This under-damped, over-screened response is responsible for negative material parameter. There is no fundamental objection to the real part of permittivity and permeability going negative. In the media at thermodynamic equilibrium there is however a restriction that imaginary part of permittivity and permeability should not be negative. This is due to the absorbed total energy in volume of medium

$$\int_V d^3r \int_{-\infty}^{+\infty} \omega \left[\mathrm{Im}(\varepsilon(\omega)) \right] \left| \vec{E}(r,\omega) \right|^2 + \mathrm{Im}(\mu(\omega)) \left| \vec{H}(r,\omega) \right|^2 \frac{d\omega}{2\pi}$$

is positive definite. Thus media with negative permittivity and negative permeability are causal, but necessarily dissipative and dispersive.

We will further give some extensions on this issue later

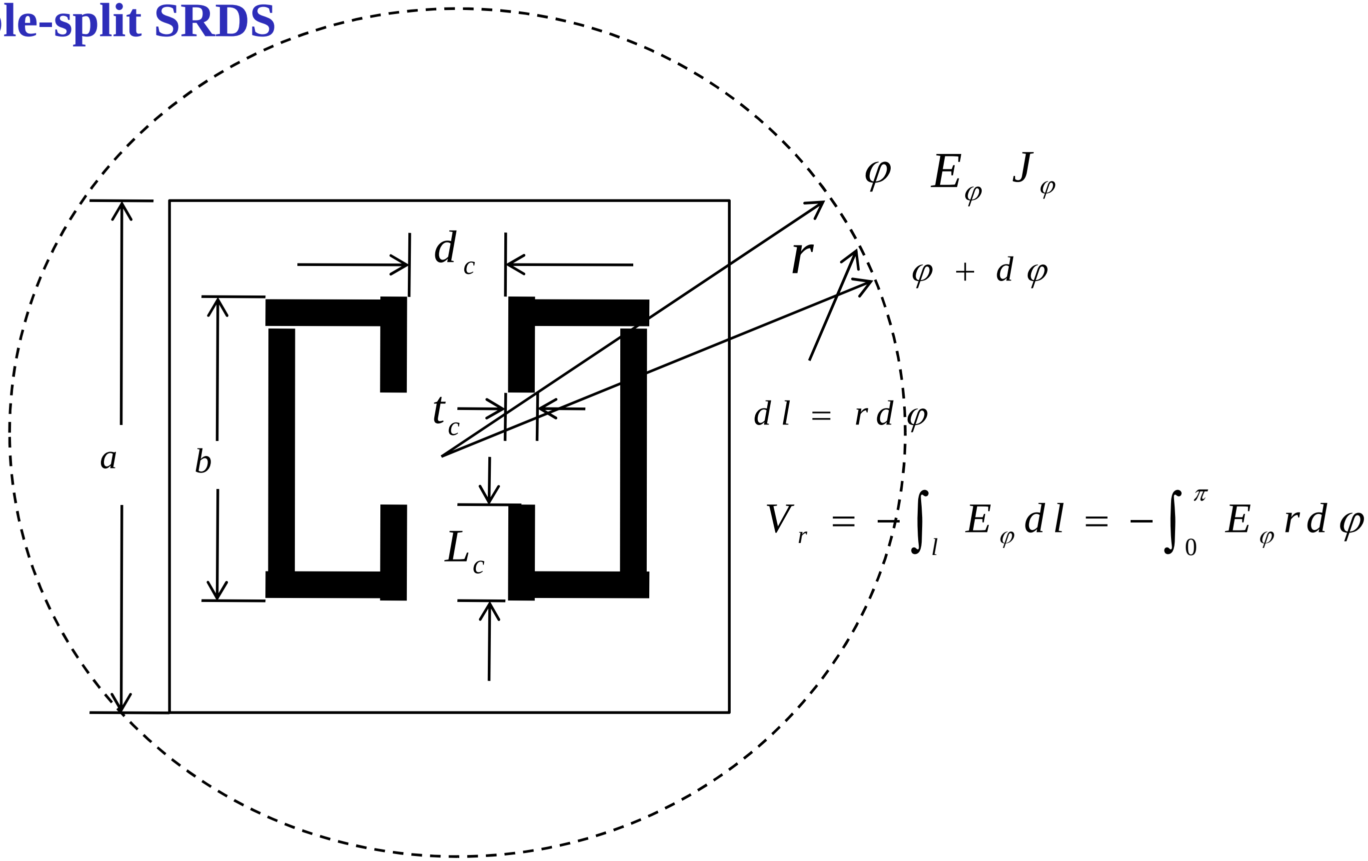
Scaling up to THz?

Although most materials exhibit good electric response, can be found at almost all the frequencies from RF to UV magnetic response of most materials is limited to low microwave (GHz) level. Magnetic polarization usually results from either unpaired electron spin or orbital electron currents, and collective excitations of these usually tend to occur at low microwave frequencies. Some materials exhibit some magnetic activity at even 100GHz, but are rare and BW is too narrow. But now possibility of artificially structuring materials at micro and nano scales enable us to generate a variety of meta-materials with magnetic activity at almost up to IR including RF the microwave frequency and THz frequency in between. Thus even magnetic activity let alone negative permeability is special at High Frequency!

Maxwell's equations appears to suggest that one can scale the phenomena by simple scaling of length--scales. However, main problem to scale to IR and optical frequency is that metals no longer behave as PEC, and EM fields penetrate considerably into metals. This means that dissipative nature of the metals must be taken into account for scaling to HF. Also technological ability on nm scales is to be overcome.

Keeping this in mind a single ring with two symmetric splits providing capacitive gaps and is suitable for THz operation, is thought off.

Single Ring with double-split SRDS



$a = 600\,nm$

$b = 312\,nm$

$L_c = 144\,nm$

$d_c = 24\,nm$

$t_c = 24\,nm$

Let us analyze HF scaling assuming strong skin depth i.e. thickness of metallic shell t_c is comparable to skin depth (20 nm at 100 THz.)

Analysis at HF for single ring with double split (SRDS)

$$J_\varphi = -\frac{d}{dt}\varepsilon_0\varepsilon_m E_\varphi = -j\omega\,\varepsilon_0\varepsilon_m E_\varphi$$

$$E_\varphi = -\frac{J_\varphi}{j\omega\,\varepsilon_0\varepsilon_m}$$

This current J_φ flows thus potential drop across each half ring is

$$V_r = \int_0^\pi E_\varphi r\,dr = -\frac{\pi r J_\varphi}{j\omega\varepsilon_0\varepsilon_m}$$

Potential drop across each capacitor gap (arm length, width and gap as L_c, t_c, d_c)

$$V_c = -\frac{1}{C}\int_0^t I\,dt$$

$$I = \int J_\varphi dA = J_\varphi (t_c)^2 \qquad C = \varepsilon_0\varepsilon_d \frac{A}{d} = \frac{\varepsilon_0\varepsilon_d L_c t_c}{d_c}$$

$$V_c = -\frac{J_\varphi t_c d_c}{j\omega\,\varepsilon_o\varepsilon_d L_c}$$

Ratio of *H* field inside and outside SRDS

Using the fact that total emf around the loop is $\oint E.dl = j\omega\mu_0 H_{int} ds$, we equate the potentials

$$2V_r + 2V_c = j\omega\mu_0\pi r^2 H_{int}$$

Using Ampere's law the magnetic fields inside and outside the cylinders can be related as:

$$H_{int} - H_{ext} = J_\varphi t_c \qquad \mathrm{A/m}$$

Yielding

$$\frac{H_{ext}}{H_{int}} = 1 - \frac{\mu_0\varepsilon_0\omega^2\pi r^2 t_c}{[(2\pi r/\varepsilon_m) + (2t_c d_c/\varepsilon_d L_c)]}$$

Averaging procedure SRDS

Averaging over line lying entirely outside the cylinders for magnetic field $H_{\text{eff}} = H_{\text{ext}}$

The average magnetic induction is $B_{\text{eff}} = (1 - F)\mu_0 H_{\text{ext}} + F\mu_0 H_{\text{int}}$ $\quad F = \pi r^2 / a^2$

Effective permeability is then:

$$\mu_{\text{eff}} = \frac{B_{\text{eff}}}{\mu_0 H_{\text{eff}}} = 1 + \frac{F\varepsilon_0\mu_0\omega^2\pi r^2}{[(2\pi r/\varepsilon_m t_c) + (2d_c/\varepsilon_d L_c)] - \varepsilon_0\mu_0\omega^2\pi r^2 t_c}$$

Assuming $\varepsilon_m \cong -\dfrac{\omega_p^2}{[\omega(\omega + j\gamma)]}$; for IR and optical region, we get same 'genetic' form as for SRR

$$\mu_{eff}(\omega) = 1 + \frac{F'\omega^2}{\omega_0^2 - \omega^2 - j\Gamma\omega}$$

Resonant frequency $\omega_0 = \dfrac{1}{\sqrt{(L_i + L_g)C}}$

Effective damping $\Gamma = \dfrac{L_i}{L_i + L_g}\gamma$ Effective filling $F' = \dfrac{L_g}{L_g + L_i}F$

Gap capacitance per unit thickness $C = \dfrac{\varepsilon_0\varepsilon_d L_c}{2d_c}$ Geometric inductance $L_g = \mu_0\pi r^2$

Additional Inductance $L_i = 2\pi r/(\varepsilon_0 t_c \omega_p^2)$

Noting that the $\omega_p^2 = Ne^2/\varepsilon_0 m_e$ we see that additional inductance is due to electron mass and hence named as INERTIAL INDUCTANCE or KINETIC INDUCTANCE.

Resonance and enhancement of field intensity and non-linearity

One of the main aspects of 'LHM' meta-material, is that resonant nature makes the 'local' microscopic fields highly inhomogeneous and fairly intense. There can be immense enhancements of local fields, and there is potential for including non-linear effects.

The *E* field in the capacitive gaps at resonance , for SRDS are:

$$E_c(\omega_0) = \frac{V_c(\omega_0)}{d_c} = \frac{\mu_0 \omega_p^2 r t_c}{2\gamma L_c}$$

For incidence radiation the energy is equally distributed between *E* and *H* fields. Hence, we obtain an enhancement factor by:

$$Q = \frac{1}{2}\frac{(1/2\varepsilon_0)\left|E_c(\omega_0)\right|^2}{(1/2)\mu_0\left|H_{ext}(\omega_0)\right|^2} = \frac{1}{8c^2}\left[\frac{r t_c \omega_p^2}{2\gamma L_c}\right]^2$$

For the energy stored in capacitive gaps at resonance, where the factor 2 is for two gaps.

This *Q* ranges from 10^4 to 10^6

It is the intense magnetic field in vicinity of the thin wires that makes them work (perhaps) as low frequency plasma.

Regardless of frequency of interest the non-linear Kerr effect in dielectric is always possible, where *n* depends on EM field (intensity). $n = \sqrt{\varepsilon} = n_0 + n_2 I_N$ Where Intensity is given by:

$I_N = (1/2)(\varepsilon/\mu)^2 \varepsilon_0 c \left|E\right|^2$ and n_2 can be positive/negative called focusing/defocusing.

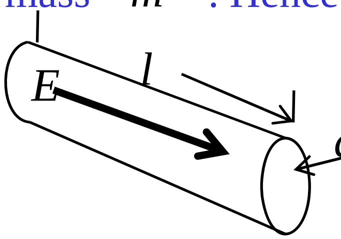
Suppose the gap be filled with Kerr dielectric then resonant frequency is highly nonlinear to applied intensity

Kinetic/Inertial Inductance

We can see that limitation to scale up to IR comes due to inertia of electrons, the limit of magnetic resonance is about 350 THz. A normal *L-R* circuit excited by step voltage has following relation

$$V = L\frac{di(t)}{dt} + Ri(t) \qquad i(t) = \frac{V_0}{R}[1 - \exp(-t/\tau)] \qquad \tau = L/R$$

The current seem to be delayed due to the time constant. We may ask a question, is there anything else in electrical circuit that cause delay? Clearly in order to have current; the charge carriers must be accelerated and it takes time to accelerate particles of mass m. Hence current will necessarily lag behind voltage causing it to rise.



$$dS = \pi r^2 \qquad V = El \qquad i = JS = NevS$$

$$eE = m\left(\frac{dv}{dt} + \frac{v}{\tau_{coll}}\right)$$

$$V = \frac{lm}{Ne^2S}\left(\frac{di}{dt} + \frac{i}{\tau_{coll}}\right)$$

Compare with *L-R* circuit equation, we have kinetic/inertial inductance and resistance, as: $L_i = \frac{lm}{Ne^2S}$ $\quad R_i = \frac{lm}{Ne^2S\tau_{coll}}$

Looking at expression of kinetic/inertial resistance, it is nothing but 'ordinary resistance'. On the other hand the expression for kinetic inductance is new. As the area of conductor becomes smaller and smaller the kinetic inductance becomes comparable to magnetic inductance. It can even become dominant inductance as in example in circuit containing nano-rods. Since $\omega_p^2 = Ne^2/\varepsilon_0 m$

$$L_i = \frac{l}{\pi r^2 \varepsilon_0 \omega_p^2}$$

Interpretation of additional inductance at very high frequency

The presence of this additional inductance can be explained by noting that at high frequencies the currents are hardly diffusssive, and almost ballistic; because the distance through which electrons moves with in a period of wave becomes comparable to the mean free path in metals. This means that if the frequencies are too high, the electrons can hardly be accelerated and the response falls. The mass of electron contribute additionally to the inductance. Current density $J = Nev \approx Ne(-j\omega eE/m)$, then the potential drop is $V \approx \{ml/Ne^2\}(\partial J/\partial t)$, implying an inductance that is proportional to electron mass.

The effective damping factor Γ also increases, becomes much larger as the size of the ring is reduced. this is due to the fact that the proportional energy in ballistic motion of the electron increases as size gets reduced and resistive losses are then very large indeed.

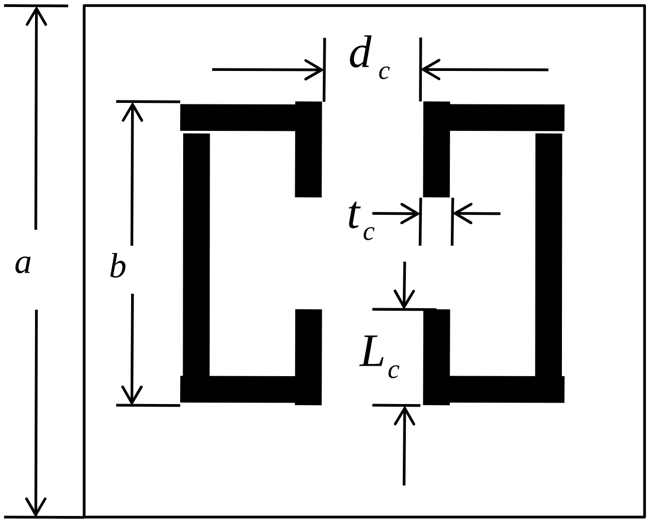
Thus even if the size of the ring were negligible the inertial/kinetic inductances would still be present preventing scaling to higher frequencies. Well, this effect of inertial inductance is also there even if super conducting SRR are employed.

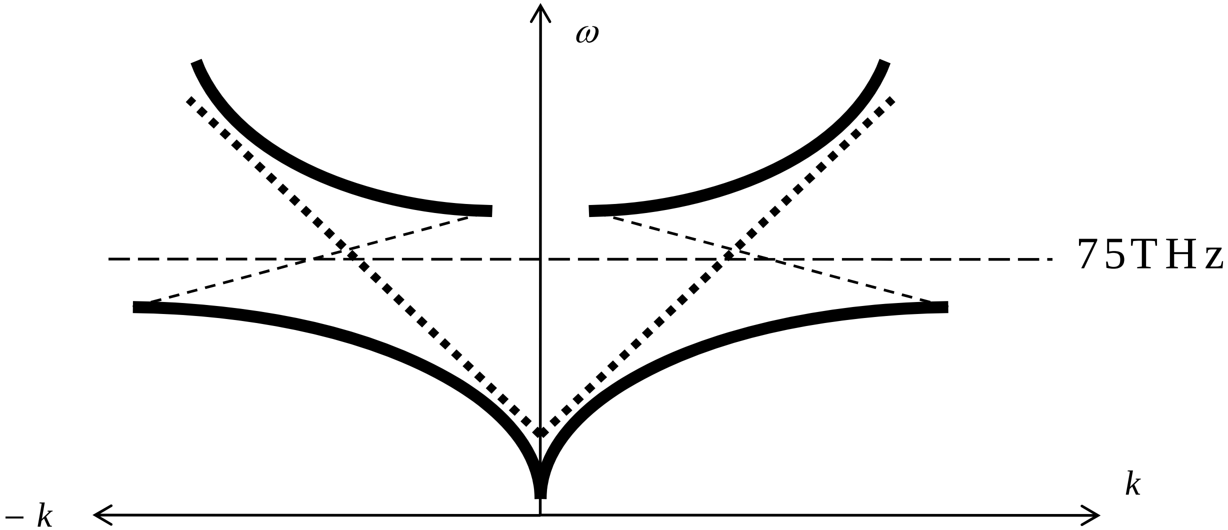
The large increase in damping as the sizes are scaled down broadens the resonance and permeability does not rapidly disperse; and the regions of MNG vanishes altogether. This increase in damping is matter of great concern for optical frequencies.

SRR with this two splits tends to tail off at wave length of 5 u-m. (IR region). By adding more capacitive gaps to lower the net capacitance and adjusting the dielectric constant of substrate MNG with this scaled down is achievable at 1.5 u-m.

Parallel metal sticks (pair of wire)of 100nm length periodically embedded in dielectric behaves as MNG at IR.

Single Ring with double-split Band Structure

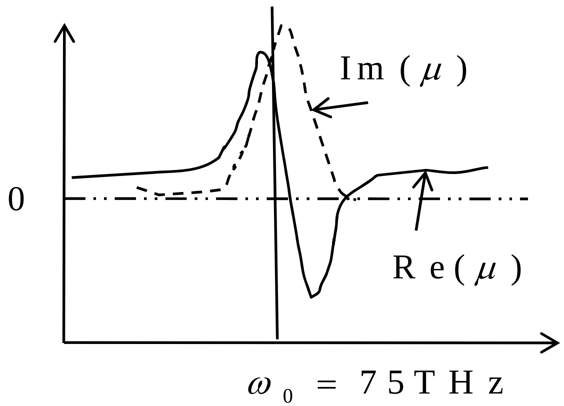


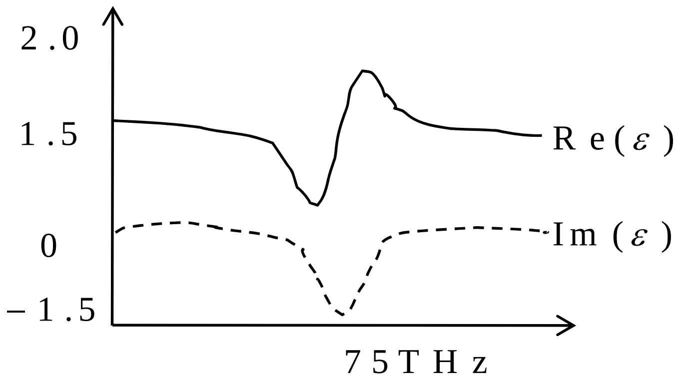


The band structure for infinite long split cylinder is depicted. The *H* is assumed to be along the axis of split cylinder.

Note that the presence of frequency gap at about 75 THz is due to MNG of structure.

Anti-Resonance!!





Clearly PEC cannot be made at HF. The figures show after extraction of permeability and permittivity, values from complex Transmission S_{12} measurements, show real part of permeability going negative near resonance. Although surprising imaginary part of permittivity going negative along with, and real part of permittivity disperse the other way around. This behavior is called ‘anti-resonance’.

Composing SRR and WA together inhomogeneous way to have NRM

Well embedding a thin wire array (WA) in a homogeneous medium of MNG may not produce NRM, and system may not have any propagating modes.

In a meta-material one cannot assume a homogeneous permittivity and permeability, due the other parts of the composite medium.

As long as WA are not placed in region where the highly inhomogeneous magnetic fields associated the SRR are present (along the axis of SRR) and SRR planes are placed such that they are at points of symmetry between the wires (where the magnetic fields associated with wires are minimal), the interference can be greatly be reduced.

We note that the magnetic fields due the wires fall of rather rapidly with distance from the wires and should not affect the SRR badly.

Then the quasi static responses derived for permittivity and permeability would be valid in negative index band, and SRR and WA would function independently as if in the air (vacuum).

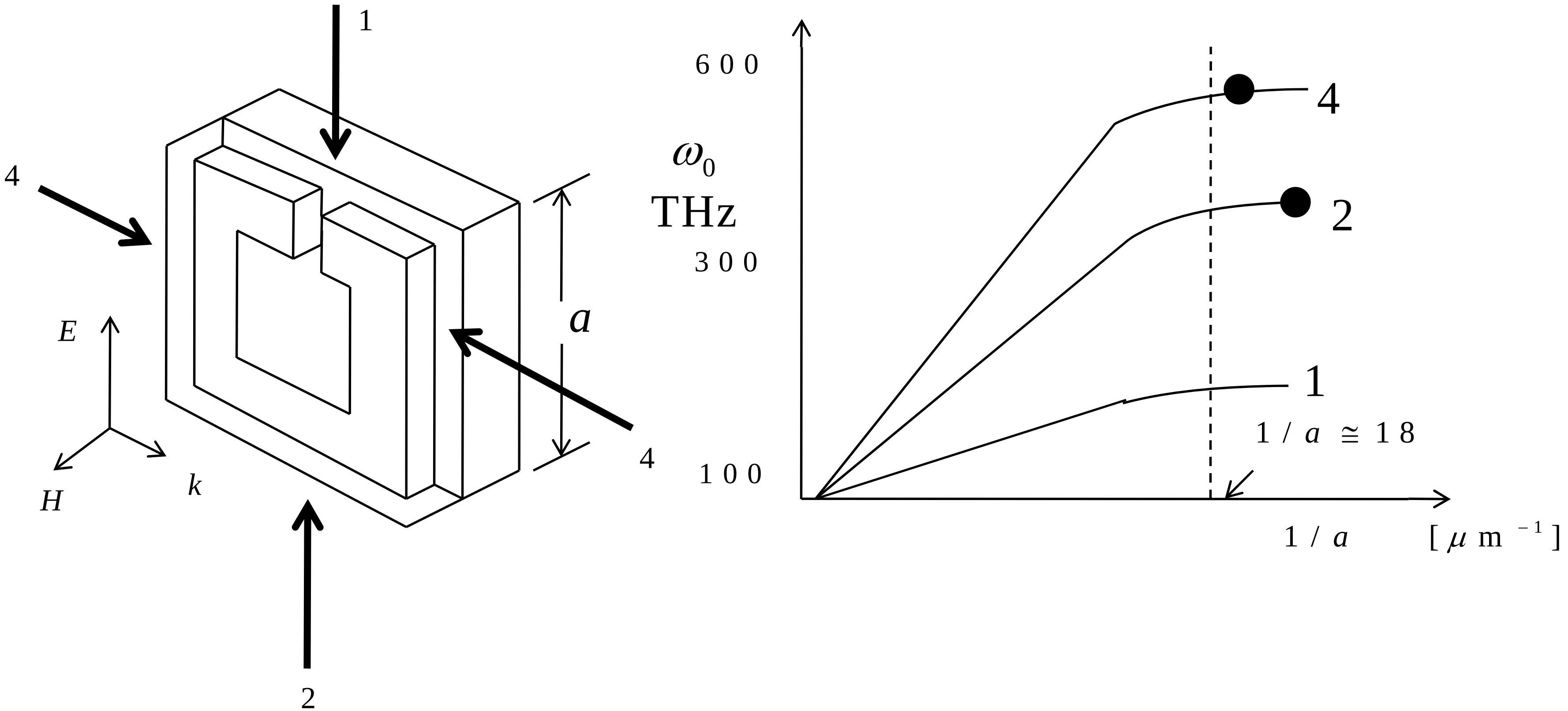
Thus relative placement of the corresponding components could be crucial and might account for differences in simulation and experiment.

A clear demonstration that the fields due to the two structures do not interfere with the functioning of each other is yet to be made.

Well due to residual interaction there is shift in resonant frequency, magnetic plasma frequency and electric plasma frequency and increased dissipation in the structures.

Further there are problems associated with simplistic approach of homogenization of composites at sub-resonance frequency where permeability is large and permittivity due to WA is negative. Then wave is highly evanescent and homogenization may not be possible.

Scaling for magnetic resonance



The scaling of magnetic resonance frequency as a function of the linear size '*a*' of unit cell for 1, 2, 4 cut SRR, show up to lower THz region the scaling is linear. The maximum attainable frequency is strongly enhanced with number of cuts in SRR ring. Black ball indication and 1/*a* = 18 line indicate MNG (negative permeability) is no longer reached, though one can continue with *LC* simulations.

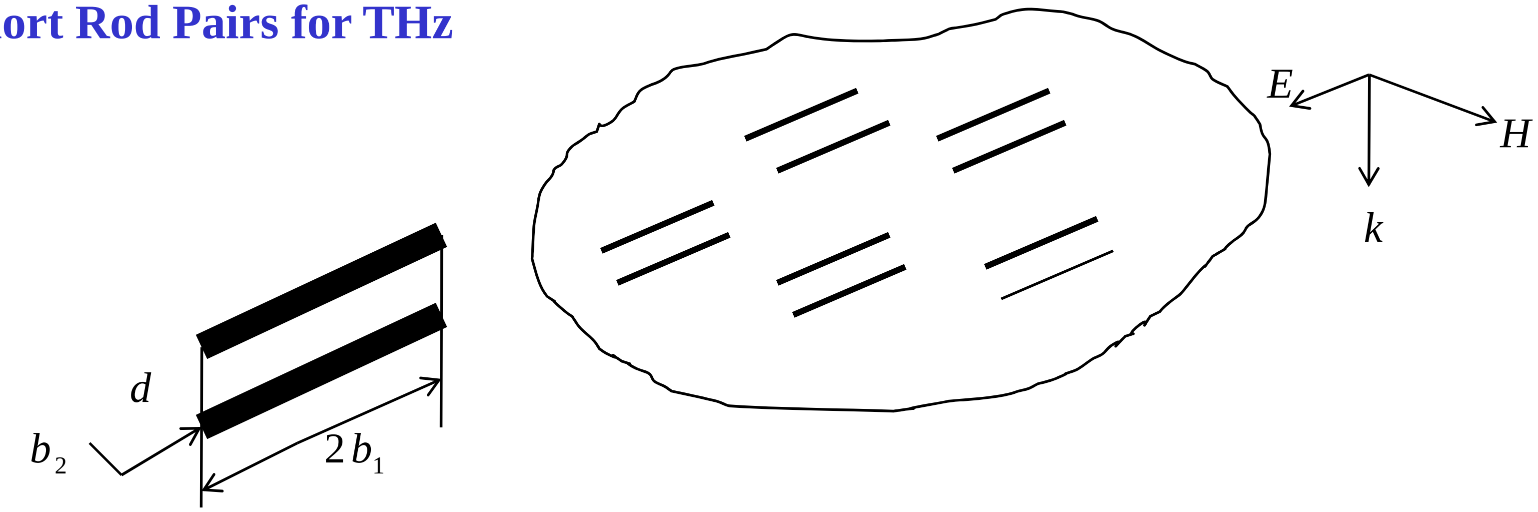
Some thoughts

SRR or its type still be preferred candidate for HF

350THz limit for magnetic resonance; but thought is plasma resonances may occur as long as magnetic resonance frequency is less that electric plasma frequency.

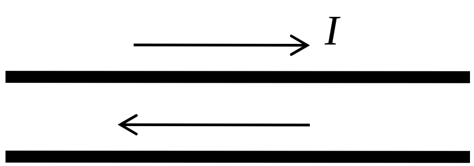
Silver strips 10 nm X 20 nm resonance frequency as high as 830 THz.

Short Rod Pairs for THz

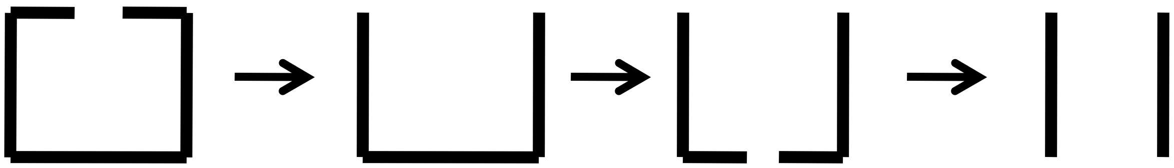


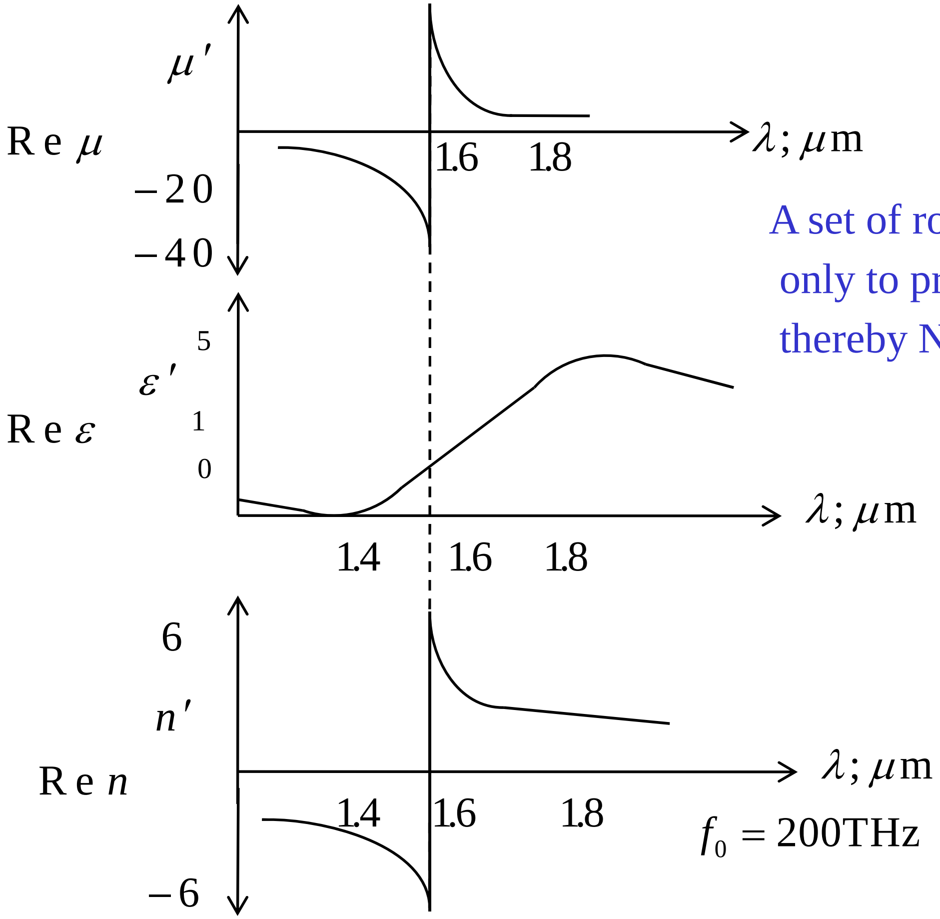
A di-electric suddenly becomes conductor when density of metallic inclusions reaches critical value. This is in context of ‘percolation thresh hold theory context.

The rod pairs they both have plasma plus *LC* resonances. Plasma resonance follows from the fact that frequency is high enough to be not too far from the metal’s plasma frequency. The *LC* resonance from simple argument, there is mutual inductance between them (rods) analogous to SRR. There are conduction currents I flowing in opposite directions and displacement currents between the rods.



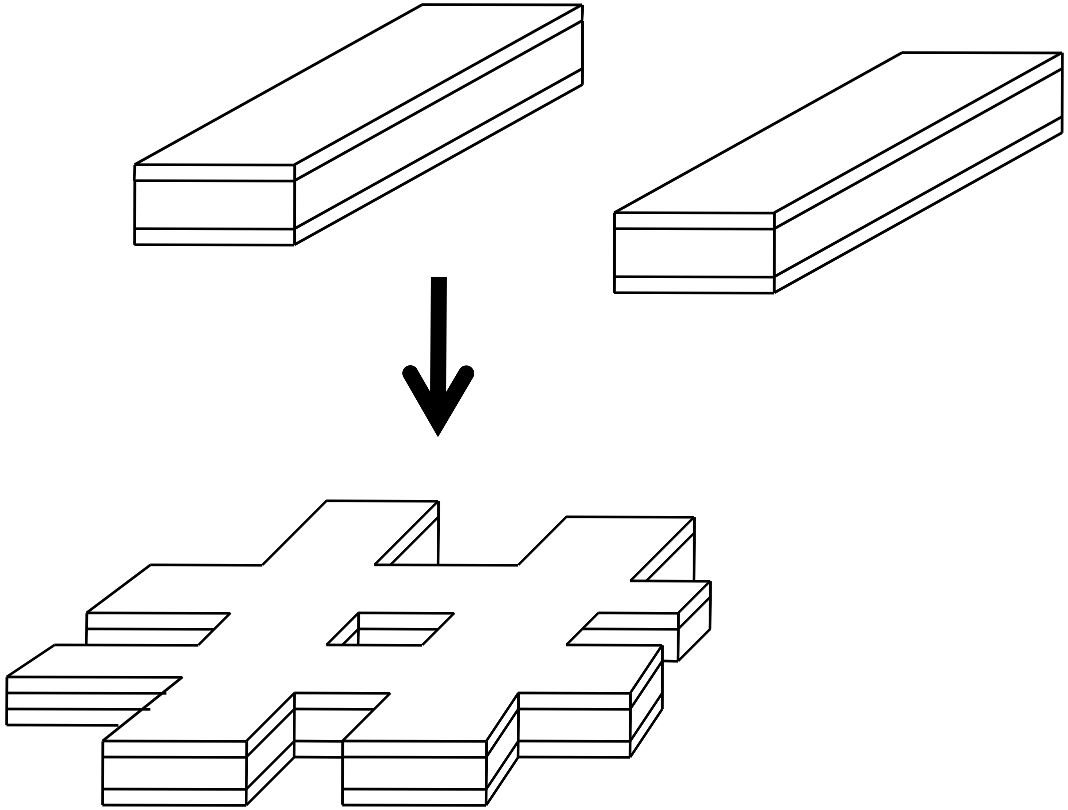
Transition from open resonators to rods





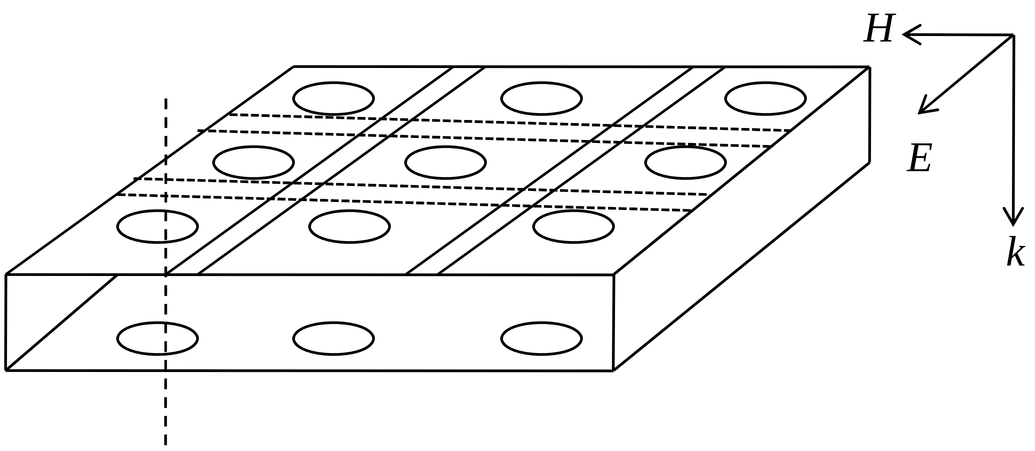
A set of rod pairs turn out to be suitable, not only to provide MNG, but also ENG, and thereby NRM

Fish net structure for THz



150 THz DNG

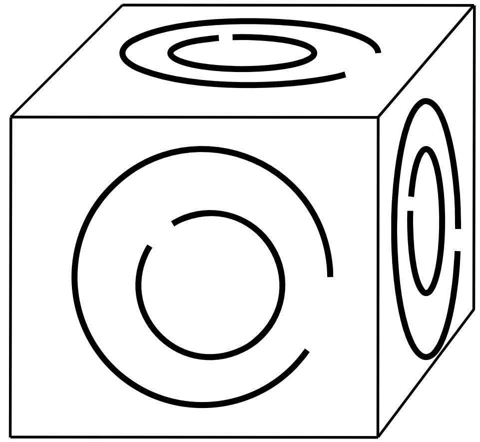
Perforated gold film for THz



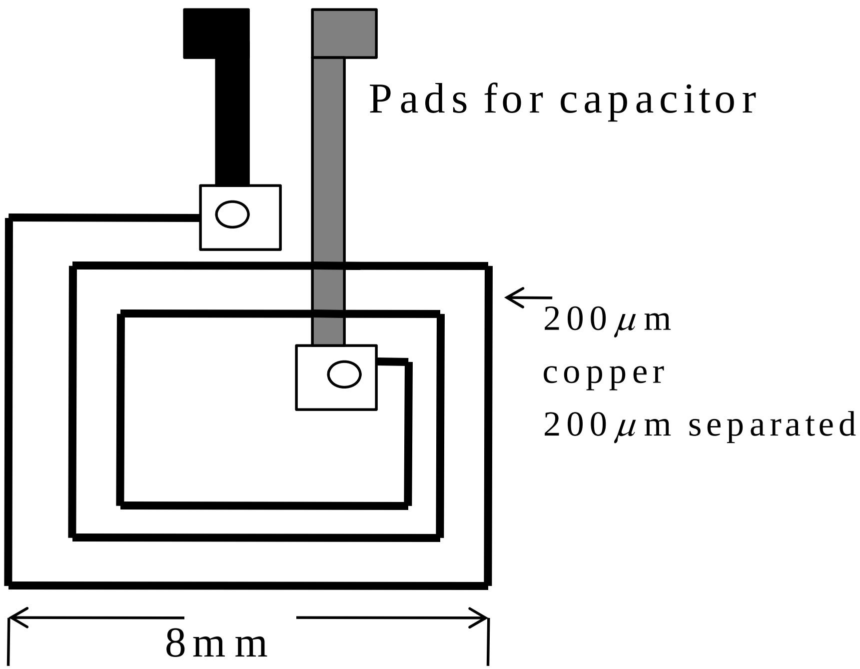
Dielectric alumina Al_2O_3 layer between two perforated gold films. Array of holes 830 nm pitch, and diameter 360 nm atop glass substrate.

Some other resonators for GHz band

Isotropic 3D -SRR

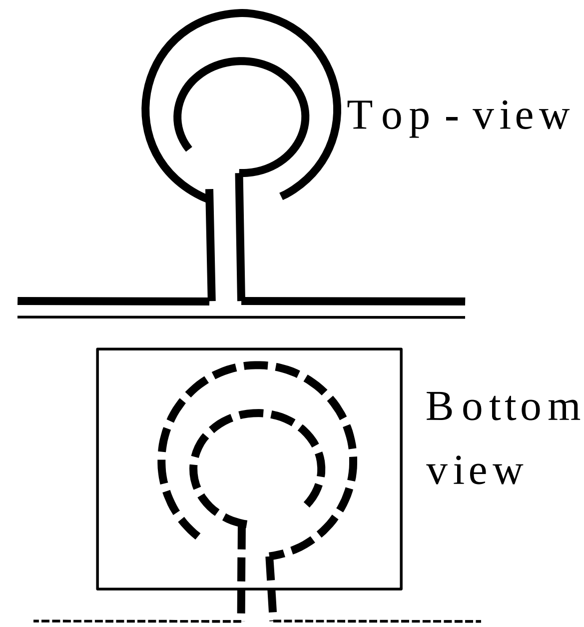


Capcitively loaded Resonator



Printed on both sides of PCB

Open-Ring Resonator



Causality Energy Density in NRM

It must be noted that dispersion less material parameters with permeability and permittivity negatives, cannot co exists.

Negative permittivity and negative permeability for static fields would for example imply that the energy density for static field (per volume) is negative

$$\mathrm{E}_{static} = \frac{\varepsilon_0 \varepsilon}{2} E_s^2 + \frac{\mu_0 \mu}{2} H_s^2 < 0$$

is not possible.

ENG, MNG are resonant effect.

These effects are necessarily dispersive and dissipative.

A non-dispersive NRM would (perhaps) imply time running backwards for the Electromagnetic excitation in that media, non causal system?

Causality further extended

Expression for energy density re-written (Taylor expansion about a frequency); in the so called transparency region for frequency far enough away from resonance, where absorption is small.

$$E_c = \frac{1}{2}\left[\frac{\partial}{\partial\omega}(\omega\varepsilon)\varepsilon_0 E^2 + \frac{\partial}{\partial\omega}(\omega\mu)\mu_0 H^2\right]$$

Thus sufficient condition for a positive energy density under these approximation is

$$\frac{\partial(\omega\varepsilon)}{\partial\omega} > 0; \quad \frac{\partial(\omega\mu)}{\partial\omega} > 0$$

This is satisfied in region of normal dispersion far from resonance.

Another general expression is:

$$E(t) = \frac{\varepsilon_0}{2|E(t)|^2} + \frac{\mu_0}{2|H(t)|^2} + \int_{-\infty}^{\infty} d\omega \left[\left|\alpha_E(\omega)\int_{-\infty}^{t} d\tau e^{j\omega\tau} E(\tau) + \alpha_H(\omega)\int_{-\infty}^{t} d\tau e^{j\omega\tau} H(\tau)\right|^2\right]$$

Where $\alpha_E^*(\omega)\alpha_E(\omega) = \omega \,\mathrm{Im}\, \varepsilon(\omega)$ and $\alpha_H^*(\omega)\alpha_H(\omega) = \omega \,\mathrm{Im}\, \mu(\omega)$

This E (t) is positive definite and applicable to cases of ENG and MNG as well.

The real part and imaginary part of physical quantity , responding to external excitation has point of view to state its causality. In simple terms real part what we measure, and the imaginary part is what we call dissipative . The physical quantity is permeability, permittivity or refractive index , responding to EM radiation, or it could be position of a body, responding to external force. In general we write the physical quantity responding as $\chi(\omega) = \mathrm{Re}\,\chi(\omega) + j\,\mathrm{Im}\,\chi(\omega)$ complex function in complex frequency plane $\omega = \mathrm{Re}\,\omega + j\,\mathrm{Im}\,\omega$. Real and Imaginary part of response is by Kramer's-Kronig law

The Kramer's-Kronig Relation (KK Relation) and causality

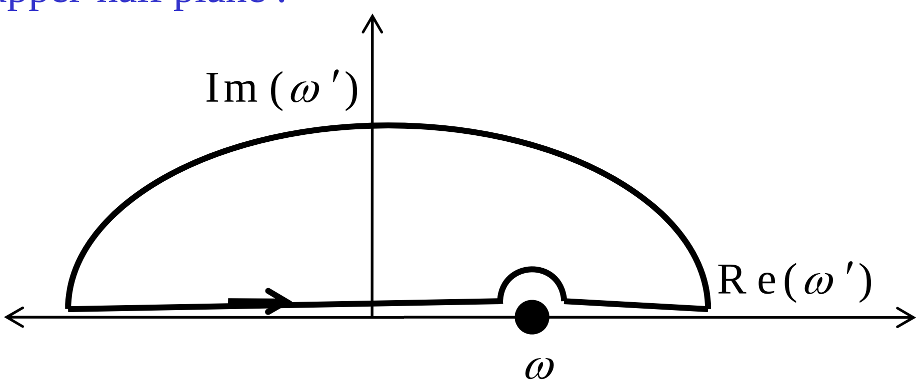
The KK relations are mathematical properties, connecting the real and imaginary part of any complex function, which is analytic in 'upper-half' plane. These relations are often used to relate response function in physical system because causality implies the analyticity condition is satisfied, and conversely analyticity implies causality of physical system. An "analytic signal" means a signal with no negative frequency terms. Analytic functions are also termed as "holomorphic" functions.

For a complex function $\chi(\omega) = \mathrm{Re}\,\chi(\omega) + j\,\mathrm{Im}\,\chi(\omega)$ of complex variable ω, analytic in the upper half plane and which vanishes faster than $|\omega|^{-1}$ as $\omega \to \infty$, then

$$\mathrm{Re}\,\chi(\omega) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\mathrm{Im}\,\chi(\omega')}{(\omega' - \omega)} d\omega' \qquad \mathrm{Im}\,\chi(\omega) = -\frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{\mathrm{Re}\,\chi(\omega')}{(\omega' - \omega)} d\omega'$$

P is Cauchy's principle value

Given any function $\chi(\omega')$ analytic in upper half plane , the function $\chi(\omega')/(\omega' - \omega)$, where ω is real is also analytic in upper-half plane .



Note the Real and Imaginary parts of frequency responses, as related of causal system as shown are Hilbert Transforms of each other (will be described in later part)

Physics and KK relation to causality

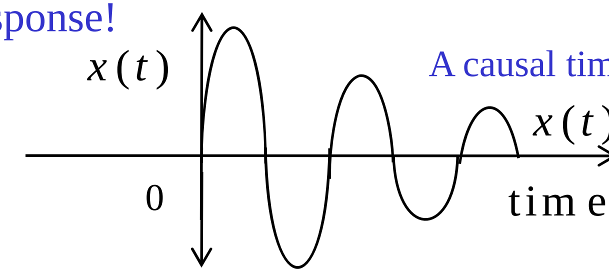
We can apply KK formalism to a response function. In physics time response function $\chi(t-t')$, describes how some property say $P(t)$ of physical system responds to an excitation force $F(t)$. The response function $\chi(t-t')$ must be zero for $t < t'$ as systems cannot respond to a force before it is applied. This causality condition implies that Fourier Transform $\chi(\omega)$ is analytic in "upper" half plane of complex frequency . Additionally if we subject the system to an oscillatory force with much higher than the highest resonant frequency, there will be no response , implies that $\chi(\omega) \to 0$ as $\omega \to \infty$
The imaginary part of $\chi(t-t')$ describes how a system dissipates energy, since it is out of phase with the driving force , (rather reactive energy) . The formulas of KK relations are not useful for reconstructing physical responses as integrals run from minus infinity to plus infinity, implying we know the responses at negative frequencies! Fortunately in most physical systems the positive frequency response determines the negative frequency response, because the $\chi(\omega)$ is Fourier Transform of real quantity $\chi(t-t')$
So $\chi(-\omega) = \chi^*(\omega)$. This means $\mathrm{Re}\,\chi(\omega)$ is even function and $\mathrm{Im}\,\chi(\omega)$ is odd function.

Understanding KK relations with simple example of causal function

The Fourier Integral, or frequency response of any arbitrary function $x(t)$ is $X(\omega)$, given as:

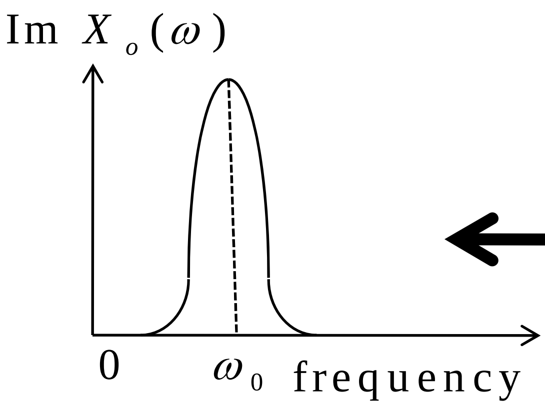
$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{+\infty} [x(t)\cos\omega t - jx(t)\sin\omega t] dt = \mathrm{Re}\, X(\omega) + j\,\mathrm{Im}\, X(\omega)$$

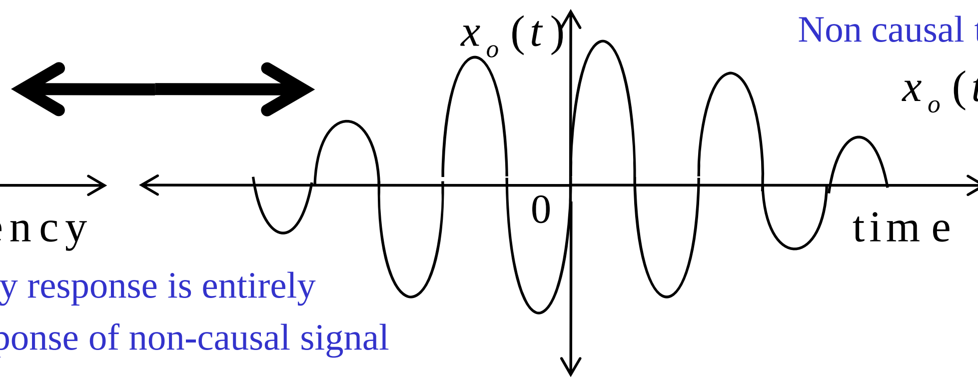
Think about a causal real value function in time say damped sine wave after time zero as; obviously its frequency response will be like a resonance response!



A causal time impulse response

$$x(t) = 0 \quad \text{for} \quad t < 0$$





Non causal time odd impulse response

$$x_o(t) = -x_o(-t)$$

Imaginary part of frequency response is entirely due to odd part of time response of non-causal signal

Remember cosines are even and sines are odd. For an odd function, the odd-even product $x_o(t)\cos\omega t$ integrates to zero, because the left and right half have equal magnitude but opposite sign and thus cancel each other. Consequently only the finite non-zero terms are imaginary $\int jx_o(t)\sin\omega t\, dt$

Real and Imaginary part of frequency response from causal time response decomposed as even and odd non-causal time responses!

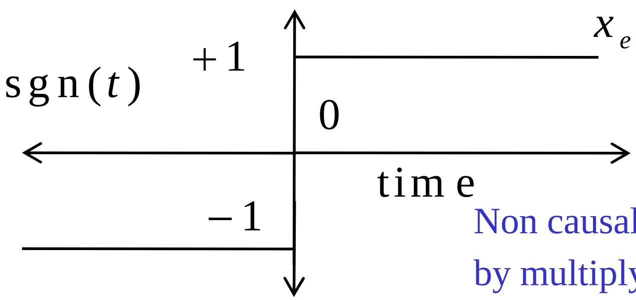
Let us see relation between an even impulse response and its Fourier integral $x_e(t) = x_e(-t)$, The Fourier integral of non causal even time signal is purely real. The even-odd product $jx_e(t)\sin\omega t$ must integrate to zero, thus only $x_e(t)\cos\omega t$ remains.

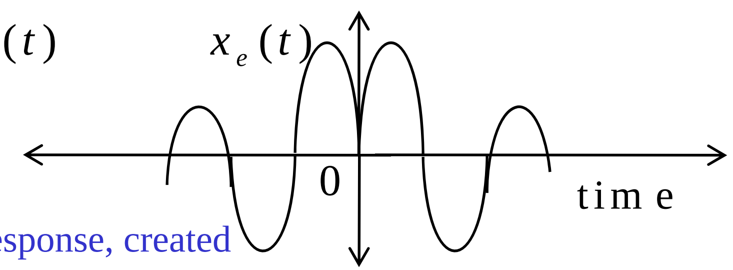
Now let us decompose a causal time function into even and odd parts; one can construct any causal or non--causal $x(t)$ out of a sum of some linear combination of even and odd components using:

$$x(t) = \frac{x(t) + x(-t)}{2} + \frac{x(t) - x(-t)}{2}$$

Consider our odd function $x_o(t)$ and multiply by $\mathrm{sgn}(t)$ this yields a new even function as

$$x_e(t) = x_o(t)\,\mathrm{sgn}(t)$$





Non causal time even impulse response, created by multiplying sgn function and odd non-causal function

Now add the odd function $x_o(t)$ to this created even function $x_e(t)$ we get $x(t)$ the original damped causal function! $x(t) = x_e(t) + x_o(t) = x_o(t)\,\mathrm{sgn}(t) + x_o(t)$

Notice that $x_e(t) = x_o(t)\,\mathrm{sgn}\,t$ will have frequency response function as Real, and $x_o(t)$ will give Imaginary frequency response; both depending on the same function i.e. odd non-causal!

$$X(\omega) = \mathrm{Re}\,X(\omega) + j\,\mathrm{Im}\,X(\omega) \qquad \text{for causal} \quad x(t)$$

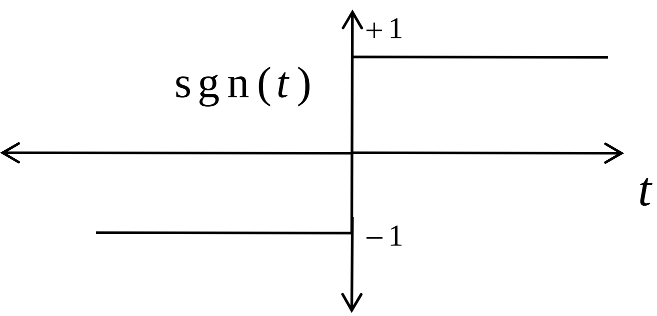
From even odd non-causal functions to Hilbert transform in causal system

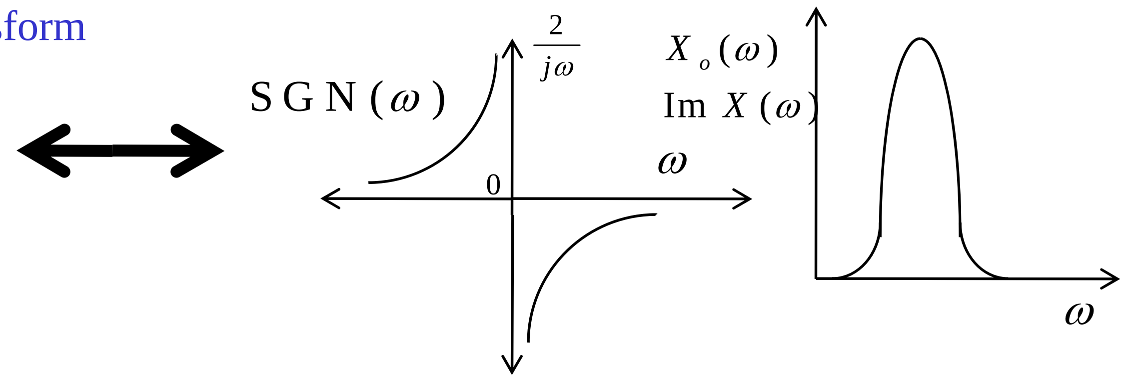
We can see thus the causality constraints means that the Real and Imaginary parts are related and contain the same information. But what is the exact relationship? Multiplication in time domain is convolution in the frequency domain

$$X(\omega) = X_o(\omega) * SGN(\omega) + X_o(\omega) = \left\{\int_{-\infty}^{+\infty} X_o(\omega')SGN(\omega - \omega')d\omega'\right\} + X_o(\omega)$$

The $sgn(t)$ is odd (non-causal) so its frequency response is purely imaginary, as $SGN(\omega) = 2/j\omega$

The convolution above is Hilbert Transform





Hilbert Transform in frequency domain is multiplication by $sgn\ t$ in time domain. Hilbert kernel is $2/j\omega$

We can therefore rewrite the real part of our frequency response $X(\omega)$ as Hilbert Transform of the imaginary part of the frequency response

$$\mathrm{Re}\,X(\omega) = X_e(\omega) = SGN(\omega) * X_0(\omega) = \int_{-\infty}^{+\infty} \frac{2}{j(\omega' - \omega)} X_o(\omega')d\omega'$$

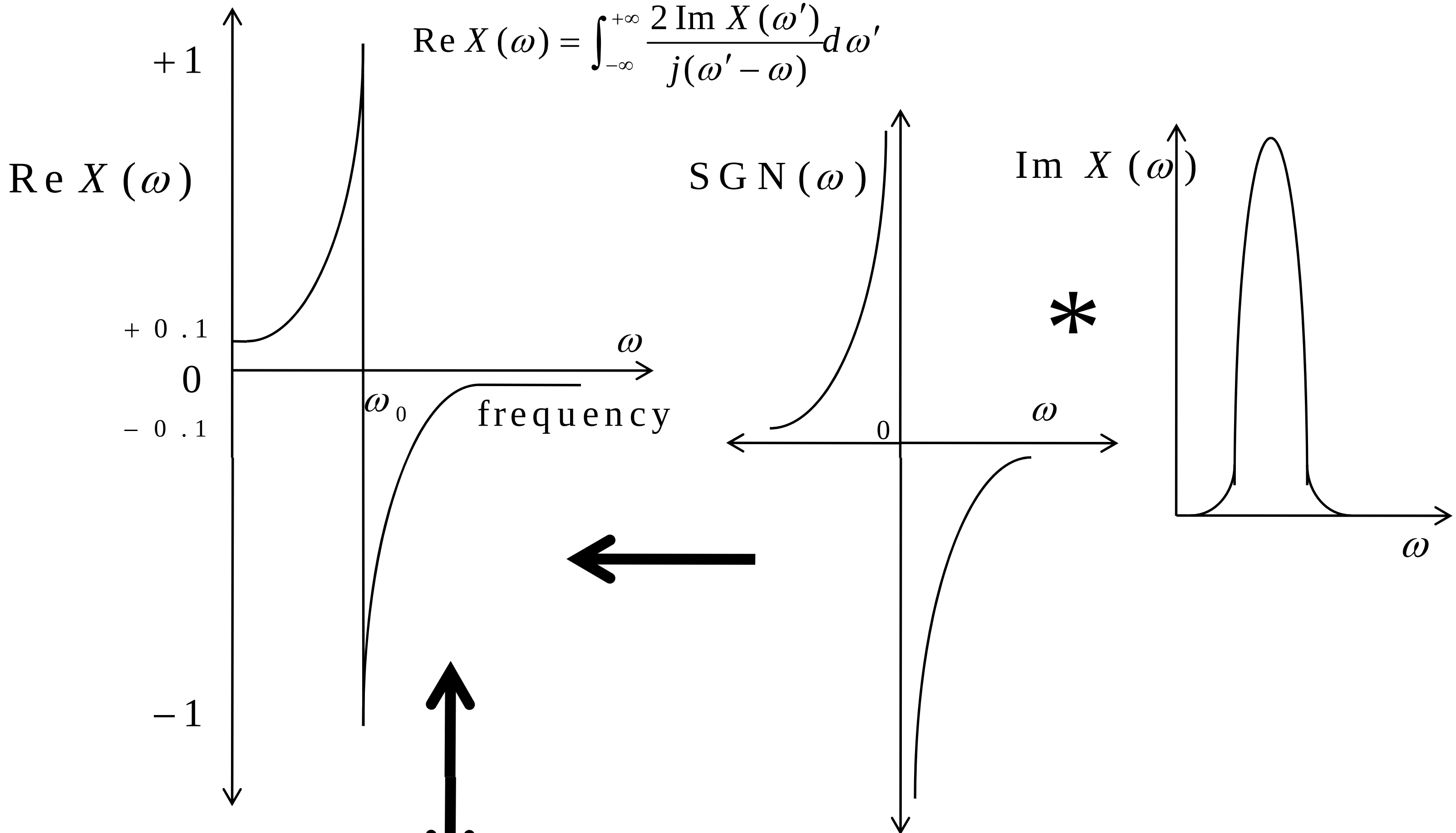
Note $X_o(\omega')$ is purely imaginary and j times imaginary is real!!

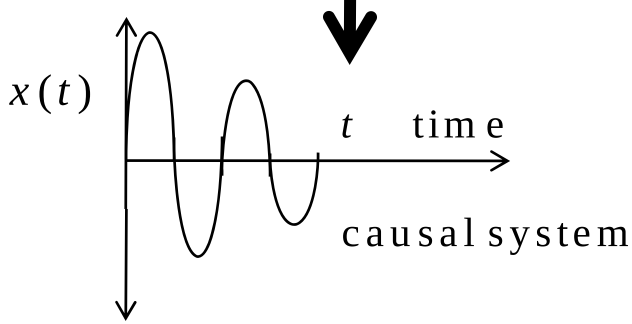
If the real and imaginary parts are Hilbert Transforms of each other then the time response is causal not otherwise; the above expression is similar to KK-relation

Hilbert Transform of Imaginary frequency response generates real part & vice-versa

$$\mathrm{Re}\,X(\omega) = X_e(\omega) = \mathrm{SGN}(\omega) * X_0(\omega) = \int_{-\infty}^{+\infty} \frac{2}{j(\omega' - \omega)} X_o(\omega')\,d\omega'$$

$$\mathrm{Re}\,X(\omega) = \int_{-\infty}^{+\infty} \frac{2\,\mathrm{Im}\,X(\omega')}{j(\omega' - \omega)}\,d\omega'$$





$$\varepsilon(\omega) = \mathrm{Re}\,\varepsilon(\omega) + j\,\mathrm{Im}\,\varepsilon(\omega) = \varepsilon' + j\varepsilon''$$

$$\mu(\omega) = \mathrm{Re}\,\mu(\omega) + j\,\mathrm{Im}\,\mu(\omega) = \mu' + j\mu''$$

$$n(\omega) = \mathrm{Re}\,n(\omega) + j\,\mathrm{Im}\,n(\omega) = n' + jn''$$

Propagation of radiation in NRM

$k \times E = \omega \mu_0 \mu H$ Phase velocity has k opposite to $S = E \times H$. We can imply $n = \sqrt{\varepsilon \mu} < 0$

$k \times H = -\omega \varepsilon_0 \varepsilon E$ thus phase is accumulating in propagation distance d, is $\phi d = -n \omega d / c$

Well $\varepsilon = \varepsilon' + j\varepsilon''$, and $\mu = \mu' + j\mu''$ the medium is absorbing if $\varepsilon'' > 0; \mu'' > 0$ and amplifying if $\varepsilon'' < 0; \mu'' < 0$. Let us consider a plane wave $\exp(jnk_0 z)$ in absorbing NRM, where

$n = \pm[(\varepsilon'\mu' - \varepsilon''\mu'') + j(\varepsilon'\mu'' + \mu'\varepsilon'')]^{1/2} \cong \pm[\varepsilon'\mu' + j(\varepsilon'\mu'' + \mu'\varepsilon'')]^{1/2}$ for small absorption

$$n \cong \pm[\sqrt{\varepsilon'\mu'}(1 + j\{\varepsilon'\mu'' + \mu'\varepsilon''\} / 2\varepsilon'\mu')] = \pm[(\varepsilon'\mu')^{1/2} + \{j/2\}\{\varepsilon'\mu'' + \mu'\varepsilon''\} / (\varepsilon'\mu')^{1/2}]$$

Now waves should decay in amplitude as it propagates in dispersive media and governed by sign of $\mathrm{Im}(k_z)$. For $\varepsilon' < 0$ and $\mu' < 0$ this demands negative root of n be taken!

Well, power radiated inside NRM shall depend on wave impedance $Z = \sqrt{\mu / \varepsilon} > 0$

$$Z = \sqrt{\mu / \varepsilon} = \sqrt{\mu\mu / \varepsilon\mu} = \mu / n > 0$$

$n > 0, \mu > 0$ For DPS and PRM and power is radiating away

$n < 0, \mu < 0$ For DNG and NRM and power is radiating away

The complex wave vector and PRM and NRM wave propagation

Consider EM wave vector $[k_x, 0, k_z]$ incident from $(-\infty < z < 0)$ on semi-infinite medium $(0 < z < \infty)$

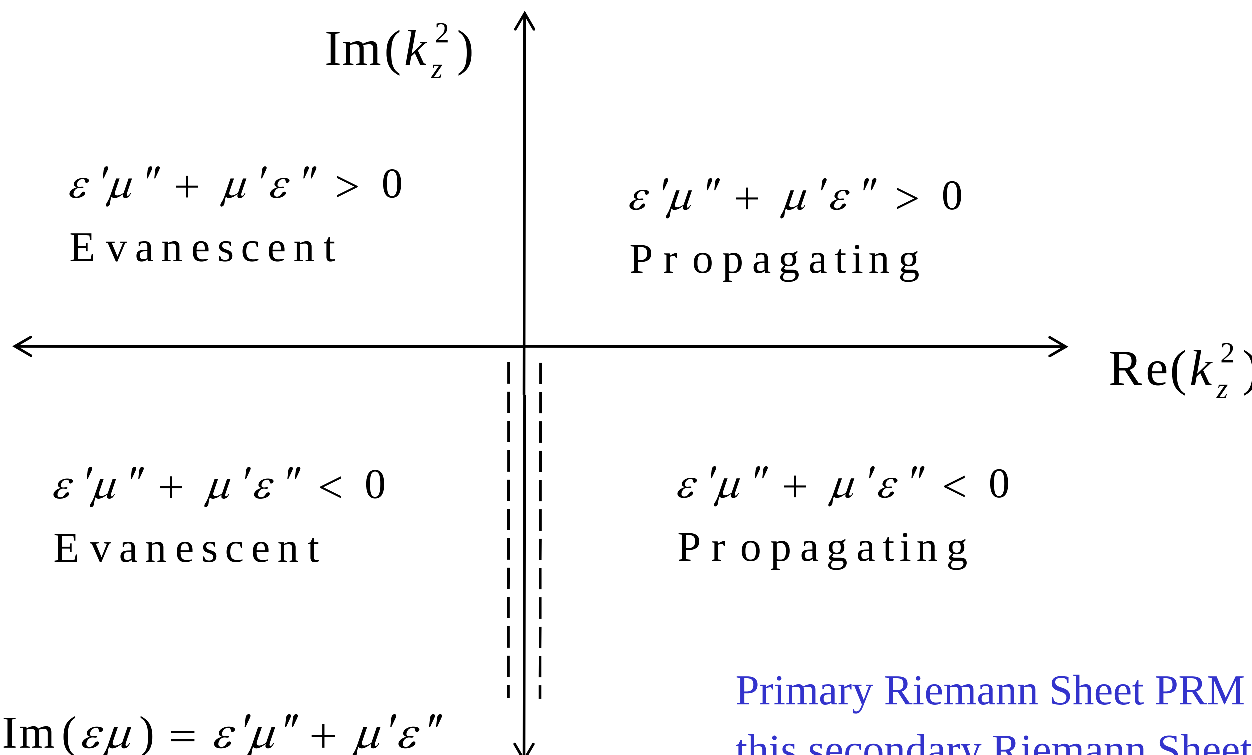
Due to *x*-invariance, the k_x is preserved across the boundary. The *z* component should be found from

$k_z = \pm\sqrt{\varepsilon\mu\frac{\omega^2}{c^2} - k_x^2}$ Where physical choice has to be again made for the square root sign

Now the second medium could be propagating $k_x^2 < \mathrm{Re}(\varepsilon\mu\omega^2/c^2)$ or evanescent $k_x^2 > \mathrm{Re}(\varepsilon\mu\omega^2/c^2)$

further the second medium could be absorbing or amplifying depending on sign of $\mathrm{Im}(n^2) = \mathrm{Im}(\varepsilon\mu)$

This enables us to divide the complex plane $k_z^2 = \mathrm{Re}(k_z^2) + \mathrm{Im}(k_z^2)$ into four quadrants.



$\mathrm{Im}(n^2) = \mathrm{Im}(\varepsilon\mu) = \varepsilon'\mu'' + \mu'\varepsilon''$

Primary Riemann Sheet PRM only, beneath this secondary Riemann Sheet give NRM

Taking square root of k_z^2 plane into two Riemann sheets

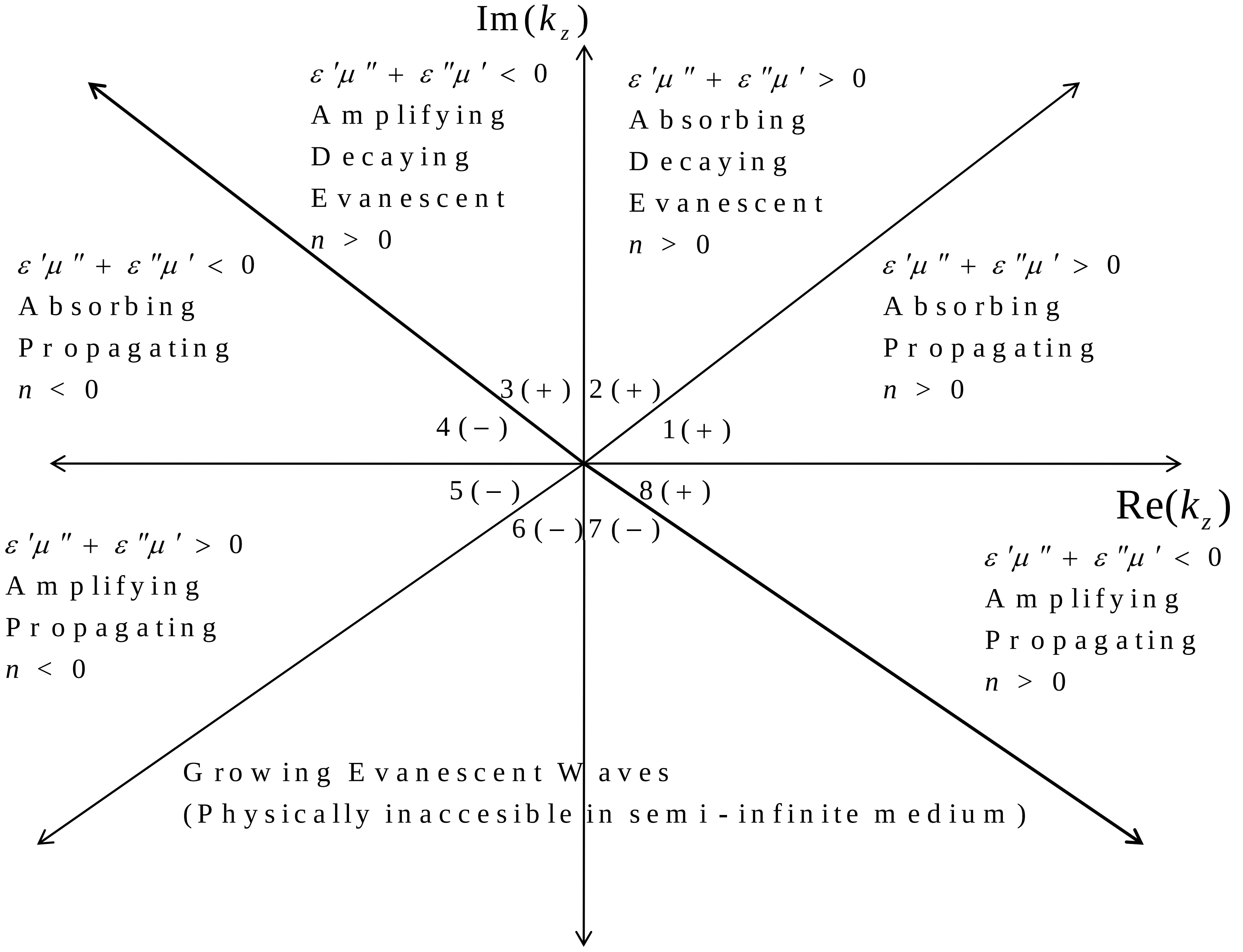
For "absorbing" medium the wave amplitudes at infinities has to obviously disappear. For "amplifying" medium one has to be more careful. The only conditions are that evanescent decaying waves remain decaying, propagating ones remains propagating and no information can flow from infinities.
This ensures that near field features of a source cannot be probed at large distances merely by embedding the source in amplifying medium.

Hence our choice was inconvenient for the branch cut, that is from 270 to -90 degrees, hence our range of argument in first Riemann sheet is $\angle k_z^2 = \theta \quad (-\pi/2 < \theta < 3\pi/2)$, argument in the second Riemann sheet is $\angle k_z^2 = \theta \quad (3\pi/2 < \theta < 7\pi/2)$, corresponding to

$$k_z = \pm\sqrt{k_z^2} = \pm\left[\left|k_z^2\right|\exp(j\theta)\right]^{1/2} = \begin{cases} \left|(k_z^2)\right|^{1/2}\exp(j\theta/2) \\ \left|(k_z^2)\right|^{1/2}\exp(j\theta/2 + j\pi) \end{cases}$$

The second Riemann sheet corresponds to NRM $\angle k_z = \theta/2 = \theta_2 \quad (3\pi/4 < \theta_2 < 7\pi/4)$

Complex wave vectors in two Riemann sheets first sheet PRM second sheet NRM



(+) Primary Riemann sheet $n > 0$, $\mathrm{Re}(k_z) > 0$, PPV, for Propagating

(−) Secondary Riemann sheet $n < 0$, $\mathrm{Re}(k_z) < 0$, NPV, for Propagating

Explanation for the two Riemann sheet zones

Region 1 and 8 corresponds to propagating waves in PRM, that are absorbing or amplifying respectively.
Region 6 and 7 corresponds to growing evanescent waves that build up at infinities, which are unphysical in the semi-infinite medium.
Decaying evanescent waves fall with region 2 if $\mathrm{Im}(n^2) = \mathrm{Im}(\varepsilon\mu) = \varepsilon'\mu'' + \varepsilon''\mu' > 0$, and in the region 3 if $\varepsilon'\mu'' + \varepsilon''\mu' < 0$
Note the Poynting vector points away from the source (interface) if medium is absorbing overall, and actually towards the source (interface) if media is amplifying overall.
For the case of evanescent waves in amplifying media our choice of Poynting vector that points towards the source (interface in this case). This however does not violate the causality as the Poynting vector energy flow decays exponentially to zero at infinity and no information flows from infinity.
The counter-intuitive behavior does not imply that source has turned into sink-rather indicates that there would be large (infinitely large unsaturated linear gain) accumulation of energy density (intense field enhancements) near a source.
Now propagating waves in ENG MNG simultaneously in region 4 and 5 depending on whether $\varepsilon'\mu'' + \varepsilon''\mu' < 0$ or $\varepsilon'\mu'' + \varepsilon''\mu' > 0$ respectively corresponding to absorbing and amplifying media respectively. In both cases negative square root need be chosen, this is start of second Riemann sheet.
In case of normal incidence the sign of wave vector (k) and sign of index of refraction (n), are same. The quantity $\varepsilon'\mu'' + \varepsilon''\mu'$ determines the energy flow. In dissipative media $\mathrm{Im}(k_z) < 0$ for propagating waves , which reduces to $\mathrm{Im}(n) > 0$ for normal incidence. Thus one can reasonable talk of NPV rather NGV

For NRM in secondary Riemann sheet absorbing and amplifying reversed

$$n \cong \pm[(\varepsilon'\mu')^{1/2} + \{j/2\}\{\varepsilon'\mu'' + \mu'\varepsilon''\}/(\varepsilon'\mu')^{1/2}] = \pm[\mathrm{Re}(n) + j\,\mathrm{Im}(n)]$$

$$n = \begin{cases} \mathrm{Re}(n) + j\,\mathrm{Im}(n) \\ -\mathrm{Re}(n) - j\,\mathrm{Im}(n) \end{cases}$$

$$\mathrm{Im}(n) \cong \varepsilon'\mu'' + \mu'\varepsilon''$$

$$\mathrm{Im}(k_z) \cong \varepsilon'\mu'' + \varepsilon''\mu' = \begin{cases} > 0 & \text{dissipative PRM} \\ < 0 & \text{amplifying PRM} \end{cases}$$

$$\mathrm{Im}(k_z) \cong \varepsilon'\mu'' + \varepsilon''\mu' = \begin{cases} < 0 & \text{dissipative NRM} \\ > 0 & \text{amplifying NRM} \end{cases}$$

For dissipative , absorbing NRM with ENG and MNG simultaneously, we get , $\mathrm{Re}(n) < 0$, $\mathrm{Re}(k_z) < 0$ $\varepsilon'\mu'' + \varepsilon''\mu' < 0$ $\mathrm{Im}(k_z) < 0$ NPV , opposite definitions of absorbing and amplifying as compared to PRM. Thus we get $\mathrm{Im}(n) > 0$

Using Real and Imaginary concepts in extraction from S12 and S11 response data

$$T = \left[\cos(nk_z d) - \frac{j}{2}\left(Z + \frac{1}{Z}\right)\sin(nk_z d)\right]^{-1} \exp(-jk_z d)$$

$$R = -\frac{j}{2}\left(Z - \frac{1}{Z}\right)\sin(nk_z d) T \exp(jk_z d)$$

Extraction is by

$$n = \pm \cos\left(\frac{1 - r^2 - t^2}{2t}\right)$$

$$Z = \pm \left[\frac{(1+r)^2 - t^2}{(1-r)^2 - t^2}\right]^{1/2}$$

$$r = R \qquad t = T \exp(jk_z d)$$

$$\varepsilon = n / Z \qquad \mu = nZ$$

The multi-valued nature of trigonometric function give rise to ambiguity in *n* and *Z* which however, can be resolved by determining the *T* and *R* at several thickness *d* of the slab, and conditioned as Re (*Z*) > 0, Im (*n*) > 0 for causal absorbing (dissipating) media.

A meta-material with $\mathrm{Re}\,\varepsilon < 0$ has $\mathrm{Im}\,\mu < 0$; similarly meta-material with $\mathrm{Re}\,\mu < 0$ has $\mathrm{Im}\,\varepsilon < 0$ for both SRR medium with WA dielectric cylinders. Although surprising this does not violet any funda- -mental as long as passive LHM is dissipative with $E_c > 0$. These negative $\mathrm{Im}\,\mu$ and $\mathrm{Im}\,\varepsilon$ are due to finiteness of unit cell.

Phase and Group velocity revisit

The wave vector $\vec{k}$ in an isotropic NRM points opposite to Poynting vector $S = E \times H$. Thus it is clear that the phase $v_{ph} = \omega / k < 0$, however, the energy flow is along the Poynting vector and away from the source. The group velocity $v_g = \nabla_k \omega(k)$ is along the direction of Poynting vector.

We should note that phase and Poynting vector do point opposite in anisotropic media

In homogeneous isotropic media $v_g = \nabla_k \omega(k) = p \dfrac{\vec{k}}{k} \dfrac{d\omega}{dk}$

For NRM say $\vec{k} = -k$ and $p = -1$ gives $v_g = \dfrac{d\omega}{dk} > 0$

For PRM say $p = +1$ and $\vec{k} = +k$ gives $v_g = \dfrac{d\omega}{dk} > 0$

This means that the group velocity can only be parallel in PRM or anti parallel in NRM.

NRM behaves as isotropic media and not anisotropic one.

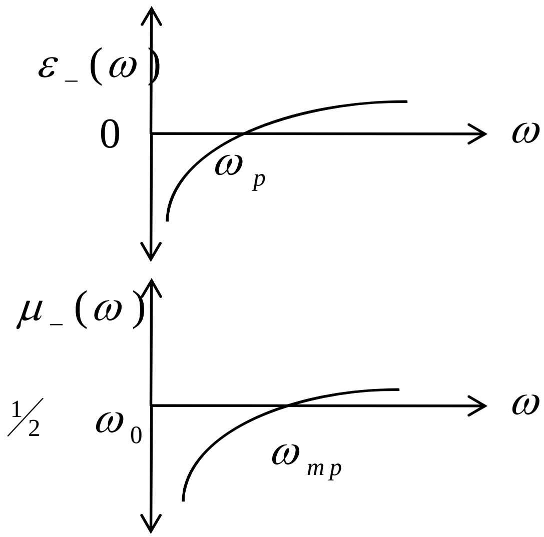
Dispersion diagram in semi-infinite NRM with ENG and MNG resonance.

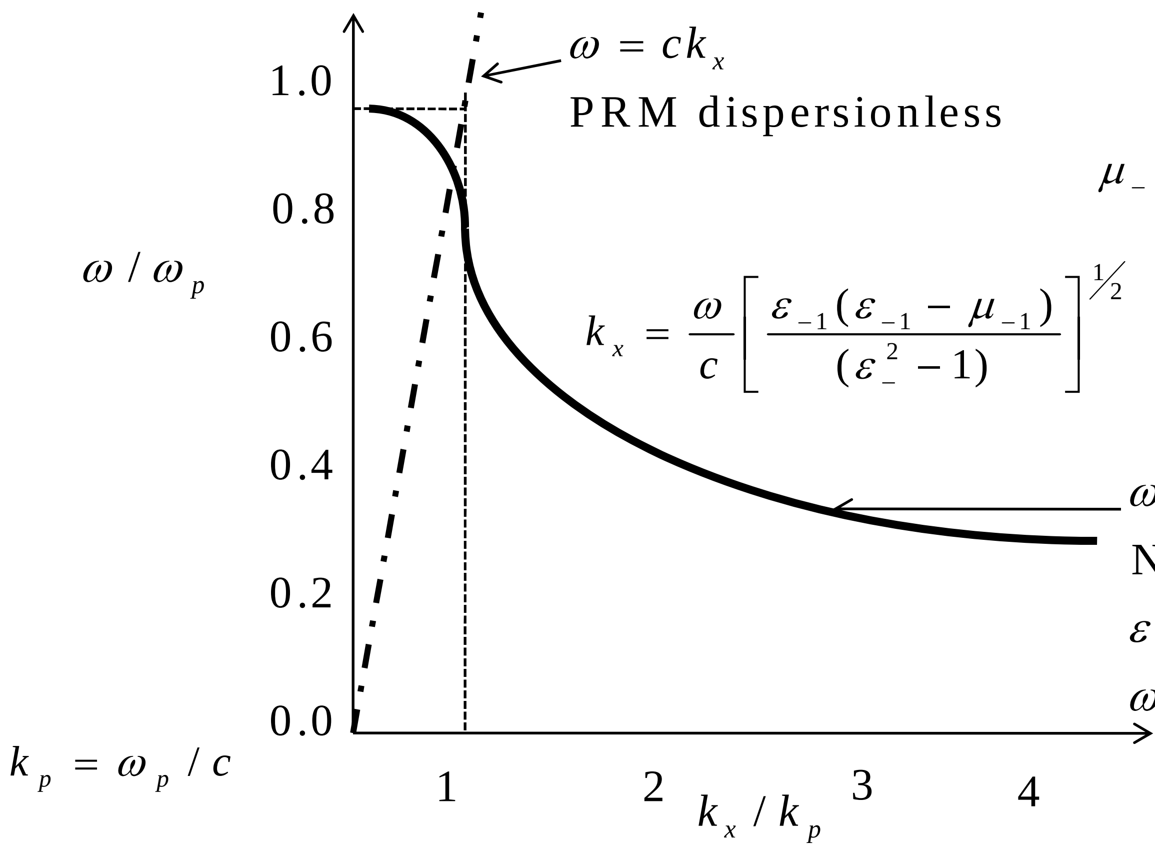
$$\omega = ck_x \quad \text{for} \quad \mu_+ = \varepsilon_+ = 1$$

$$\omega = ck_x / \sqrt{\varepsilon_- \mu_-} = -ck_x / n(\omega) \quad \text{for} \quad \text{ENG } \varepsilon_-(\omega) < 0, \text{MNG } \mu_-(\omega) < 0$$

$$\varepsilon_-(\omega) = 1 - (\omega_p^2 / \omega^2)$$

$$\mu_-(\omega) \cong 1 - (\omega_{mp}^2 / \omega^2)$$



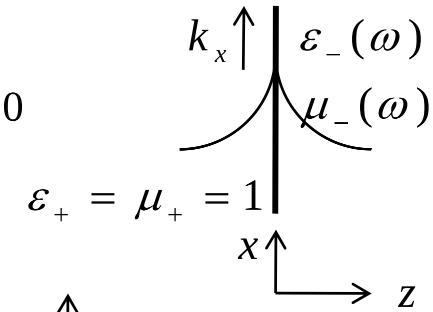


The NRM diagram gives negative transverse group velocity, actually to have NPV, one should take It to negative k_x axis, rotate the figure.

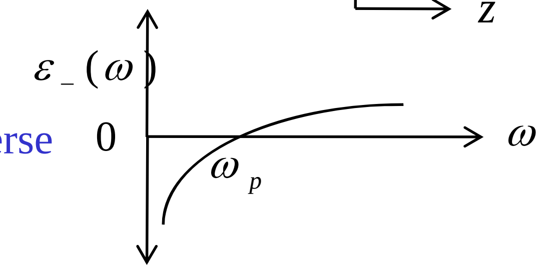
Surface Plasmon dispersion in semi-infinite NRM

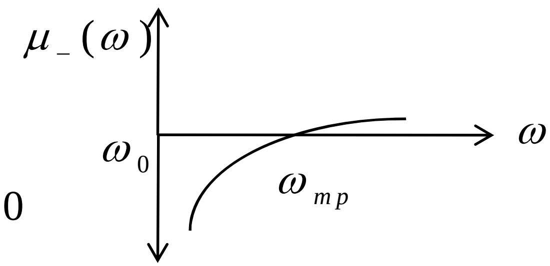
For p-polarized TM case for an interface, the SPP condition is $\frac{k_{z1}}{\varepsilon_+} + \frac{k_{z2}}{\varepsilon_-} = 0$

giving: $k_x = \frac{\omega}{c}\sqrt{\frac{\varepsilon_-(\varepsilon_- - \mu_-)}{(\varepsilon_-^2 - 1)}}$ (Surface Waves Part-3)

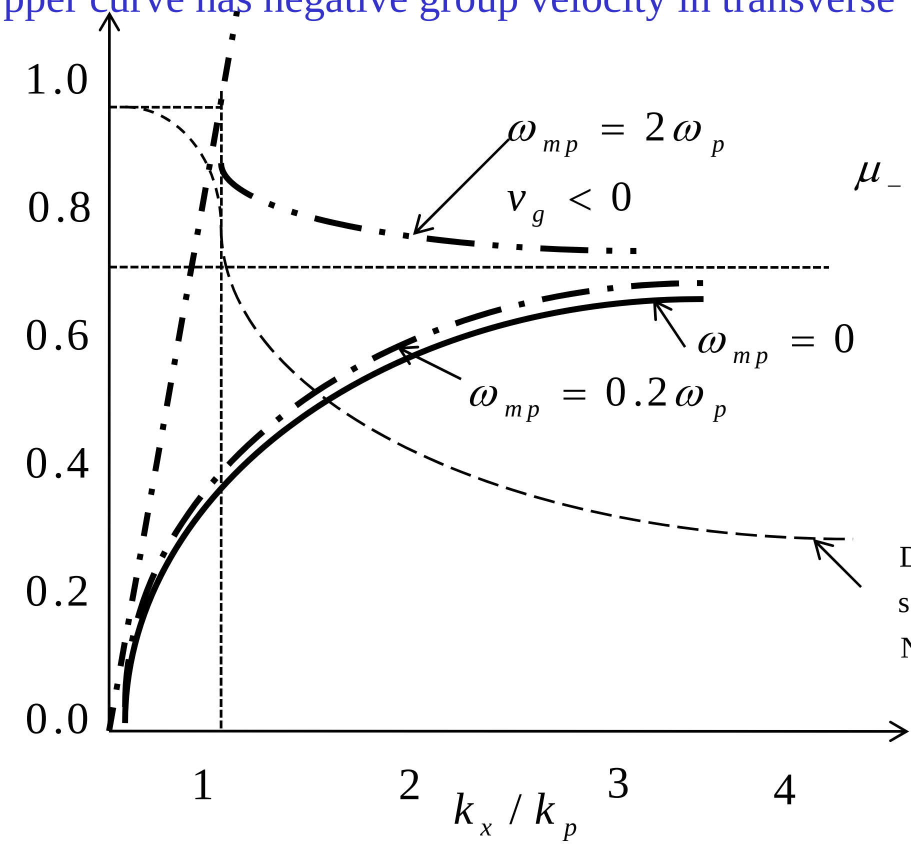


Plasmon dispersion takes different forms $\omega_{mp} > \omega_p$ is upper curve $\omega_{mp} < \omega_p$ is lower. As $k_x \to \infty$ degenerates, plasmon 'stops' at $\omega = \omega_p / \sqrt{2}$. Upper curve has negative group velocity in transverse Direction, this is due to MNG presence





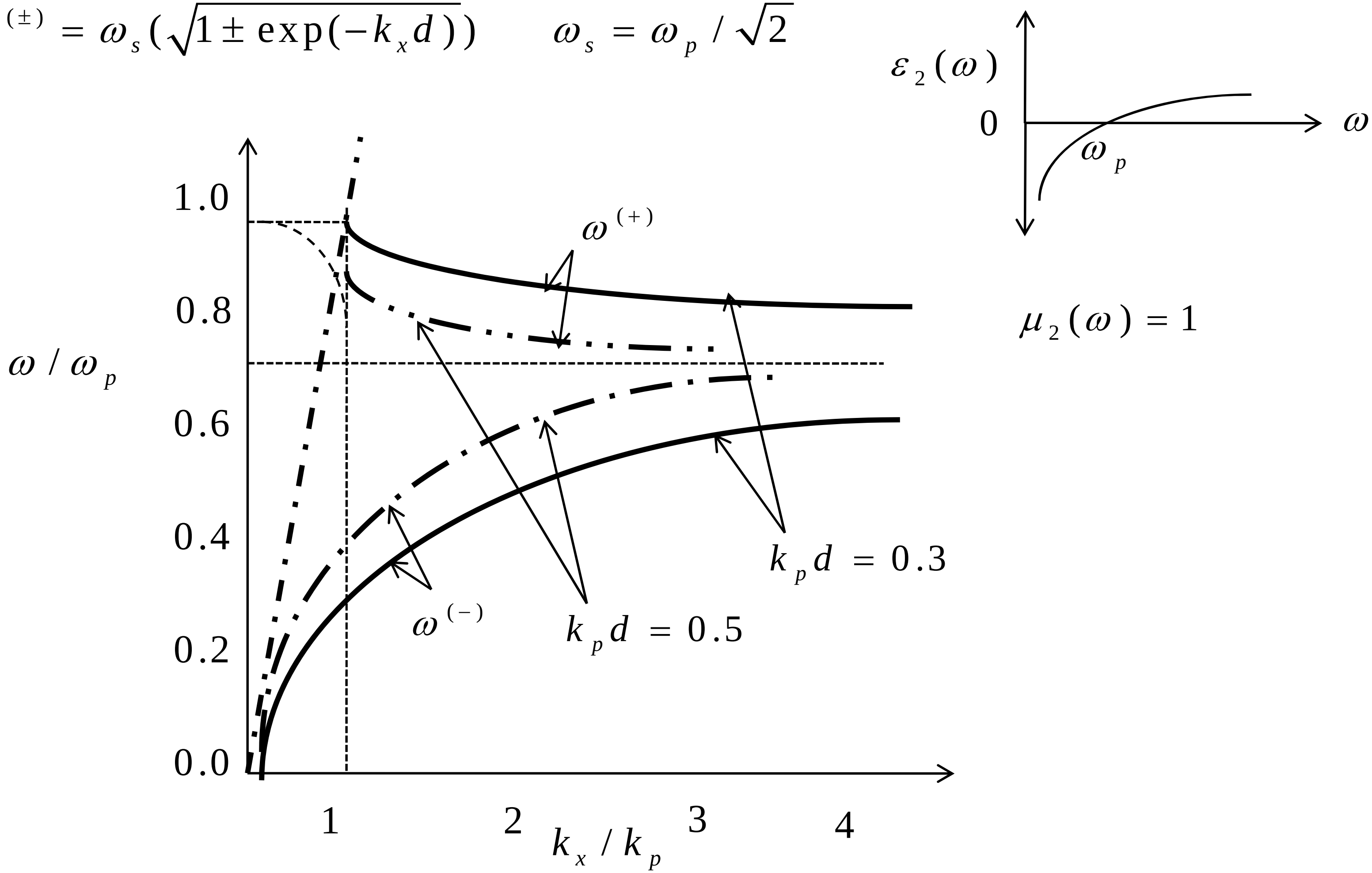
ω / ω_p



For s-polarized should it be other way around i.e. $\omega_{mp} < \omega_p$ upper and $\omega_{mp} > \omega_p$ lower curve.

Surface Plasmon Polariton for slab for ENG-the Slab Plasmon Poalritons

$$\omega^{(\pm)} = \omega_s\left(\sqrt{1 \pm \exp(-k_x d)}\right) \qquad \omega_s = \omega_p / \sqrt{2}$$



Slab-Plasmon-Polariton for NRM with ENG and MNG

The condition for p-polarized TM case are $\tanh(k_z^{(2)} d/2) = -(\varepsilon_2 k_z^{(1)} / k_z^{(2)})$ for symmetric $\omega^{(+)}$ and $\coth(k_z^{(2)} d/2) = -(\varepsilon_2 k_z^{(1)} / k_z^{(2)})$ for anti symmetric $\omega^{(-)}$ mode. Where the wave propagation in *z*-direction is for PRM (1) and NRM (2) is $k_z^{(i)} = \sqrt{k_x^2 - (\varepsilon_i \mu_i \omega^2 / c^2)}$. Again we assume resonance $\varepsilon_2(\omega) = 1 - (\omega_p^2 / \omega^2)$ and $\mu_2(\omega) = 1 - (\omega_{mp}^2 / \omega^2)$. We have seen in earlier section that the finite frequency changes from electrostatic limit we have split curves as

$$\omega^{(\pm)} = \omega_s(\sqrt{1 \pm \exp(-k_x d)}) \qquad \omega_s = \omega_p / \sqrt{2}$$

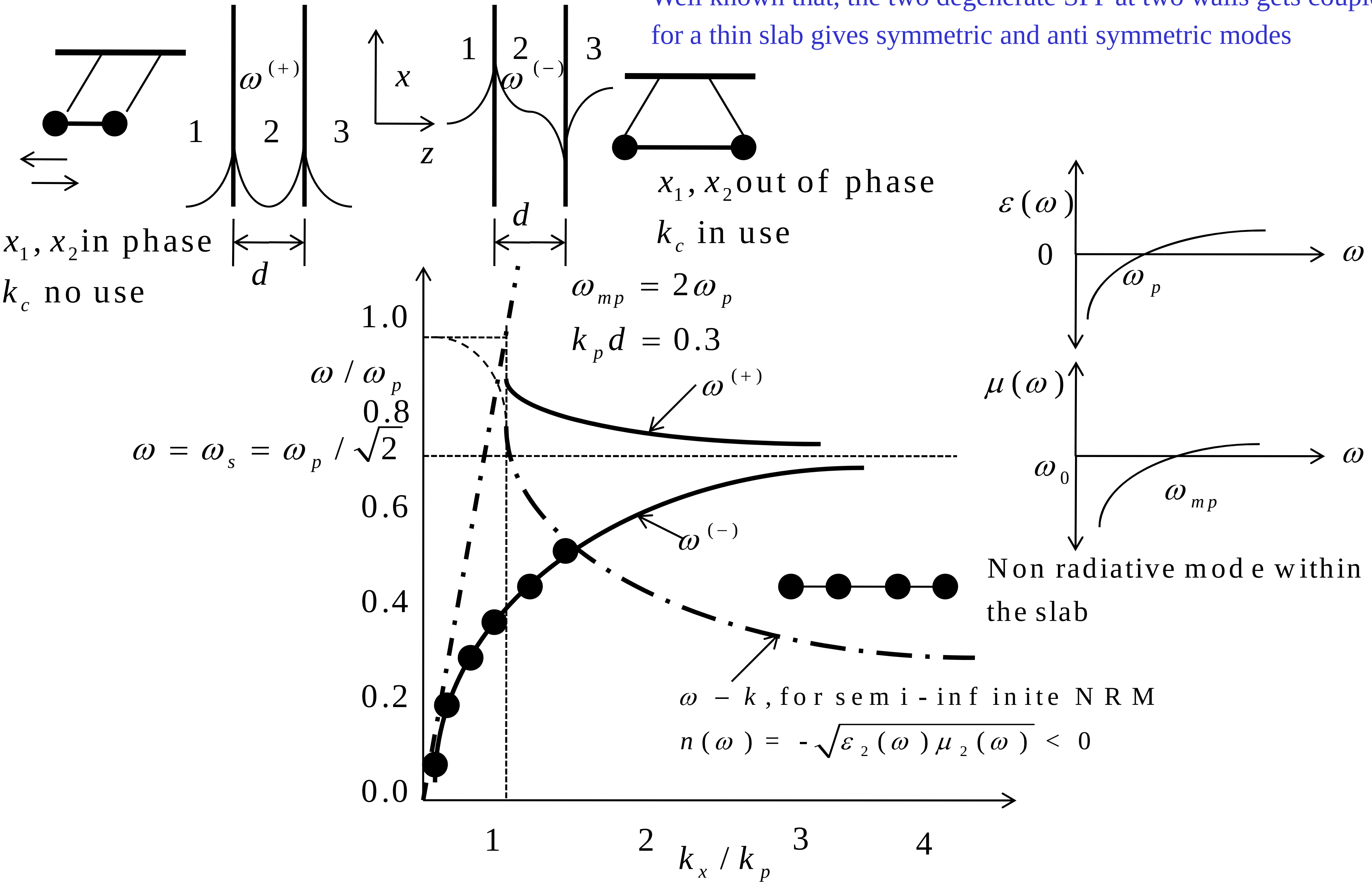
The dispersion relations are qualitatively different for $\omega_{mp} < \omega_p$ or $\mu_2(\omega_s) > -1$ and the other case $\omega_{mp} > \omega_p$ or $\mu_2(\omega_s) < -1$, where $\omega_s = \omega_p / \sqrt{2}$.
Physically, the behavior of both the symmetric and anti symmetric modes at large k_x, have to tend to the uncoupled plasmon dispersion as for single interface.

Interestingly we can have wave guide mode in the slab when ENG and MNG and $|n|$ slab is greater than one. These new wave-guide modes are absent with only ENG or MNG.

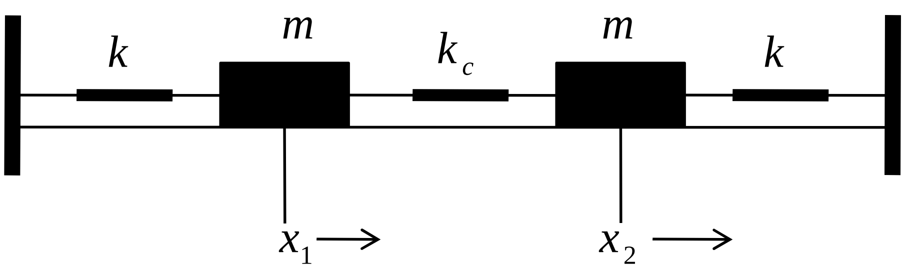
The SPP can have great effect on the scattering properties of materials on the radiation in the vicinity of these surface. The dispersion of these surface modes are function of geometry of the surfaces, sphere NRM will have different modes , than these slab plasmon polaritons.

Nature of Surface Plasmon in thin slab NRM & resonances call it Slab-Plasmon-Polariton

Well known that, the two degenerate SPP at two walls gets coupled for a thin slab gives symmetric and anti symmetric modes



Slab Plasmon Polariton a case of coupled oscillator



$$m\ddot{x}_1 + 2\gamma\dot{x}_1 + kx_1 + k_c(x_1 - x_2) = 0$$
$$m\ddot{x}_2 + 2\gamma\dot{x}_2 + kx_2 + k_c(x_2 - x_1) = 0$$

For a single oscillator $kc = 0$

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$$

$$x_T(t) = x(t) = \begin{cases} Ae^{-\gamma t}\cos(\omega_\gamma t + \delta) & \text{for} \quad \omega_0^2 > \gamma^2 \\ (C_1 + C_2)e^{-\gamma t} & \text{for} \quad \omega_0^2 = \gamma^2 \\ C_1 e^{-\gamma_+ t} + C_2 e^{-\gamma_- t} & \text{for} \quad \omega_0^2 < \gamma^2 \end{cases}$$

$$\omega_\gamma^2 = \omega_0^2 - \gamma^2 \qquad \gamma_\pm = \gamma \pm \sqrt{-\omega_\gamma^2}$$

A, δ, C_1, C_2 depends on initial condition

Driven single oscillator is:

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = A_1 \sin\omega_1 t$$

$$x(t) = x_T(t) + \frac{A_1 \sin(\omega_1 t + \beta)}{\sqrt{(\omega_0^2 - \omega_1^2)^2 + (2\gamma\omega_1)^2}}$$

$$\beta = \arctan\left[(2\gamma\omega_1)/(\omega_0^2 - \omega_1^2)\right]$$

for steady-state

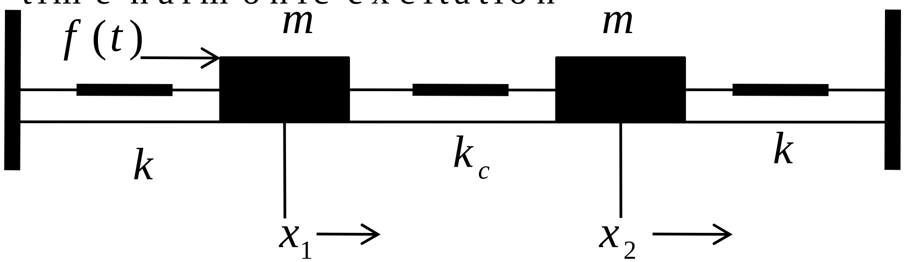
$$x(t) = A\sin(\omega_1 t + \beta) \qquad A = \frac{A_1}{\sqrt{(\omega_0^2 - \omega_1^2)^2 + (2\gamma\omega_1)^2}}$$

phase β is not by initial-condition, but by $\omega_0, \gamma, \omega_1$

Slab Plasmon Polariton a case of coupled oscillator, the first wall is driven

$f(t) = f_0 \exp(-j\omega t)$

time harmonic excitation



$$\ddot{x}_1 + \gamma \dot{x}_1 + \omega_0^2 x_1 + k_c x_2 = f(t)$$

$$\ddot{x}_2 + \gamma \dot{x}_2 + \omega_0^2 x_2 + k_c x_1 = 0$$

steady - state

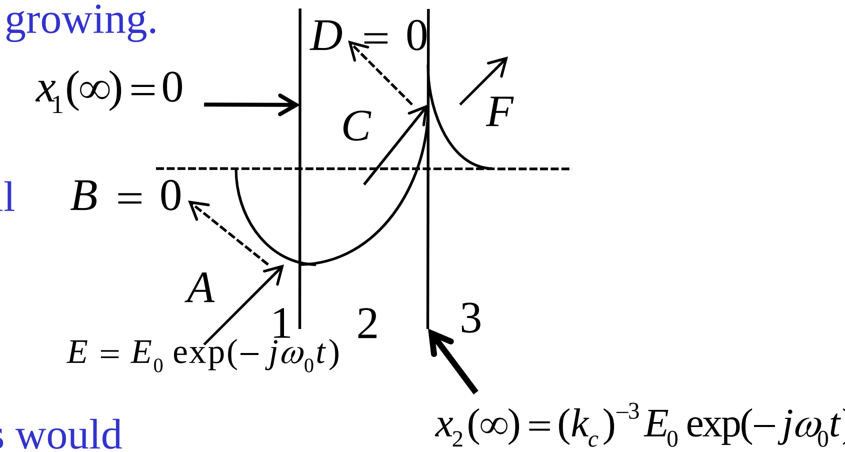
$$x_1 = \frac{(\omega_0^2 - \omega^2 + j\gamma\omega) f_0}{(\omega_0^2 - \omega^2 + j\gamma\omega)^2 - k_c^4}$$

$$x_2 = \frac{-k_c f_0}{(\omega_0^2 - \omega^2 + j\gamma\omega)^2 - k_c^4}$$

Note that forcing the system at resonance ($\omega = \omega_0$), without damping makes second oscillator x_2 non-zero steady state. The x_1 first oscillator does not move at all; and all the excitation is passed to the second oscillator. This corresponds to second wall strongly excited, with zero reflection in our case-ideal case for evanescent waves growing.

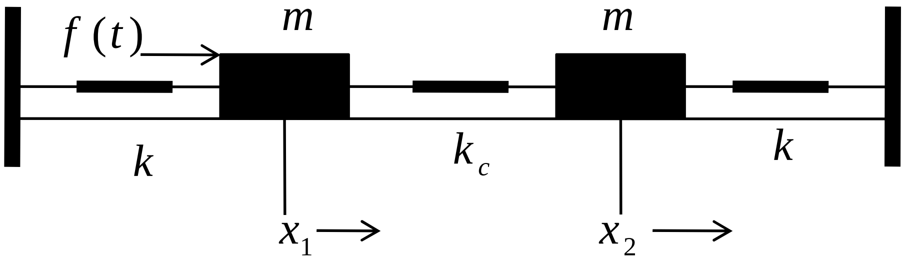
For non-zero dissipation γ the first wall of the slab x_1 is also excited. For very large dissipation γ the second wall x_2 is no longer excited, this corresponds to non-effective growth of the evanescent waves in the NRM slab.

Note that without absorption γ the slab plasmon polaritons would ring indefinitely.



Ideal case of evanescent - wave growth inside NRM

Energy transfer time, temporal evolution of image through NRM



From this scheme we can also consider temporal evolution of the imaging process by considering the rate of energy transferred between the two walls 1 and 2 (equivalent to the two masses coupled here). Note that slab plasmon polariton modes are decoupled from single plasmon case.

Meaning that from single SPP dispersion curve, we have two decoupled curves symmetric and anti symmetric Thus a source with sharp onset has a frequency bandwidth, and would excite them (SPP's). The transmitted field across a loss-less slab then has an amplitude (Gomez-Santos G, 2003 Phys. Rev. Letts.)

$$E_t(t) = \Theta(t) A(t) \exp(-j\omega_0 t) \exp(k_x d) \quad \text{where} \quad A(t) \cong (1/2)(\Delta\omega_{kx} t)^2$$

Where $\Delta\omega_{kx}$ is frequency spacing between slab plasmon polaritons $\omega^{(+)}$, and $\omega^{(-)}$ at a particular k_x. From the curves of slab we see that spacing $\Delta\omega_{kx}$ decreases exponentially as k_x increases. Thus large wave-vector component (corresponding to small wavelength sub-wave length feature of source), are small at short-times; and cut-off times could be approximately defined as $\Delta\omega_{kx} t_{kx} \sim 1$, for every wave vector k_x, before which it does not appreciably contribute to image formation!

Thus initially the image field will be ill defined and will become better and better with time till it reaches optimal resolution (defined by TF cut-off). Also without absorption the SPP will ring indefinitely never allowing the formation of image!

Revisiting perfect flat lens

NRM with ENG and MNG not only propagates radiation but also ‘focuses' evanescent components the (non-propagating ones) of the radiation that are usually confined to immediate vicinity of object (source). The later one with spatial features of the source at a sub wavelength scale and the focusing action of the evanescent is accomplished through surface states (SPP) that reside on NRM surface (walls). Thus there is big role that SPP plays in ideal lens, that surface modes are crucial in the action of perfect lens through NRM. The perfect lens conditions are $\varepsilon_- = -\varepsilon_+$ and $\mu_- = -\mu_+$ for p-polarized TM and s-polarized TE cases. In a specific case at particular ω when both are satisfied, the condition for the surface mode for semi-infinite NRM is:

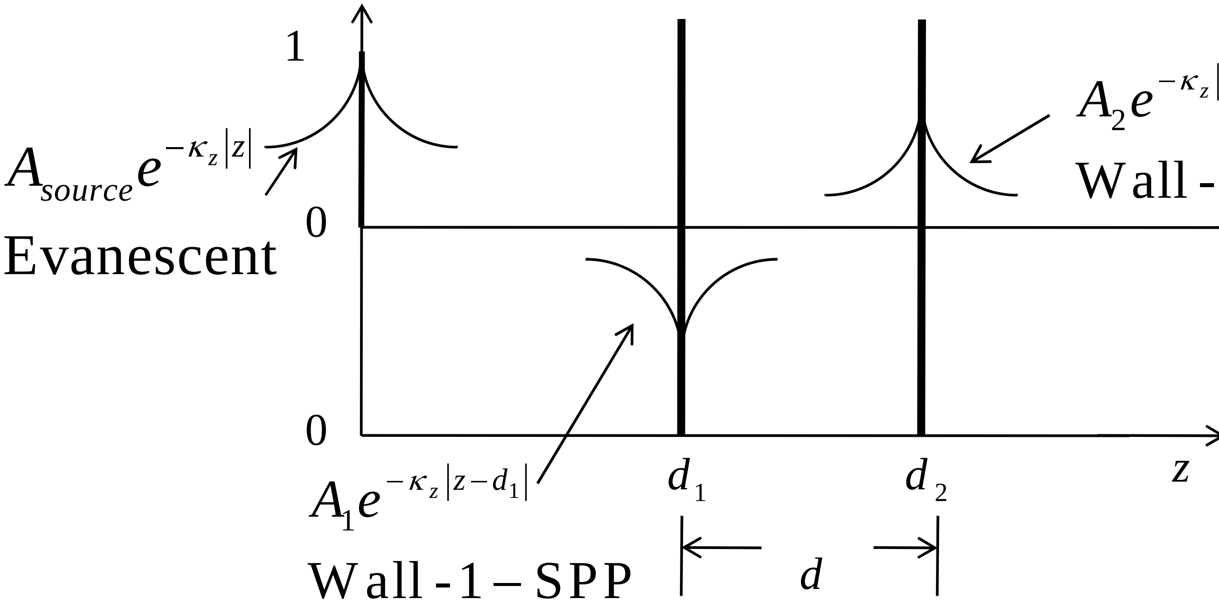
$$\frac{\sqrt{k_x^2 - (\varepsilon_+ \mu_+ \omega^2)/c^2}}{\varepsilon_+} + \frac{\sqrt{k_x^2 - (\varepsilon_- \mu_- \omega^2)/c^2}}{\varepsilon_-} = 0$$

$$\frac{\sqrt{k_x^2 - (\varepsilon_+ \mu_+ \omega^2)/c^2}}{\mu_+} + \frac{\sqrt{k_x^2 - (\varepsilon_- \mu_- \omega^2)/c^2}}{\mu_-} = 0$$

holds for all k_x and SPP type (p-electric and s-magnetic) becomes degenerate at ω . These dispersion less modes has group velocity zero. The total field can be written as sum of field due to source and fields due to these surface modes excitation at any wave vector $j\kappa_z$.

$$E(z) = A_{source} e^{-\kappa_z |z|} + A_1 e^{-\kappa_z |z - d_1|} + A_2 e^{-\kappa_z |z - d_2|}$$

Field summation from source plus SPP of front wall and rear wall of NRM slab



$$A_{source} = 1$$

$$A_1 = -e^{-\kappa_z d_1} = -e^{-\kappa_z d/2}$$

$$A_2 = +e^{-\kappa_z (d_2 - 2d)} = +e^{-\kappa_z d/2}$$

$$d_1 = d/2 \qquad d_2 = 3d/2$$

$$E(z) = A_{source}e^{-\kappa_z|z|} + A_1 e^{-\kappa_z|z-d_1|} + A_2 e^{-\kappa_z|z-d_2|}$$

The presence of other surface (wall-2) ‘detunes’ the SPP resonance on each interface so that they are excited to correct degree, so as to make the field of the surface plasmon exactly cancel the incident field for the zone the $z > d_1$. On the left side of slab the fields of the incident evanescent exactly cancels the field due to the first wall SPP for zero reflectivity. Here the object field is also disturbed at $z = 0$

$$E(0) = 1 - e^{-\kappa_z d/2} e^{-\kappa_z d/2} + e^{-\kappa_z d/2} e^{-3\kappa_z d/2}$$

$$= 1 - e^{-\kappa_z d} + e^{-2\kappa_z d} \neq 1 = A_{source}$$

Field values at several points of the NRM slab

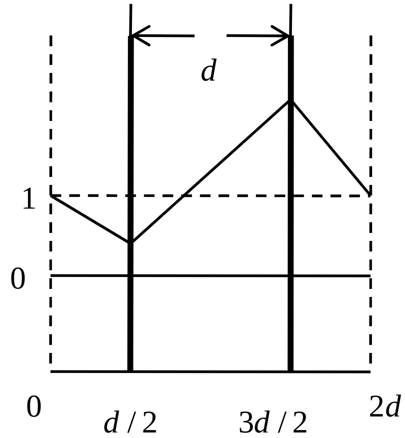
$$E(0) = 1 - e^{-\kappa_z d} + e^{-2\kappa_z d}$$

$$E(d_1) = e^{-3\kappa_z d/2}$$

$$E(d_2) = e^{-\kappa_z d/2}$$

$$E(2d) = e^{-\kappa_z d}$$

$$E(0) = 1 - e^{-\kappa_z d} + e^{-2\kappa_z d} \cong 1 - (1 - \kappa_z d) + (1 - 2\kappa_z d) = 1 - \kappa_z d \cong e^{-\kappa_z d} = E(2d)$$



The object and image plane has same evanescent field, evanescent restoration!

The wall-1 SPP cancels the evanescent field from source inside the NRM slab.

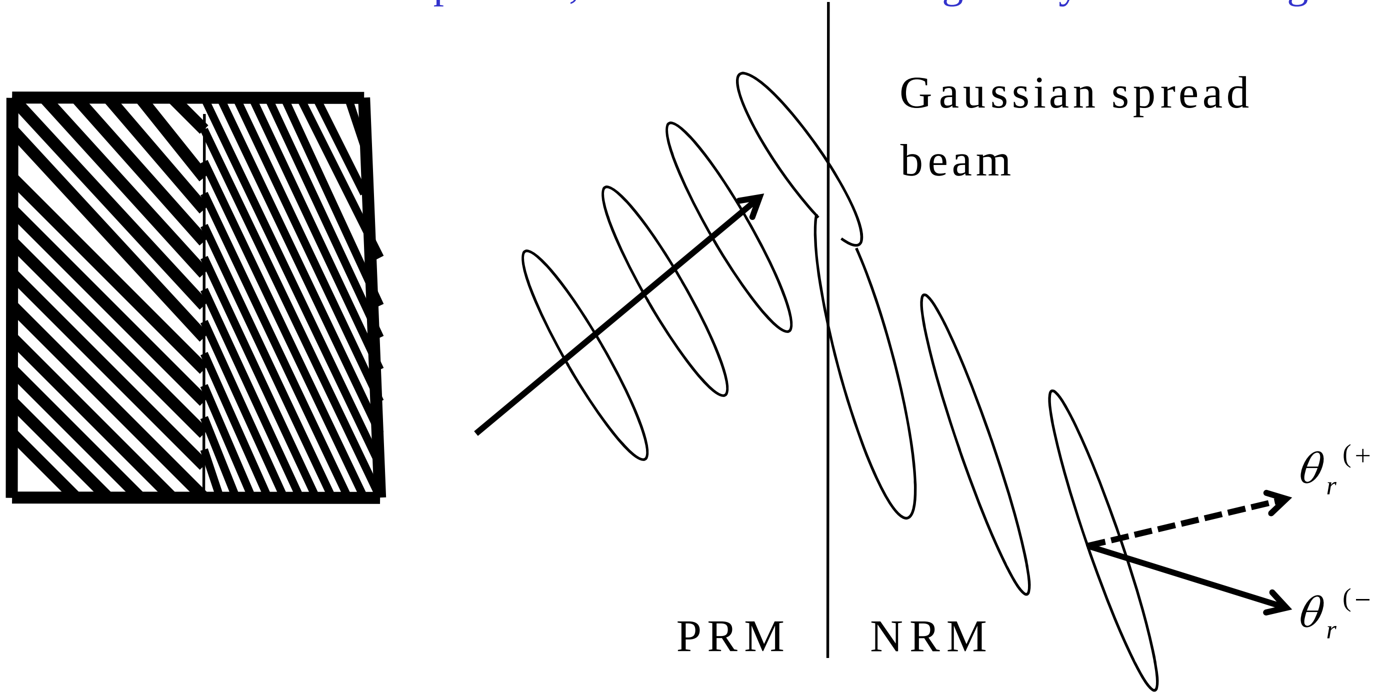
The summing up shows that resultant at the wall-1 is solely due to the SPP at wall-2, and as though the SPP at wall-1 is not existing. ($A_2 e^{-\kappa_z d}$)

It is this coherent action of the SPP that is responsible for perfect lens action

In the ideal case too the source field is disturbed.

NRM and interference front

Refraction of interference front in NRM is positive, while the beam is negatively refracted-a good case of experimentation



The velocity of the interference fronts would coincide with the group velocity only if the waves have wave vector in the same direction, with slightly different frequencies. Due to dispersion in NRM $\varepsilon_-(\omega);\mu_-(\omega)$, the two waves with different ω would refract at slightly different angles. Now the constitutive waves of interference patterns in NRM no longer point in the same direction and corresponding interference front no longer propagates along the direction of group velocity.

Beams suffer negative refraction while the interference patterns suffers positive refraction

Some related topics for NRM research or formulation

1. Energy velocity formulation $v_E = S / u$, with $S = E \times H$ and *u* as average EM Energy Density; difficult to define in dispersive media like NRM.

2. Negative Refraction of beam (energy) experience a long delay at the interface; one has to give time for the both the wall's surface modes to excite!!.

3. Discontinuity in the phase front at the interface but the discontinuity formation needs be causal!!

4. Group delay of pulse propagating through a slab of NRM, advance retard? For a narrow band pulse group delay is $\tau_\Phi = \partial \Phi(\omega) / \partial \omega \big|_{\omega_c}$ where $\Phi(\omega)$ is phase of transmission or reflection coefficient and ω_c is the carrier frequency . At what say $\omega_{mp} < \omega < \omega_p$ or $\omega_p < \omega < \omega_{mp}$ the delay is superluminal!! or negative group delay , the physics of negative group delay in NRM.

End of Part-6