Wave-Mechanics and Circuit Theory
for
Left Handed Maxwell Systems

Part-2

Few Terms for Left Handed Maxwell Systems

NPV: **Negative Phase Velocity**

DNG: **Double Negative Material**

LHM: **Left Handed Material**

NGV: **Negative Group Velocity**

NRI: **Negative Refractive Index**

NGD: **Negative Group Delay**

PRI/NRI: **Positive Refractive Index/Negative Refractive Index**

METAMATERIALS: **Composite structures made of naturally occurring materials, specifically designed to obtain 'anomalous' Electromagnetic Properties, not found commonly in nature.**

LEFT HANDED MAXWELL SYSTEMS
is an Young Subject
recently born in XXI century-will make several new theories

Right Handed Maxwell System

For $\mu > 0, \varepsilon > 0$

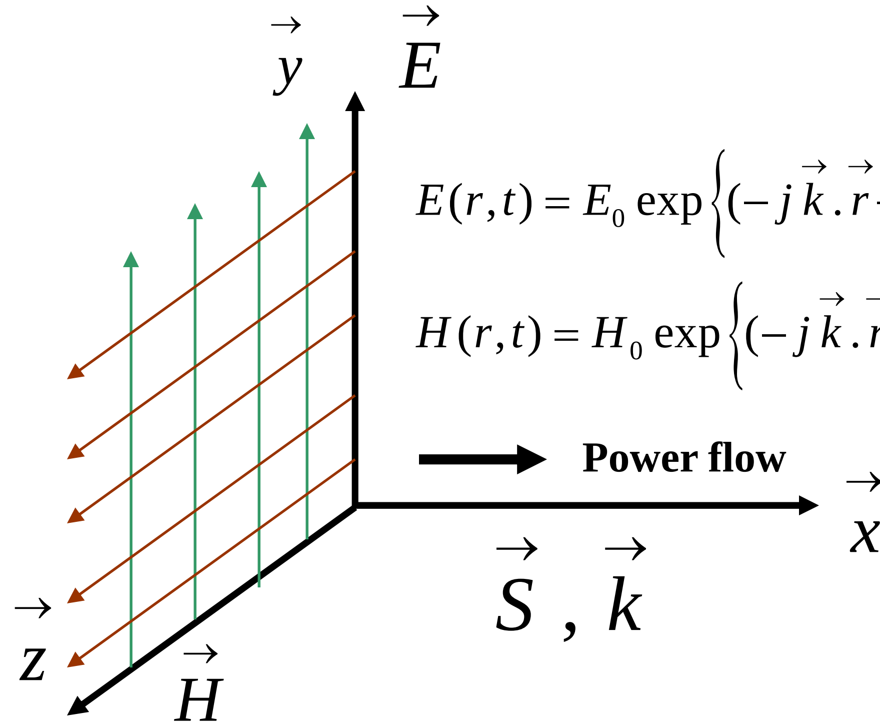
$$\nabla \times \vec{E} = -j\omega\mu \vec{H}$$

$$\nabla \times \vec{H} = j\omega\varepsilon \vec{E}$$

$$\vec{S} = \left(\vec{E} \times \vec{H}\right)$$

$$\vec{k} \times \vec{E} = +\omega\mu \vec{H}$$

$$\vec{k} \times \vec{H} = -\omega\varepsilon \vec{E}$$



$$E(r,t) = E_0 \exp\left\{(-j\vec{k}.\vec{r} - j\omega t)\right\}$$

$$H(r,t) = H_0 \exp\left\{(-j\vec{k}.\vec{r} - j\omega t)\right\}$$

k is propagation eigen mode (vector) from which attenuation & phase constants appear in TL theory.

$\vec{k}$ & $\vec{S}$ are in same direction. Thus phase velocity & Poynting Vector are in same direction. An effective medium explanation in large scales

Left handed Maxwell System

For $\mu < 0, \varepsilon < 0$

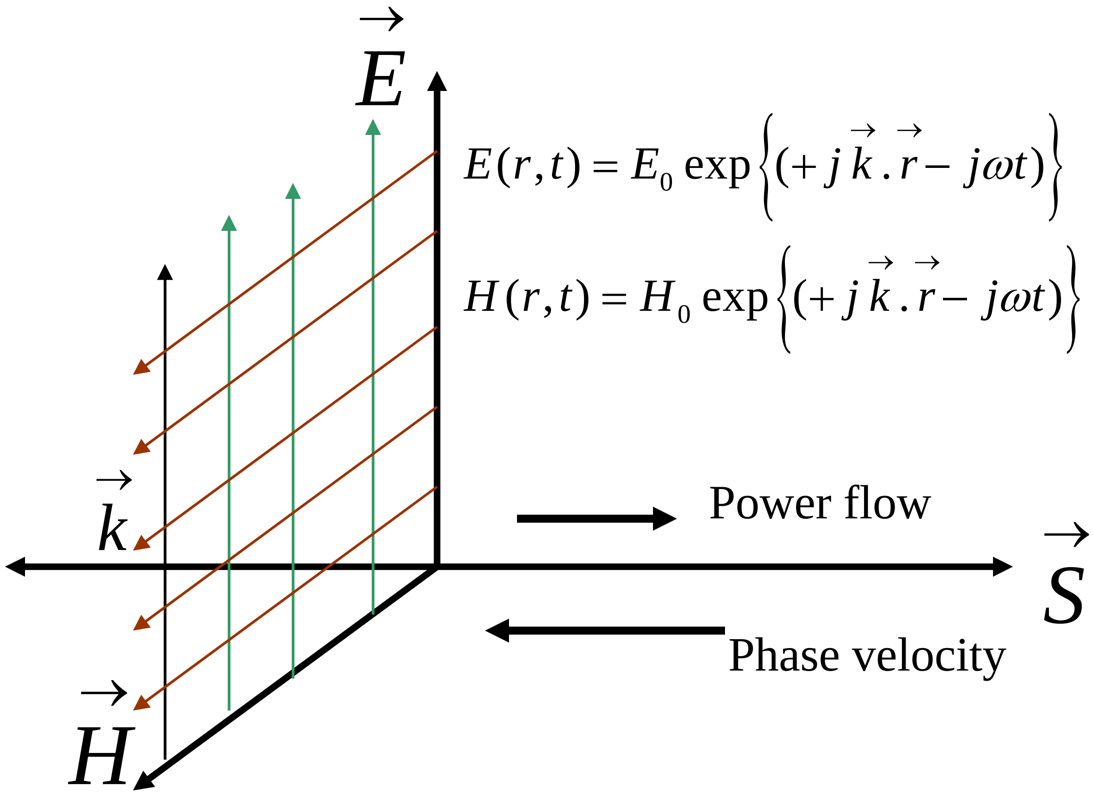
$$\vec{k} \times \vec{E} = -\omega \mu \vec{H}$$

$$\vec{k} \times \vec{H} = +\omega \varepsilon \vec{E}$$

$$\vec{S} = \frac{1}{2}\left(\vec{E} \times \vec{H} \right)$$

$$\nabla \times \overline{E} = +j\omega \mu \overline{H}$$

$$\nabla \times \overline{H} = -j\omega \varepsilon \overline{E}$$



$\vec{k}$ (hence the phase velocity)is opposite to the direction of power flow $\vec{S}$.
These system (LHM) support waves with phase velocity negative, or backward waves, also are DNG Doubly Negative Material

…..AMBIDEXTEROUS……

DNG Interpretation macroscopically

1. We may admit that the properties of substance are actually not affected by simultaneous change of sign of effective permeability & permittivity.
2. It might be for permeability & permittivity to be simultaneously negative contradicts some fundamental laws of nature. Therefore no substance of DNG type exists.
3. It could be admitted that substances with DNG type have some different properties (different from substances with permeability & permittivity, positive)

Taking point 3 the result of backward wave where plane wave propagation is, opposite to the direction of energy flow, does not follow from the wave equation, which remains unchanged for DNG

$$\nabla^2 \begin{pmatrix} \overline{E} \\ \overline{H} \end{pmatrix} + \overline{k}^2 \begin{pmatrix} \overline{E} \\ \overline{H} \end{pmatrix} = 0$$

Where $k^2 = \omega^2 \mu \varepsilon$

But comes individual Curl equations, can be formed by Left-Hand

$$\nabla \times \overline{E} = + j\omega\mu \overline{H} \qquad \vec{k} \times \vec{E} = -\omega \mu \vec{H}$$

$$\nabla \times \overline{H} = - j\omega\varepsilon \overline{E} \qquad \vec{k} \times \vec{H} = +\omega \varepsilon \vec{E}$$

Poynting vector tells direction of power flow (group-velocity) and wave vector tells the direction of phase velocity. Opposite means “Backward-Wave”

Is there fundamental contradiction with Maxwell, For DNG substances?

The dot product of Poynting Vector and wave-vector is negative, for LHM

That is $\overline{S}\,.\overline{k} < 0$;Poynting vector is $\overline{S} = \overline{E} \times \overline{H}$

However, it is to be noted that there is no fundamental contradiction as Maxwell's equations do not indicate the dot product of the two must be Positive, thus it is possible to have DNG and a Left Handed Maxwell System.

DNG system with $\mu < 0$ and $\varepsilon < 0$ gives $n < 0$ where $n = \pm\sqrt{\varepsilon\mu}$

Negative refractive index gives interesting " Canceling Scattering-Properties" of other materials as gives reversal of path rays of EM signals.

For experimentation and studies on negative refraction-salient points

1 . Dispersion is the basic concept. In any Wave phenomena one must know about plot of wave number VS frequency, $\omega - k$ diagram. The '**wave number**', (k) is called wave number, complex propagation-constant, wave vector.

2. Talking about dispersion necessitates the introduction of forward and "backward" waves. In classical electrodynamics the properties of "backward-waves" rarely were emphasized or studied.

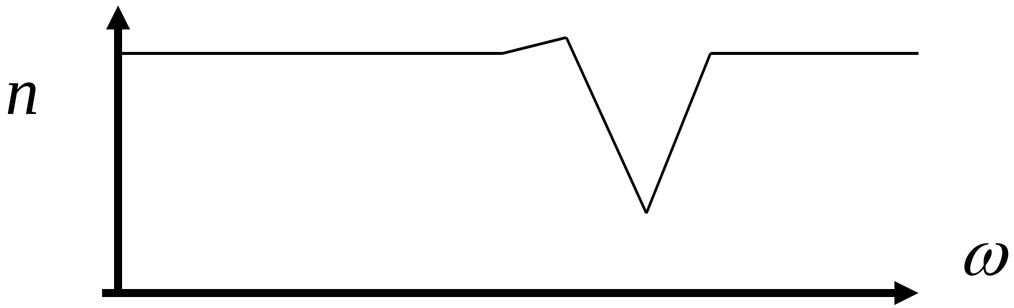
3. In LHM the "back-ward" wave has risen to fame!!

4. Very often we shall have to look at the LHM phenomena both from point of view of "field-theory" and "circuit-theory". SRR-WA from field theory and PLTL from circuit theory, in realizing hardware of LHM.

5. The building blocks in most of the cases are "resonant-elements" much smaller in wavelength of EM Wave shining it.

Anomalous Dispersion

This is what is seen in our negative refraction systems with LHM. The anomalous dispersion is old phenomena, from 19^{th} century it is observed, when absorption spectra of various materials were studied in optical region, where variation of refractive index close to absorption peaks show slope as negative.



To have NGV can this negative slope be sufficient ? Not necessarily.

$$n = \frac{c}{v_p} = \frac{c}{\omega / k} = \frac{ck}{\omega}$$

$$\frac{dn}{d\omega} = -\frac{ck}{\omega^2} + \frac{c}{\omega}\frac{dk}{d\omega} = \frac{c}{\omega}\left(-\frac{k}{\omega} + \frac{dk}{d\omega}\right)$$

$$v_g = \frac{d\omega}{dk} = \frac{c}{n + \frac{dn}{d\omega}}$$

Not only slope *dn/dw* should be negative but should be negatively large.

Possible to have NGV/NGD in PLTL

Newton's expression for electrical conductivity

The most basic is the equation of motion of Newton, for an electron of mass *m* and charge *e*, with a damping τ term in electric field *E* is:

$$m\left(\frac{dv}{dt}+\frac{v}{\tau}\right)=eE \qquad v=\frac{e}{m}\frac{E}{j\omega+\frac{1}{\tau}}$$

With temporal variation re-written as $d/dt \rightarrow j\omega$, i.e. temporal form varying as $\exp(j\omega t)$ and the current density as $J=\sigma E$ which may be written as

$$J=Nev=\frac{Ne^2\tau}{m}\frac{E}{1+\omega\tau}$$

Where

$$\sigma=\frac{\sigma_0}{1+j\omega\tau};\sigma_0=\frac{Ne^2\tau}{m}$$

Maxwell's equations are

$$\nabla\times\overline{H}=\overline{J}+j\omega\overline{D}$$

$$\nabla\times\overline{H}=J+j\omega\varepsilon\overline{E}=j\omega\varepsilon_{eff}\overline{E}$$

$$\nabla\times\overline{E}=-j\omega\overline{B}$$

$$\nabla.\overline{D}=\rho;\nabla.\overline{B}=0$$

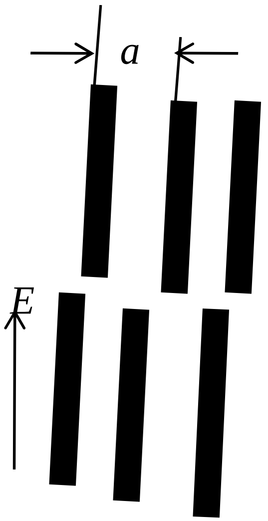
$$\overline{D}=\varepsilon\overline{E},;B=\mu\overline{H}$$

$$\varepsilon_0=8.85\times10^{-12}\,\mathrm{As/Vm};\mu_0=4\pi\times10^{-7}\,\mathrm{Vs/Am}$$

The effective epsilon in presence of current and effective epsilon for wire medium

Write the first Maxwell equation RHS as $J + j\omega\varepsilon\bar{E} = j\omega\varepsilon_{eff}\bar{E}$ with which effective ε_{eff} is defined. But $J = \dfrac{\sigma_0}{1+\omega\tau}\bar{E}$; which when substituted above we find that

$$\varepsilon_{eff} = \varepsilon + \frac{\sigma_0}{j\omega}\frac{1}{1+j\omega\tau}$$



In low frequency limit we get: $\varepsilon_{eff} = \varepsilon - j\sigma_0/\omega$

In high frequency limit we get : $\varepsilon_{eff} = \varepsilon_0(1-\omega_p^2/\omega^2);\ \omega_p^2 = Ne^2/\varepsilon_0 m$

(by taking $\varepsilon = \varepsilon_0$)

An incident electric field, *E* parallel to the wires, separated by gap *a* will give

$$I = \frac{Ea}{Z_w} \qquad Z_w = R_w + j\omega L_w$$

Average current density A/cm*cm, in this unit cell that has an area is:

Using above method of finding the effective epsilon

$$\varepsilon_r = 1 + \frac{1}{j\omega}\varepsilon_0(R_w + j\omega L_w)a \qquad J_{av} = \frac{E}{(R_w + j\omega L_w)a}$$

Defining now

$\omega_p^2 = 1/\varepsilon_0 a L_w$, we may write $\varepsilon_r = 1 + \dfrac{\omega_p^2}{\omega^2 - j(\omega/\tau_w)}$, where losses are characterized as time constant

$$\tau_w = L_w/R_w$$

Expression for $R_w = a/\pi r_w^2\sigma_0;\ L_w = (\mu_0/2\pi)\left[\ln(2a/r_w) - 0.75\right]$

$$a = 6\,\mathrm{mm};\ r_w = 0.03\,\mathrm{mm};\ \sigma_0 = 5.8\times10^7\,\mathrm{S/m}$$

$$f_p = 8.73\,\mathrm{GHz}$$

$$\tau_w = 2.24\times10^{-8}\,\mathrm{s}$$

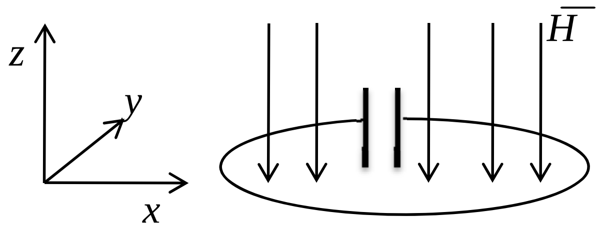
Plasma frequency from different angle

The plasma frequency $\omega_p^2 = 1/\varepsilon_0 a L_w$ for group of wires by physicists is written in form as $\omega_p^2 = e^2 N_{eff} / \varepsilon_0 m_{eff}$ It is same as $\omega_p^2 = N e^2 / \varepsilon_0 m$ but instead of actual electron density *N* , and actual electron mass *m*, the physical quantities appearing here are average electron density N_{eff} in the unit cell and the effective mass of the electron, m_e so that $\omega_p^2 = 1/\varepsilon_0 a L_w$ is satisfied.

According to Pendry et al. the reduction of plasma frequency may be interpreted as due partly to a decrease in effective electron density and partly to an increase in effective mass, and that increase may amount to four orders of magnitude. It is an interesting proposal but there is some inconsistency in it.

If the mass of the electron has increased reducing thereby the plasma frequency then increased mass must have also reduced the conductivity. A decrease in conductivity by four orders magnitude would make the losses enormous .

The effective permeability (resonator)



The flux threading the loop is $\mu_0 S H$ with S as area. Then the average voltage excited in loop is $-j\omega\mu_0 S H$

Current is $I = -j\omega\mu_0 S H / Z$, with loop impedance

$$Z = j\omega L + (1/j\omega C) + (1/R)$$

The induced magnetic moment is $m = \mu_0 S I = -(j\omega\mu_0^2 S^2 H)/Z$

polarizability electric/magnetic definition is: $\overline{p} = \alpha_e \overline{E}$ and $\overline{m} = \alpha_m \overline{H}$

For N such rings

$$M_m = N m = N \alpha_m H \quad \ldots\ldots\ldots\ldots(1)$$

Therefore $\alpha_m = -(j\omega\mu_0^2 S^2)/Z \quad \ldots\ldots\ldots\ldots(2)$

Definition of relative permeability is:

$$\mu_r = \frac{B}{\mu_0 H} = \frac{\mu_0 H + M_m}{\mu_0 H} = 1 + \frac{M_m}{\mu_0 H} \quad \ldots\ldots\ldots\ldots(3)$$

With aid of (1) (2) (3)

$$\mu_r = 1 - \frac{\mu_0 N S^2}{L\left(f_r - \frac{j}{Q}\right)} = 1 - \frac{F}{f_r - \frac{j}{Q}}$$

$$Q = \frac{\omega L}{R};\ f_r = 1 - \frac{\omega_0^2}{\omega^2},\ \omega_0^2 = \frac{1}{LC};\ F = \frac{\mu_0 N S^2}{L}$$

With little extra algebra another form is $\mu_r = (1 - F)\dfrac{\omega^2 - \omega_F^2}{\omega^2 - \omega_0^2}$ $\quad \omega_F = \omega_0 / \sqrt{1 - F}$

The losses in the loop (resonator)

Magnetic polarizability in the loop is $\overline{m} = \alpha_m \overline{H}$

$$\left(\frac{1}{\alpha_m}\right) = \left(\frac{1}{\alpha_m}\right)_{lossless} + \left(\frac{1}{\alpha_m}\right)_{ohmic} + \left(\frac{1}{\alpha_m}\right)_{radiation}$$

$$\left(\frac{1}{\alpha_m}\right)_{lossless} = -\frac{f_r L}{\mu_0^2 S^2} \qquad f_r = 1 - \frac{\omega_0^2}{\omega^2}$$

$$\left(\frac{1}{\alpha_m}\right)_{ohmic} = -\frac{jL}{Q\mu_0^2 S^2} \qquad Q = \frac{\omega L}{R}$$

$$\left(\frac{1}{\alpha_m}\right)_{radiation} = -\frac{jk_0^3}{6\pi\mu_0}$$

Measure of radiation due to magnetic dipole

The radiation loss is also called radiation damping, similar to electric dipole radiation damping which is $-\dfrac{jk_0^3}{6\pi\varepsilon_0}$. Radiation loss can be taken as radiation resistance

$$R_S = \frac{\pi}{6}\eta_0\left(\frac{2\pi r_0}{\lambda}\right)^4$$

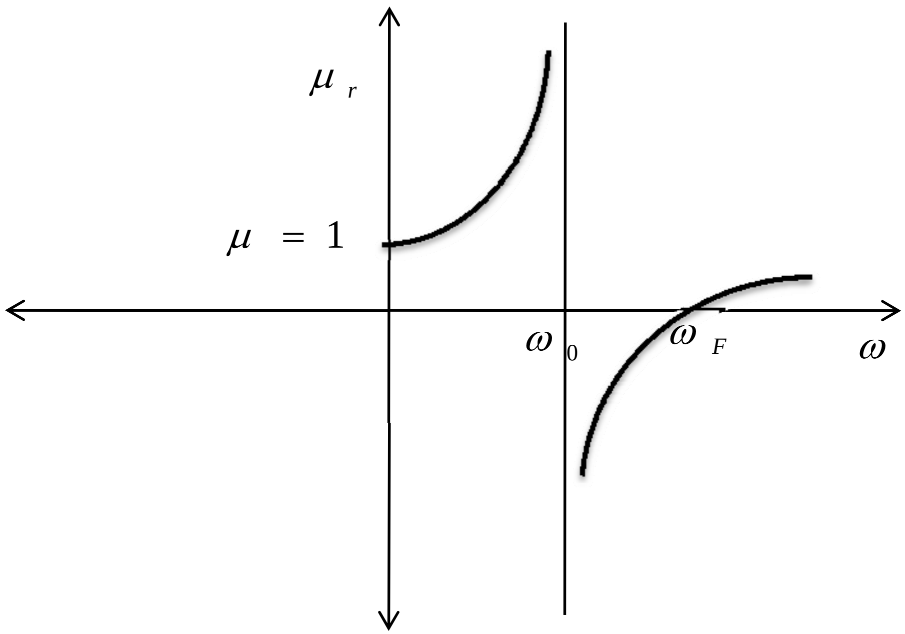
r_0 Loop radius; be added to ohmic R

The dimensions of meta-materials are small relation wavelength, hence not so significant.

The negative effective permeability of resonator

$$\mu_r = (1 - F)\frac{\omega^2 - \omega_F^2}{\omega^2 - \omega_0^2} \qquad \omega_F = \omega_0 / \sqrt{1 - F}$$

The observation in above obtained expression, is pole at ω_0 and a zero at ω_F .



The interesting fact here is that between the poles and zero the permeability is negative. How wide is the range:

$$\Delta\omega_{neg.\mu} = \omega_0\left(\frac{1}{\sqrt{1-F}} - 1\right) \cong \omega_0 \frac{F}{2}$$

In this discussion, the effect of mutual inductances of other elements ignored.

The negative effective permeability of resonator with mutual inductances

Consider into account that all other elements also contribute to the magnetic flux at element *n*. The total flux is:

$$\Phi = \mu_0 S H + I \sum M_{nn'}$$

The $M_{nn'}$ is mutual inductance between n and n' elements and I is assumed to be same in all the elements. The corresponding current must satisfy:

$$I = -\frac{j\omega}{Z}\left[\mu_0 S H + I \sum M_{nn'}\right]$$

Effective permeability is:

$$\mu_r = 1 - \frac{F}{f_r + \Delta f_r - \frac{j}{Q}} \qquad \Delta f_r = \frac{1}{L}\sum M_{nn'}$$

Split-ring resonator and Wire-Array based structures in LHM

Negative Permittivity: An array of thin strips of metallic wires by virtue of collective plasma like behavior produce effective negative dielectric permittivity. This wire array (WA) gives negative permittivity below plasma frequency.

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}, \omega_p = \frac{Ne^2}{\varepsilon_0 m_e} \qquad \omega < \omega_p, \varepsilon < 0$$

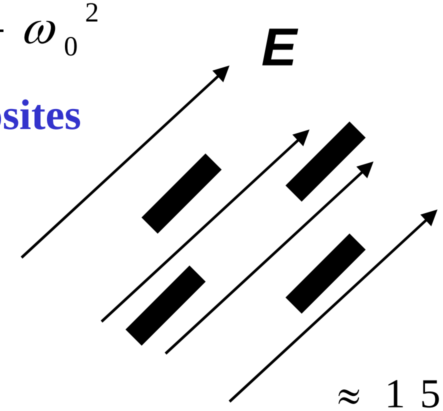
Negative Permeability: By C-shaped rings where the ring acts as inductance and open section acts as capacitance; results in magnetic resonance. This split ring resonator (SRR) may be imagined as magnetic charges exhibiting magnetic plasma frequency , below which permeability is negative. (In only a band)

Rather all resonances exhibit negative permeability or permittivity

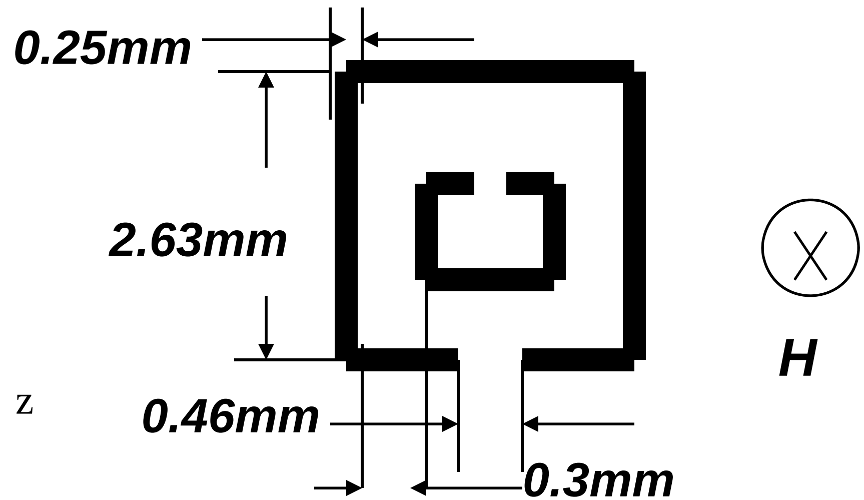
$$\mu_r = (1 - F)\frac{\omega^2 - \omega_F^2}{\omega^2 - \omega_0^2}$$

Metamaterial composites should be accessible to *E* & *H* of the EM Wave shining it.

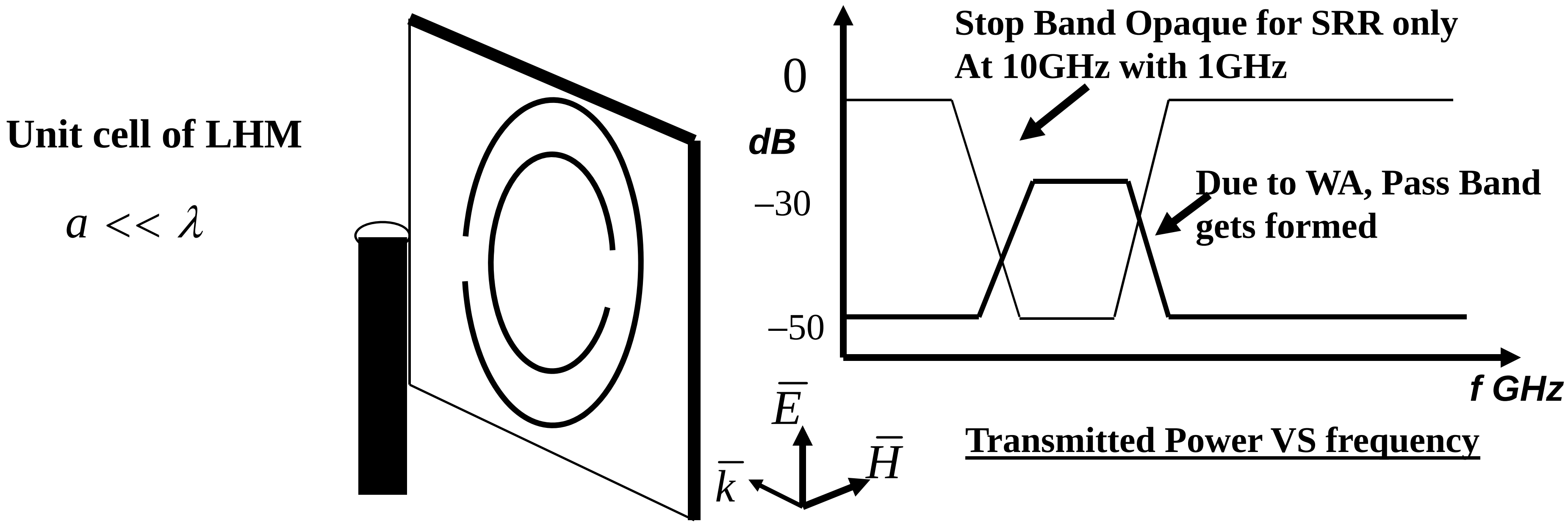
$$\lambda >> a$$



$\approx 15\,GHz$



Stop-Band of Resonance of SRR to Pass Band Change for EM Signals With the WA in proximity



Stop band due to SRR (only), resonance turns into pass band, when WA is at proximity. Note that attenuation of Transmitted signal is high in total Pass-Band, is due to considerable absorption by metal rods.

However, stop-band turning into pass band proves that material with DNG can propagate EM Waves.

Explanation from Wave Equation

From Maxwell's curl equation one can derive Wave-Equation for a source free zone:

$$\nabla^2 \begin{pmatrix} E \\ H \end{pmatrix} + \varepsilon \mu \frac{\omega^2}{c^2} \begin{pmatrix} E \\ H \end{pmatrix} = 0$$

Similar to Schrödinger's Wave Equation in Potential free zone:

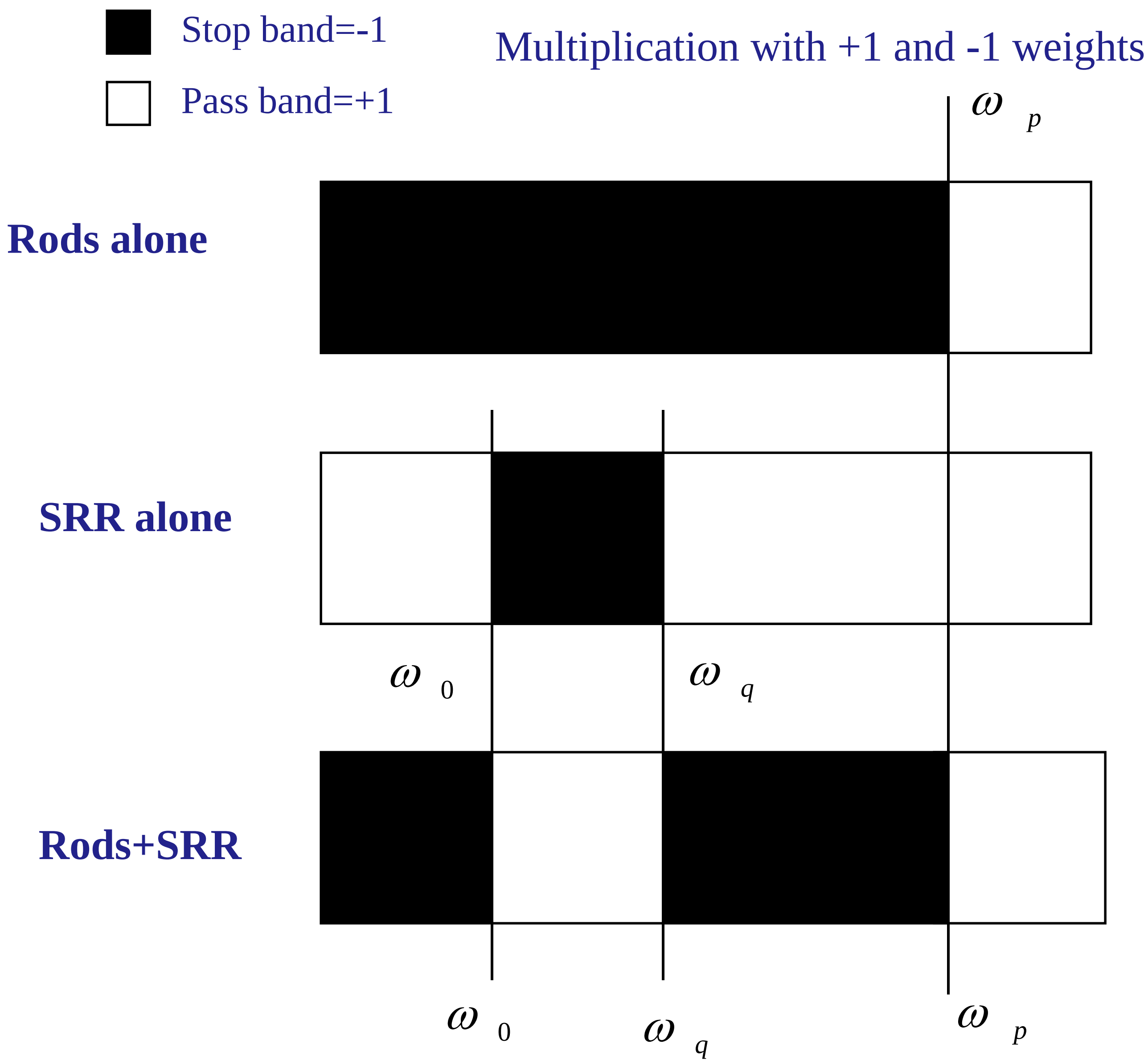
$$\frac{\hbar}{2m} \frac{\partial^2 \psi}{\partial x^2} + E\psi = 0 \qquad \text{for} \qquad V(x) = 0$$

If the product of permittivity and permeability is negative then Traveling wave Solution, does not exist. The absence of Harmonic Solution($Ae^{(\pm jk.r)}$)means waves do not propagate-the solution is exponential then-($Ae^{(\pm k.r)}$).This means Non-Radiating Bound State having no propagating component, in' E'X'H' direction?

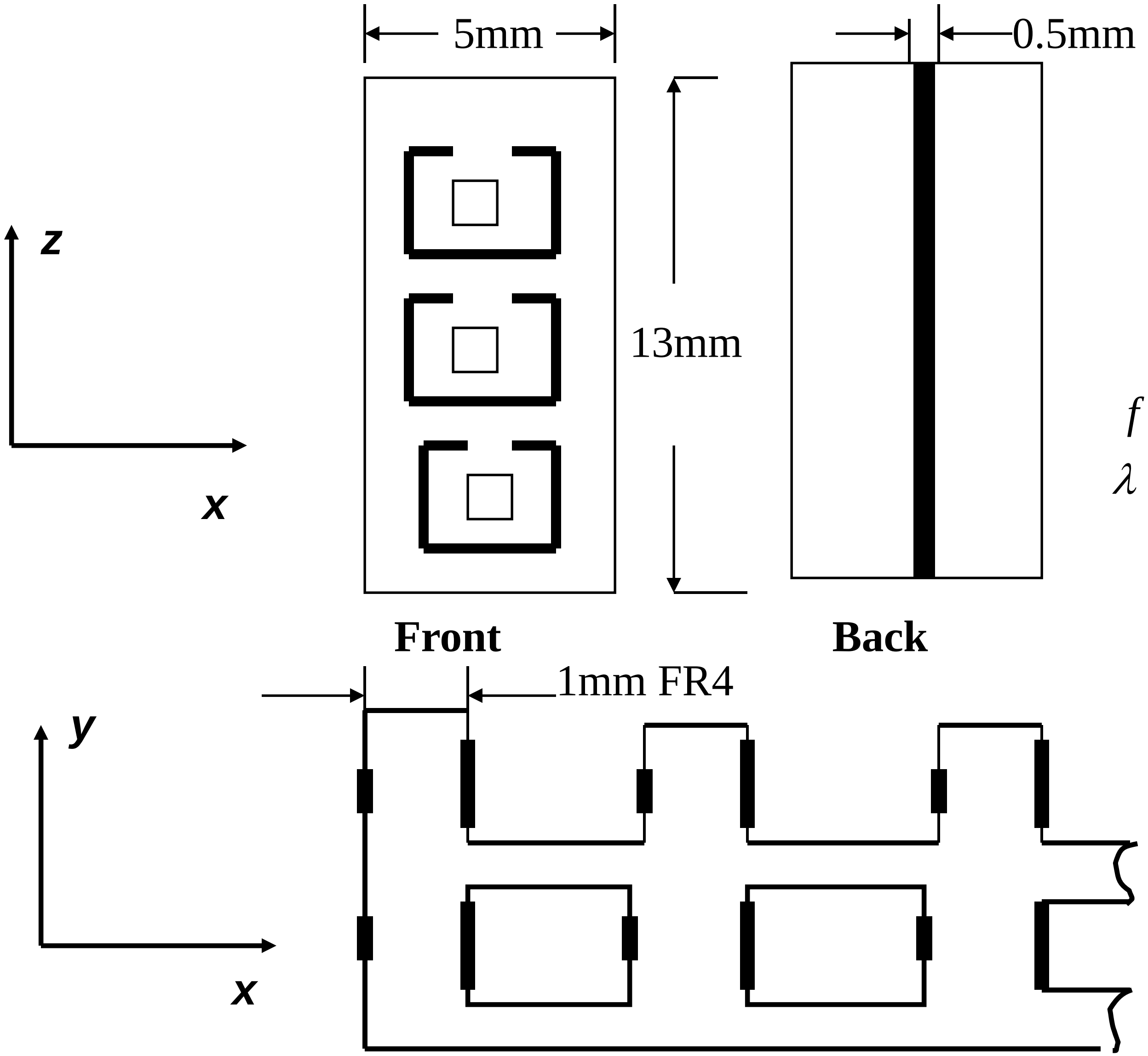
In Transmission characteristics, of only SRR a sudden drop in the pass band is due to non-propagation of waves, over the frequency range. Material becomes opaque to EM Signal at that frequency zone. So SRR experiences a drop (as Stop-band) gives permeability as negative. For WA ,when the frequency less than plasma frequency stop band exists, and the wave propagation is stopped due to product of permittivity & permeability as negative there; due to negative effective permittivity. But when both are Negative then the traveling wave solution (Harmonic solution) appears.

This is similar to $E<0$ in Schrödinger's wave equation-Bound State

Interesting Rule from observation



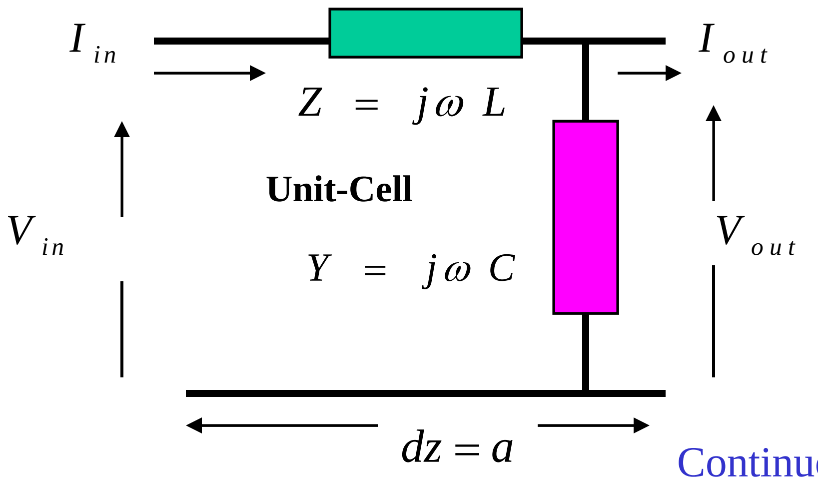
SRR-WA Metamaterial composite structure for LHM for EM Crystal



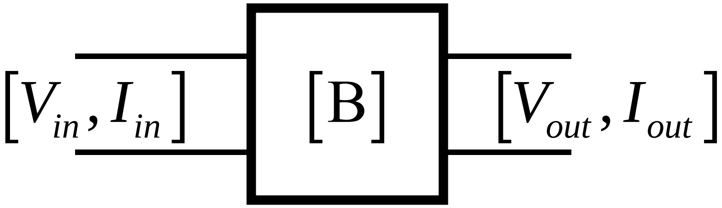
$f \approx 15\,GHz$

$\lambda \approx 20\,mm$

Transmission line approach



$$L_u = \frac{\mu_0}{\pi}\ln\left(\frac{d}{r_w}\right), C_u = \frac{\pi\varepsilon_0}{\ln\left(\frac{d}{r_w}\right)}$$



Discrete Case:

$$V_{in} = I_{in} Z + V_{out}$$

$$V_{out} = \frac{I_{in} - I_{out}}{Y}$$

$$\begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix} = \begin{pmatrix} 1 & -Z \\ -Y & 1 + YZ \end{pmatrix} \begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix}$$

$$[V_{out}, I_{out}]^T = [B][V_{in}, I_{in}]^T$$

$$b_{11} = 1; b_{12} = -Z; b_{21} = -Y; b_{22} = 1 + YZ$$

Continuous Case:

$$V(z+dz) - V(z) = -Z_u I(z) dz$$

$$I(z+dz) - I(z) = -Y_u V(z+dz)$$

$$\frac{dV}{dz} = -Z_u I; \frac{dI}{dz} = -Y_u V$$

$$\frac{d^2 V}{dz^2} + Y_u Z_u V = 0$$ Wave-Equation

$$k^2 = Y_u L_u$$

$$Z_u = j\omega L_u; Y_u = j\omega C_u$$

$$k^2 = \omega^2 L_u C_u = \omega^2 \mu_0 \varepsilon_0 = \frac{\omega}{c}$$

Waves in Transmission Line

$$\frac{d^2V}{dx^2} + Y_u Z_u V = 0$$

A characteristic of wave is that for same interval (spatial in this case), 'phase of the wave always changes by the same amount'. Thus if we have chain of unit cells the phase change between Input and Output quantities (*E*,*H*,*V*,*I*) should be by factor, $\exp(\pm jka)$ where a can be regarded as physical length of unit.

$$V_{out} = \{\exp(-jka)\} \times V_{in}$$

$$I_{out} = \{\exp(+jka)\} \times I_{in}$$

Put the above phasor expression in:

$$\begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} = \begin{pmatrix} e^{-jka} V_{in} \\ e^{+jka} I_{in} \end{pmatrix} = \begin{pmatrix} e^{-jka} & 0 \\ 0 & e^{+jka} \end{pmatrix} \begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix}$$

Solution exists when $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} e^{-jka} & 0 \\ 0 & e^{jka} \end{pmatrix}$ also is reciprocal $b_{11}b_{22} - b_{21}b_{12} = 1$

From above we have $b_{11} + b_{22} = 2\cos ka$

This is now a entirely general dispersion equation.

For TL case applying above we get:

$$2\cos ka = 1 + \frac{YZ}{2}$$

$$4\sin^2\frac{ka}{2} = -YZ$$

For $Z = j\omega L; Y = j\omega C$

Dispersion is:

$$\cos ka = 1 - \frac{1}{2}\omega^2 LC$$

$$4\sin^2\frac{ka}{2} = \omega^2 LC$$

Discrete and Continuous cases observations

In the case of wave equation we get, dispersion relation as plane wave propagation as:

$$k^2 = \omega^2 L_u C_u$$

Whereas in the case with Two-Port Network B Matrix we get, dispersion expression as:

$$4 \sin^2 \frac{ka}{2} = \omega^2 LC$$

Earlier we have assumed line is continuous where as the net-work B matrix is actually discrete. We can off course still affect the conversion from discrete to continuous by stating

$$(ka) << 1$$

The discrete case then reduces as:

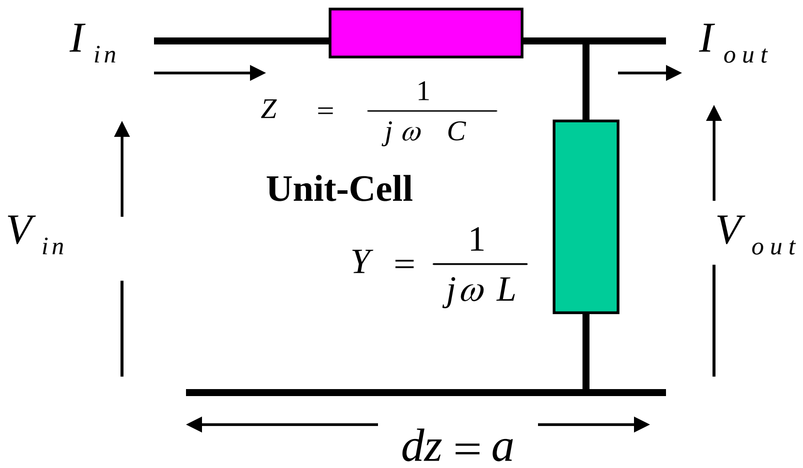
$$4 \sin^2 \frac{ka}{2} \approx (ka)^2$$

$$k^2 a^2 = \omega^2 LC$$

$$L_u = L/a ; C_u = C/a$$

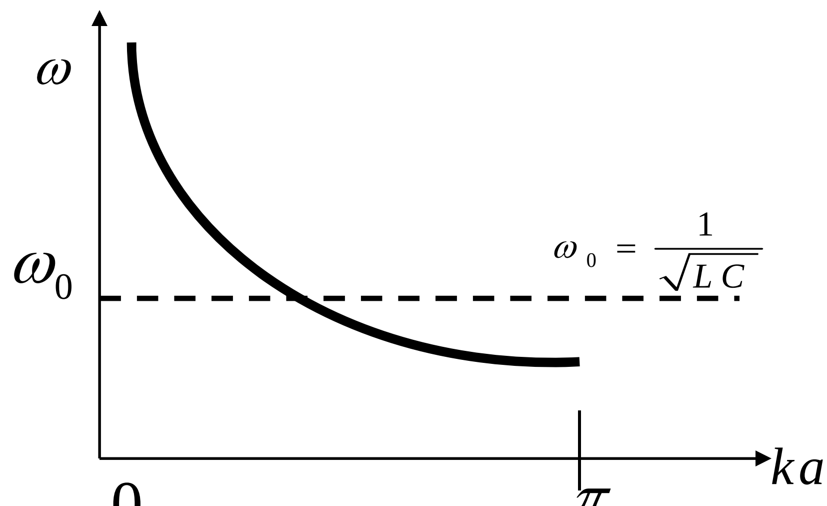
$$k^2 = \omega^2 L_u C_u$$

Realizing the Negative Group Velocity by Transmission line approach



In the transmission line have series inductance and shunt capacitance (HPF); the dispersion is:

$$\cos ka = 1 + \frac{YZ}{2} = 1 - \frac{1}{2\omega^2 LC}$$



$\omega - k$ **Dispersion diagram**

Backward wave

$$v_{ph} = \frac{\omega}{k} > 0$$

$$v_{group} = \frac{d\omega}{dk} < 0$$

Periodic Loading of Transmission Line

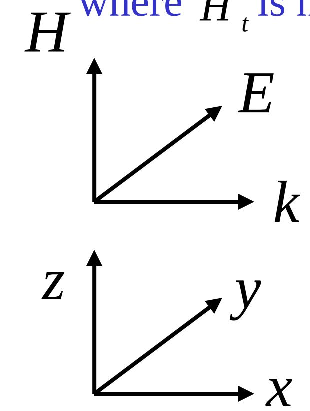
Transmission line (TL) represents a plane wave as we have seen by TL arguments. Well we can argue that in the presence of metamaterials the plane wave is not free to propagate. In other words it can propagate in a "jerky manner"- it bumps regularly (periodically) into obstacles presented by lattice of metamaterial (SRR and Rods). It propagates a little then meets an element, by which it (EM wave) gets affected, it again propagates further, it meets the next element by which it is again affected,…………………… …………………………………….and so on and so forth.

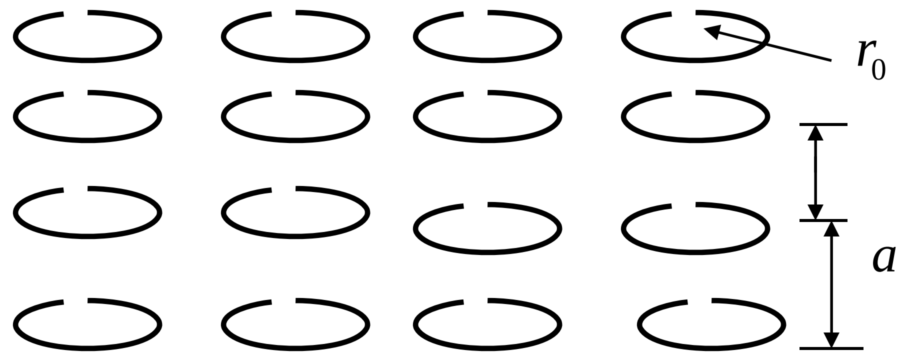
Transmission line that is plane propagating wave is loaded by load in periodicity as:

1. Resonant Elements (SRR)
2. Rods (WA)
3. Both SRR and WA

A. Loaded by resonant element SRR

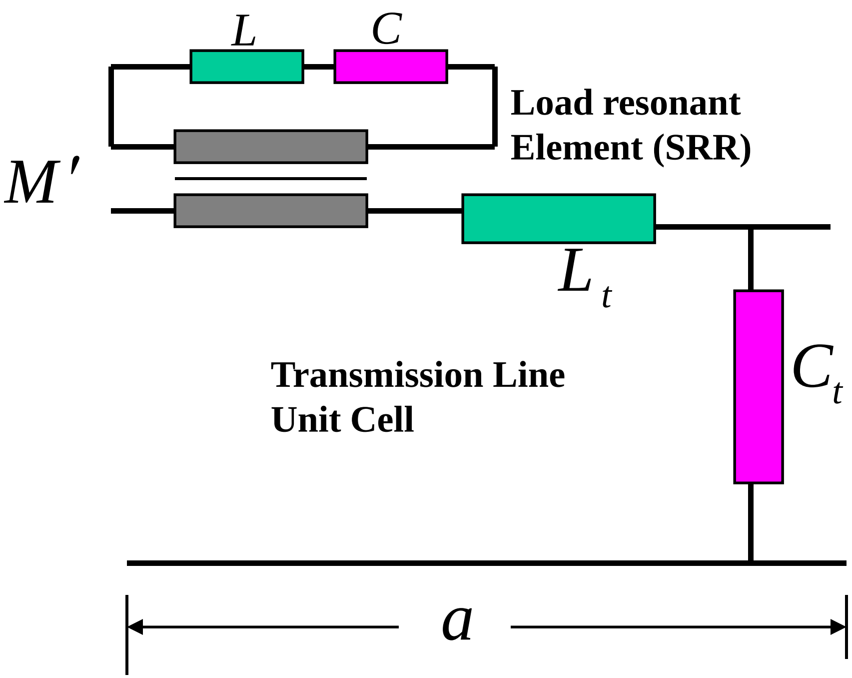
Voltage induced in resonant element $\approx j\omega M' I_t$ where I_t is the current flowing in TL (wave). The same voltage can be expressed as field quantity as, $\omega r_0^2 \mu_0 H_t$ where H_t is magnetic field of EM traveling wave





$$H_t = I_t / a$$

$$M' = \frac{\pi r_0^2 \mu_0}{a}$$



For load

$$b_{11}^L = 1, b_{21}^L = 0, b_{22}^L = 1$$

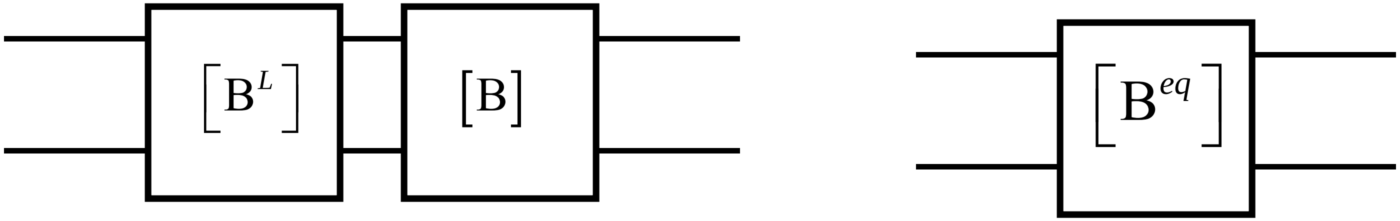
$$b_{12}^L = \frac{j\omega M'^2}{L} \frac{1}{1 - \frac{\omega_0^2}{\omega^2}}; \omega_0^2 = \frac{1}{\sqrt{LC}}$$

For TL

$$b_{11} = 1; b_{12} = -j\omega L_t$$

$$b_{21} = -j\omega C_t; b_{22} = 1 + \omega^2 L_t C_t$$

Composite B matrix and dispersion relation for resonant load on TL

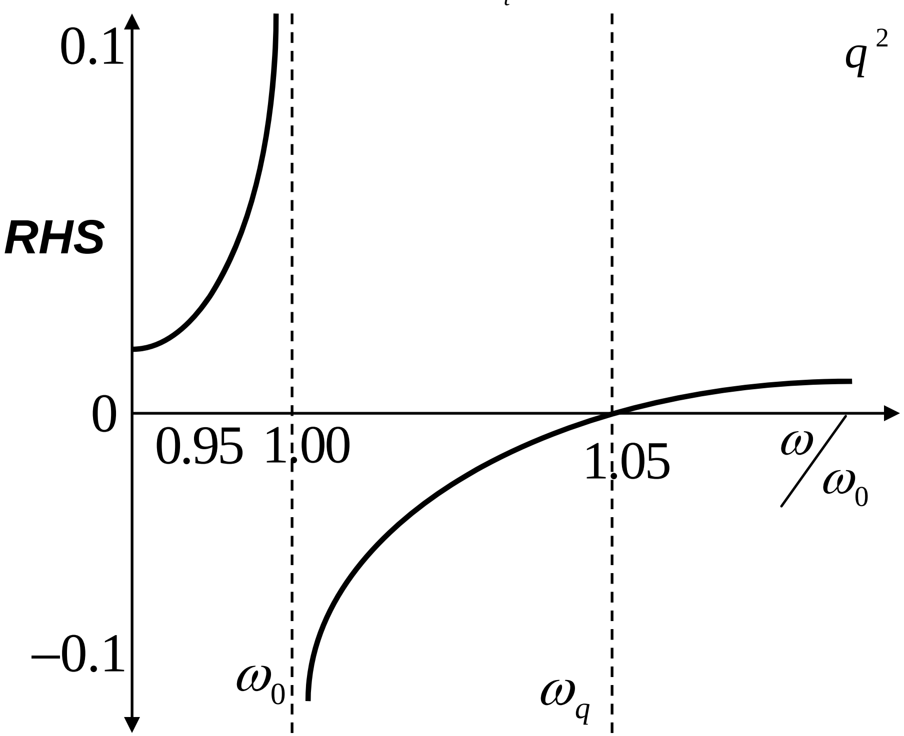


After constructing $\left[B^{eq}\right] = \left[B^{L}\right]\left[B\right]$ and from its diagonal elements the dispersion relation is:

$$4\sin^2\frac{ka}{2} = (1-q^2)\frac{\omega^2}{\omega_t^2}\frac{\omega^2-\omega_q^2}{\omega^2-\omega_0^2}$$

$$q^2 = \frac{M'^2}{LL_t};\omega_t^2 = \frac{1}{L_tC_t};\omega_0^2 = \frac{1}{LC};\omega_q^2 = \frac{\omega_0^2}{1-q^2}$$

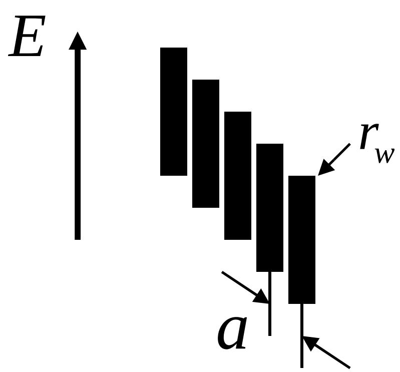
Plot of RHS for $q^2 = 0.1; \frac{\omega_0}{\omega_t} = 0.1$



Pass Band is $0 - \omega_0$ and $\omega > \omega_q$

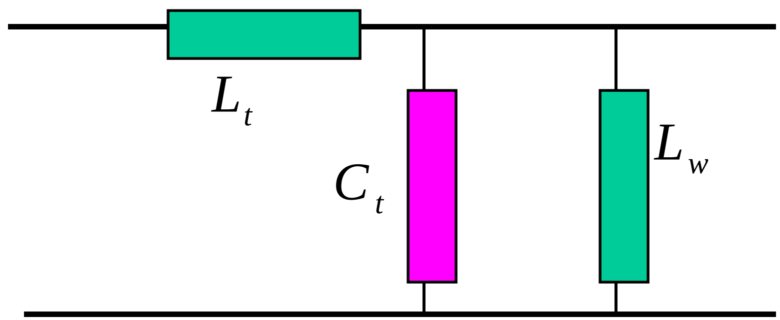
Stop Band is $\omega_0 < \omega < \omega_q$

B. Loading by rods WA



$$R_w = \frac{a}{\pi r_w^2 \sigma_0}; \; L_w = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{2a}{r_w}\right) - \frac{3}{4}\right]$$

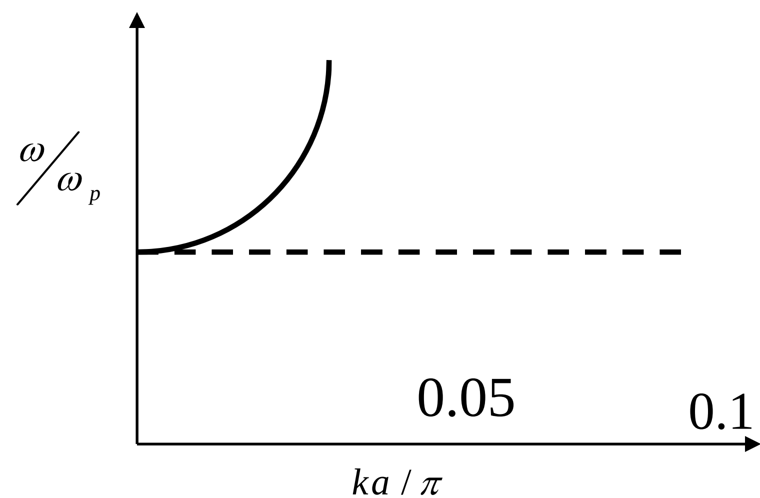
Rods placed along *E* of traveling TEM plane wave, will shunt TL, by its self inductance. However, rods are not resonant elements they give negative permittivity below the "plasma" frequency.



$$Y = j\omega C_t + \frac{1}{j\omega L_w} = j\omega C_t\left(1 - \frac{\omega_p^2}{\omega^2}\right); \omega_p = \frac{1}{\sqrt{L_w C_t}}$$ with $Z = j\omega L_t$ gives

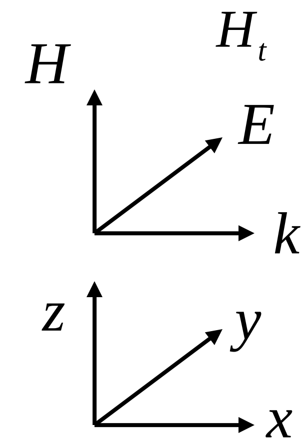
dispersion as:

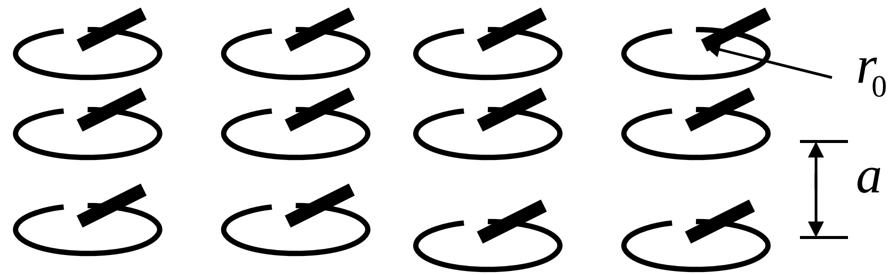
$$4\sin^2\frac{ka}{2} = \frac{\omega^2 - \omega_p^2}{\omega_t^2}; \omega_t = \frac{1}{\sqrt{L_t C_t}}$$



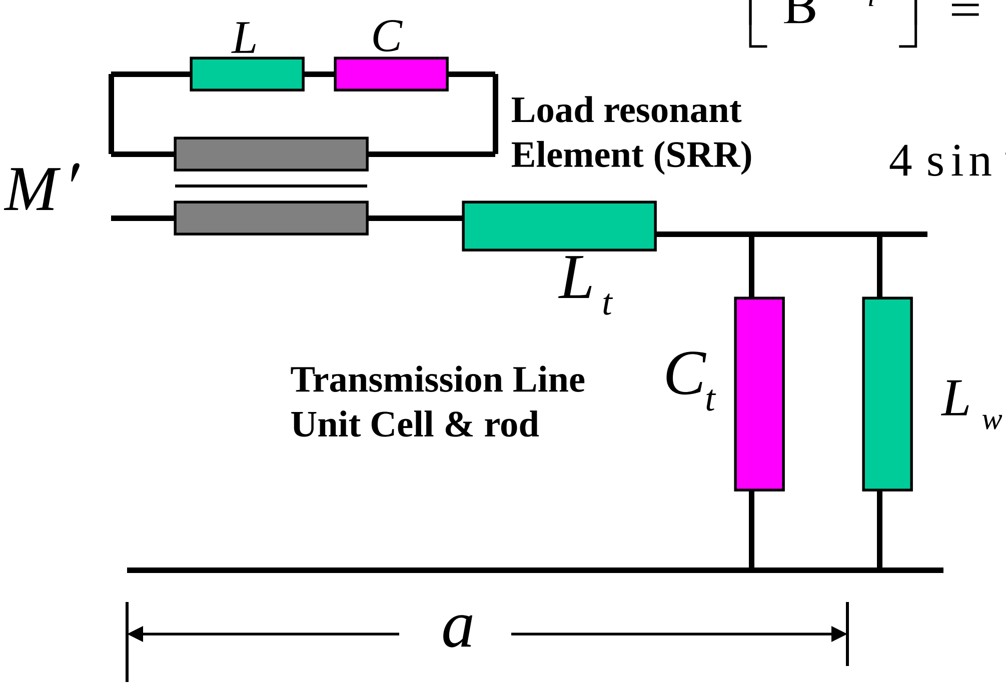
Clearly no solution for $\omega < \omega_p$ and is thus Stop Band

C. Loaded by resonant element SRR and Rods WA





$$[B^{eq}] = [B^{L}][B^{R}][B \quad]$$



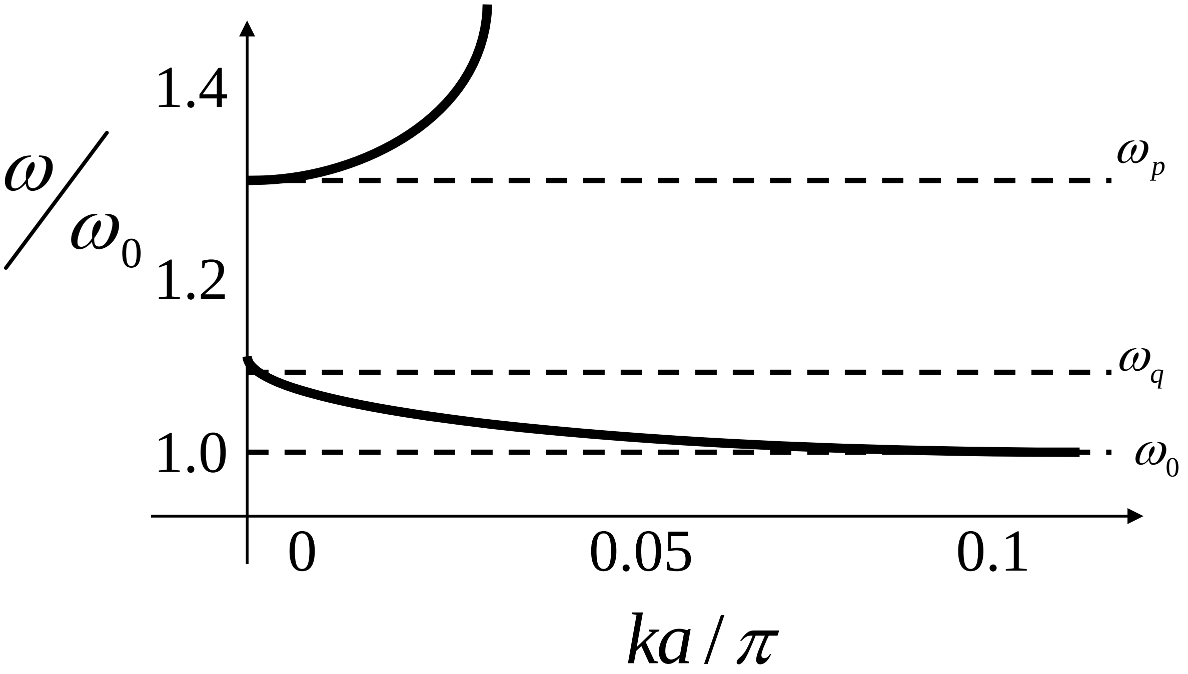
$$4\sin^2\frac{ka}{2} = (1-q^2)\,\frac{\omega^2-\omega_p^2}{\omega_t^2}\,\frac{\omega^2-\omega_p^2}{\omega^2-\omega_0^2}$$

$$q^2 = \frac{M'^2}{L L_t};\ \omega_t^2 = \frac{1}{L_t C_t}$$

$$\omega_0^2 = \frac{1}{LC};\ \omega_q^2 = \frac{\omega_0^2}{1-q^2}$$

$$\omega_t = \frac{1}{\sqrt{L_t C_t}}$$

Dispersion diagram for loading TL with SRR and WA



$\omega < \omega_0$	Stop Band
$\omega_0 < \omega < \omega_q$	Pass Band LHM with DNG
$\omega_q < \omega < \omega_p$	Stop Band
$\omega > \omega_p$	Pass Band

Negative Group Velocity (NGV)/Negative Group Delay (NGD)

Group Velocity is
$$\frac{1}{v_g} = \left[\frac{\partial \beta}{\partial \omega}\right]_{@\ \omega = \omega_0}$$

Propagation constant or phase constant is function of frequency. It corresponds to the propagation speed of an envelope of signal whose spectrum is limited with in short interval

Group Delay is $t_d = -\left[\frac{\partial \Phi}{\partial \omega}\right]_{@\ \omega = \omega_0}$ $\quad \omega_0 \pm \Delta\omega, \Delta\omega \to 0$

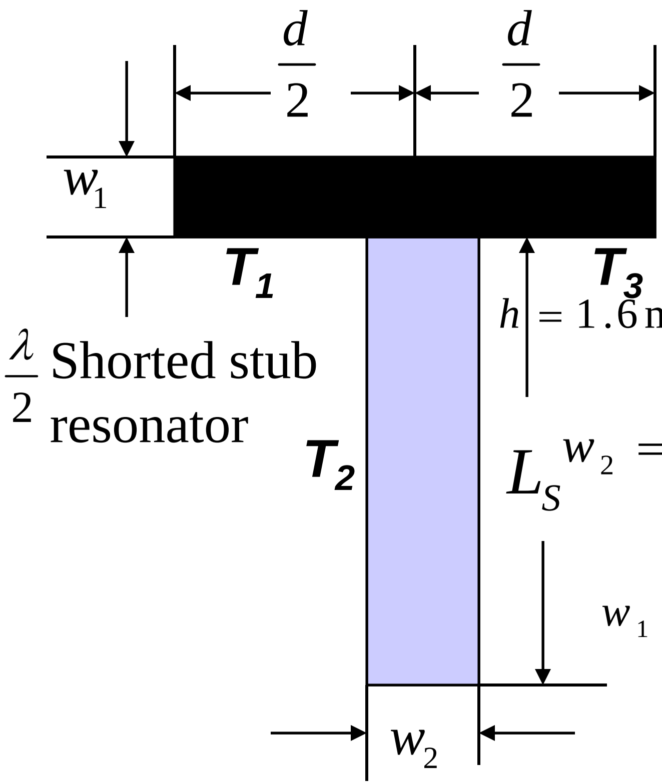
Phase shift is frequency dependent, Phase shift corresponds to temporal shift of envelope of band limited signal passing through a system.

$$\Phi = -\beta L$$

$$t_d = \frac{\partial(\beta L)}{\partial \omega} = L\frac{\partial \beta}{\partial \omega} = \frac{L}{v_g}$$

For $t_d < 0$ (NGD), then $v_g < 0$ (NGV)

Realizing NGV/ NGD in strip-line transmission line



<u>Substrate</u> FR4 ,

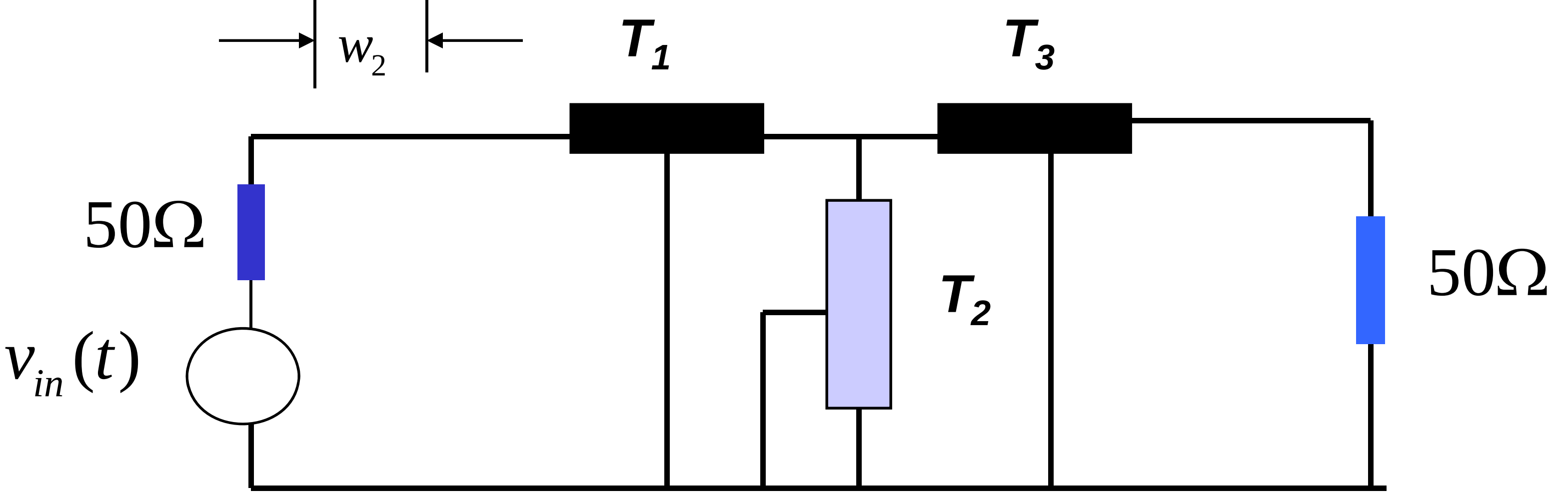
$h = 1.6\,mm, \varepsilon_r = 4.4, \tan\delta = 0.00148, d = 2\,cm, f_0 = 0.9\,GHz$

<u>Stub</u>

$w_2 = 1.39\,mm, \varepsilon_{eff} = 3.144, Z_s = 75.6\,\Omega, L_S = 9.4\,cm$

<u>Line dimensions</u>

$w_1 = 3.33\,mm, \varepsilon_{eff} = 3.0239, Z_0 = 50\,\Omega, \frac{d}{2} = 1\,cm$

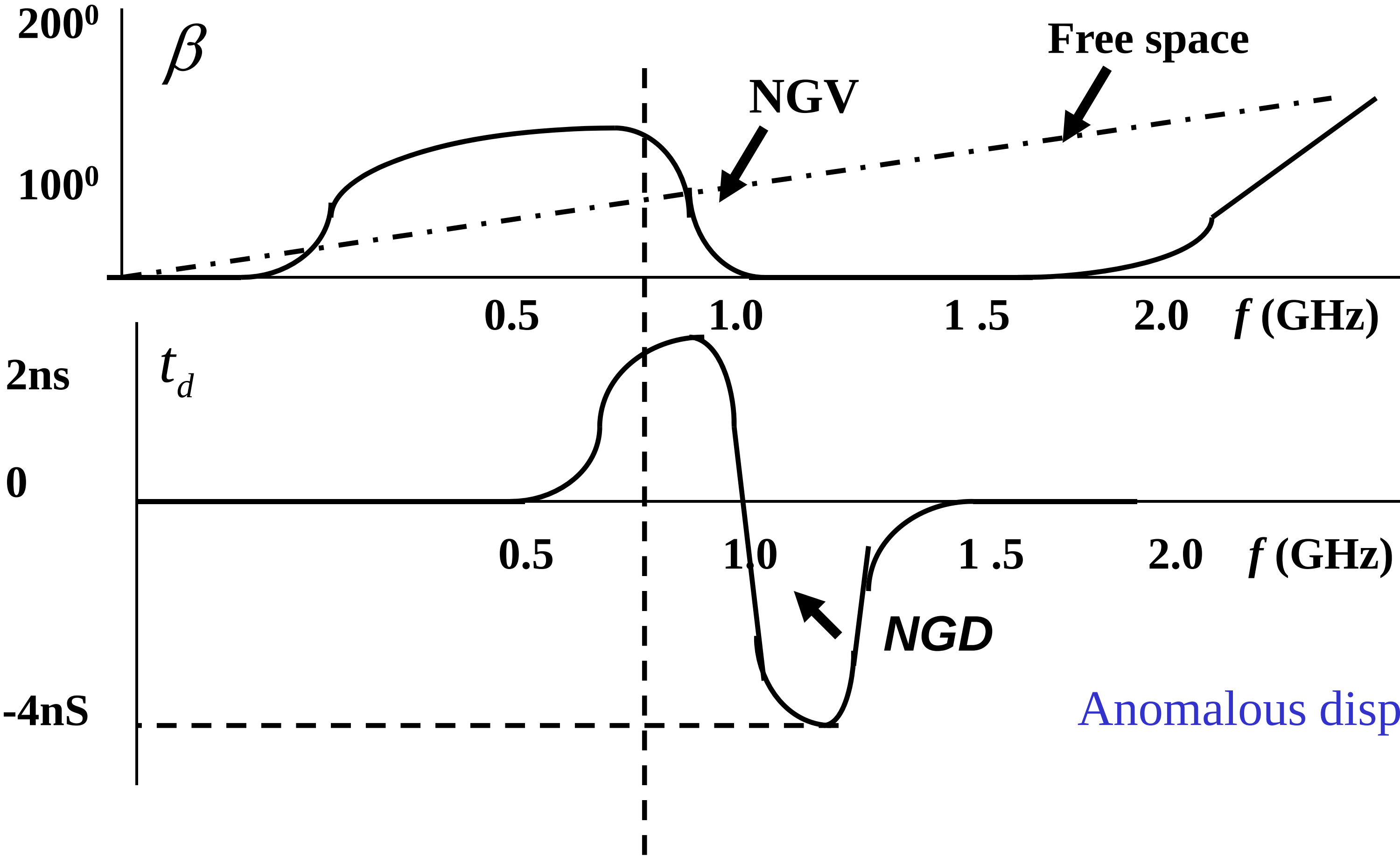


$\sum (f_1 = 0.89\,GHz), (f_2 = 0.9\,GHz), (f_3 = 0.91\,GHz)$

Values of electrical parameters of the strip-line sections

Line	*R* Ω / m	*L* $\mathrm{nH/m}$	*G* $\mu\mathrm{S/m}$	*C* $\mathrm{pF/m}$	length (cm)
T_1	5.0928	304.15	10200	121.66	1.000
T_2	11.079	446.84	6500	78.18	9.4
T_3	5.09 28	304.15	10200	121.66	1.000

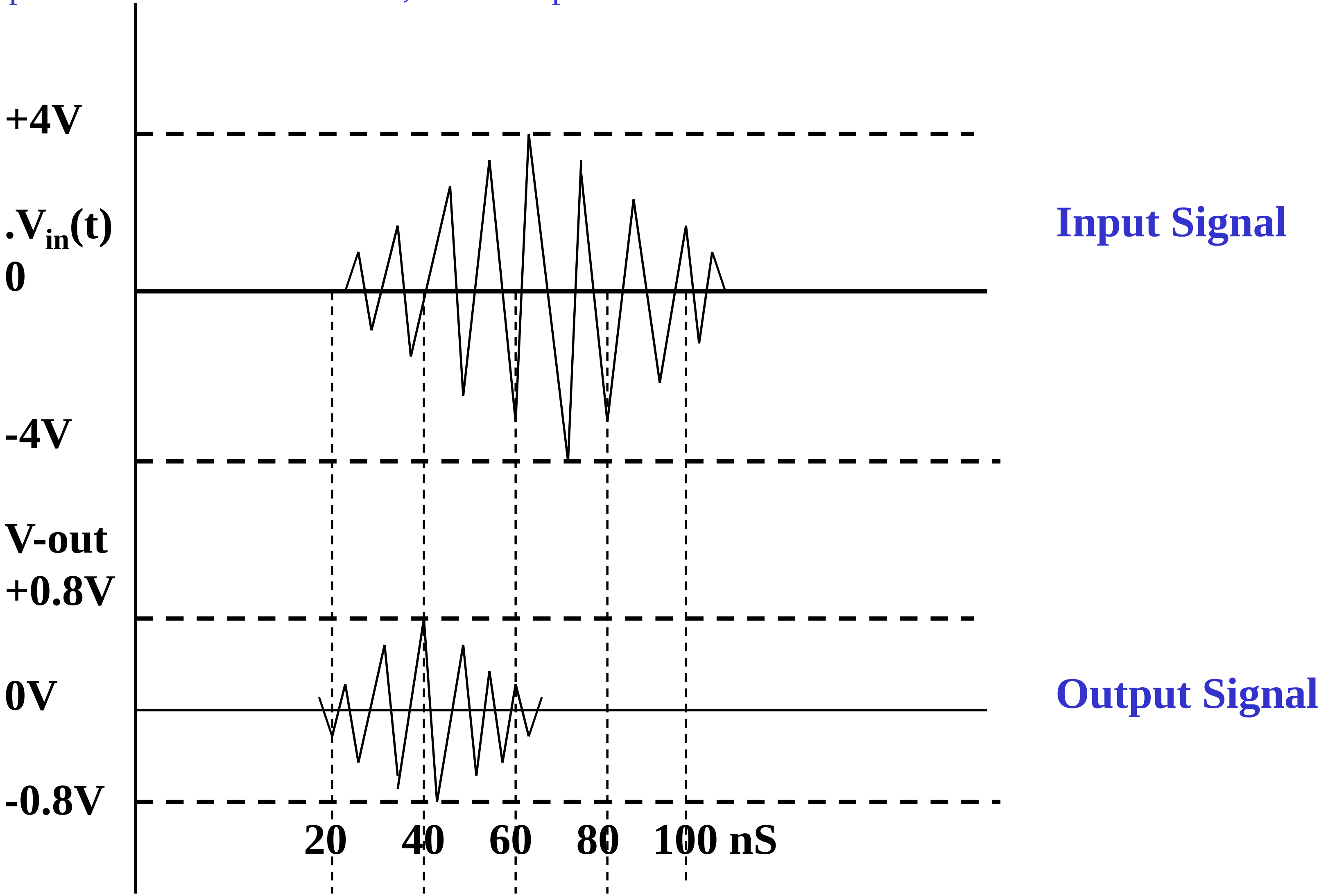
NGV /NGD from dispersion diagram of TL-RLC Shunt structure



Anomalous dispersion

Electrical Pulse in envelope when passed through a Transmission line with Doubly Negative parameters giving (i.e. $n < 0$)

Negative Group Velocity/ Negative Group Delay in time domain, is observed. The, envelope seems to advance in time, with compression too.



Remarks

Causality is not violated as “pulse front” gets positively delayed, irrespective of negative delay suffered by the envelope.

Pulse shape is exactly not preserved. Front travels with positive delay and envelope travels with negative delay.

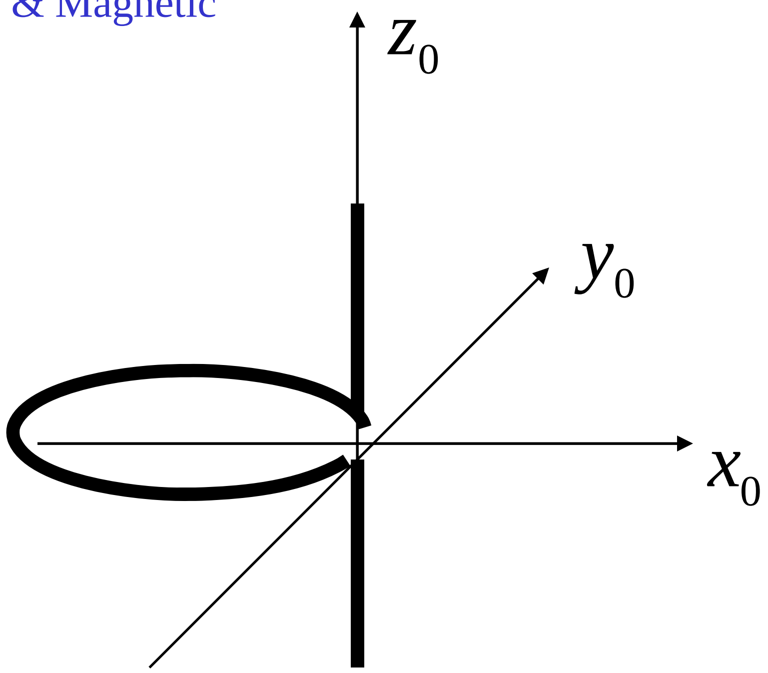
Pulse is compressed.

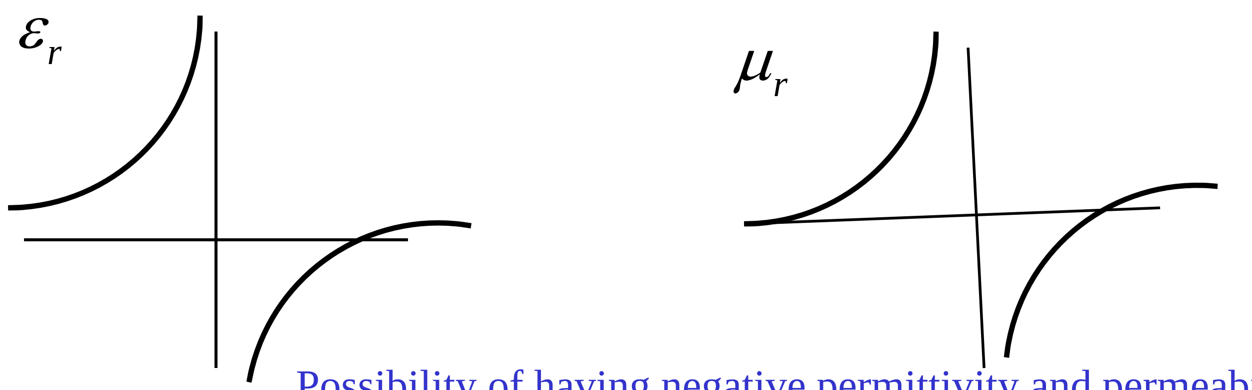
Distortion can be controlled by controlling loss in the stub (R), however signal strength reduces.

Quantum laws, and Quantum prescriptors are not yet applied in the structures of LHM a good future field of ongoing research in LHM.

Omega Particle

A combination of Electric & Magnetic dipole a Chiral Element?





Possibility of having negative permittivity and permeability by same element

End of Part-2