FRACTIONAL ORDER CONTROL SYSTEM
$PI^{\alpha}D^{\beta}$

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HOW & WHEN GOT THE FEELING FOR NON-PID?

IDEA OF HAVING NON-INTEGER ORDER PID, GOT ENHANCED AFTER UTILIZING TRANSCENDENTAL CONTROL LAWS FOR REACTOR CONTROL OF PHWR540 MW. THESE GOVERNANCE LAW GIVE CONCEPT OF **FUEL EFFICIENT REACTOR**.

$$EPE_{lin}(t) = \frac{P(t)e^{t/T}}{P_d(t)e^{t/T_d}}$$

$$EPE_{\log}(t) = K_1[\log P(t) - \log P_d(t)] + K_2[\log(1 + \frac{<\log R>}{100} - \log(1 + \frac{M}{100f})]$$

EPE: EFFECTIVE POWER ERROR

CONCEPT OF **RATIO CONTROL** & **EXPONENT SHAPE GOVERNOR** GIVE CLOSER TRACKING OF NEUTRONICS HENCE EFFICIENCY IN CONTROLS.

QUESTINED SELF

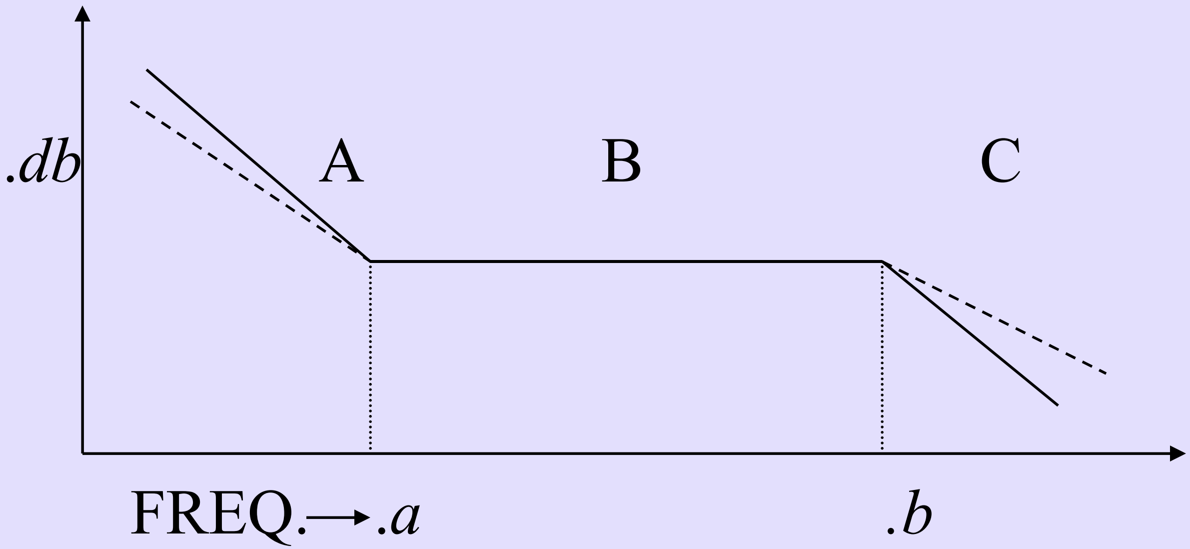
IS PID PANACEA FOR ALL ROBUST / OPTIMAL CONTROLS?

ANSWER GOT

IN HAVING FRACTIONAL ORDER CONTROLLER

BEYOND PID

EXPERIMENTAL DETERMINATION OF TRANSFER FUNCTION:



SOLID LINE SECTION A SLOPE –20 *db*/DECADE
SECTION C –20*db*/DECADE
DASHED LINE SECTION A SLOPE –15 *db*/DECADE,
SECTION C –5*db*/DECADE
.*a* & *b* ARE BREAK FREQUENCIES.

SOLID LINE CHARACTERISTIC EQUATION:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{K_1(s+a)}{s(s+b)}$$

$$\frac{d^2 y(t)}{dt^2} + b\frac{dy(t)}{dt} = K_1\frac{dx(t)}{dt} + K_1 a x(t)$$

DASHED LINE CHARACTERISTIC EQUATION:

$$G(s) = \frac{Y(s)}{X(s)} = K_2\frac{(s^{\mu 1} + a)}{s^{\mu 1}(s^{\mu 2} + b)}$$

$$\frac{d^{\mu 1+\mu 2} y(t)}{dt^{\mu 1+\mu 2}} + b\frac{d^{\mu 1} y(t)}{dt^{\mu 1}} = K_2\frac{d^{\mu 1} x(t)}{dt^{\mu 1}} + K_2 a x(t)$$

$$\mu 1 = \frac{15}{20} = 0.75, \mu 2 = \frac{5}{20} = 0.25$$

FRACTIONAL ORDER SYSTEM

STUDY AREAS & RESEARCH ACTIVITY

1. **MATHEMATICS:** DEFINIG FRACTIONAL ORDER DERIVATIVE AND OBTAINING ANALYTICAL/NUMERICAL SOLUTION FOR FRACTIONAL ORDER DIFFERENTIAL EQUATIONS

2. **PHYSICS:** EXPRESSING COMPLICATED PHYSICAL SYSTEMS IN FRACTIONAL ORDER DIFFERENTIAL EQUATIONS. DISTRIBUTED SYSTEMS ARE USUALLY EXPRESSED WELL BY FRACTIONAL ORDER SYTEM DESCRIPTION.

3. **CONTROLS:** EXPRESSING CONTROL THEORY OF INTEGER ORDER SYSTEM IN FRACTIONAL ORDER CONTROL THEORY.

MOST POPULAR DEFINITIONS IN CONTROL SCIENCE

REIMANN-LIOUVILLE (RL) / CAPUTO:

$$J^{\alpha} f(x) = \frac{d^{-\alpha} f(x)}{dx} = \frac{1}{\Gamma(\alpha - 1)} \Gamma \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau$$

$${}_0 D^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau, (m-1) < \alpha < m$$

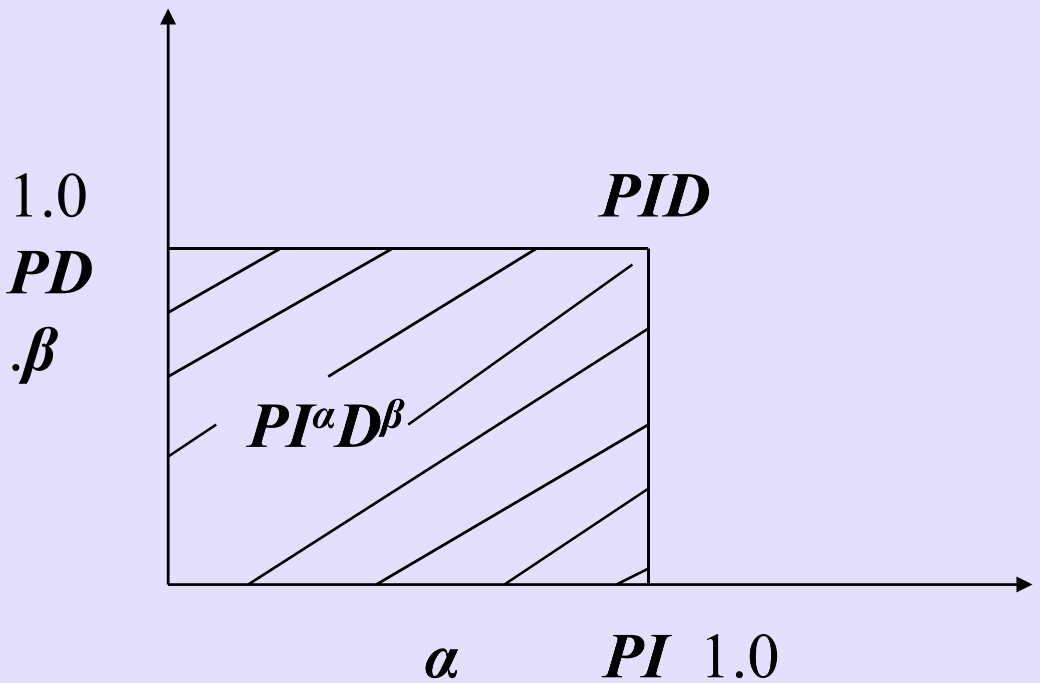
GRUNWALD-LETNIKOV:

$$\frac{d^{\alpha} f(x)}{dx^{\alpha}} = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{m=0}^{[\frac{t-a}{h}]} (-1)^m \frac{\Gamma(\alpha+1)}{m! \Gamma(\alpha - m + 1)} f(x - mh)$$

$$[\frac{t-a}{h}] \rightarrow INTEGER\_PART$$

FLOOR-FUNCTION

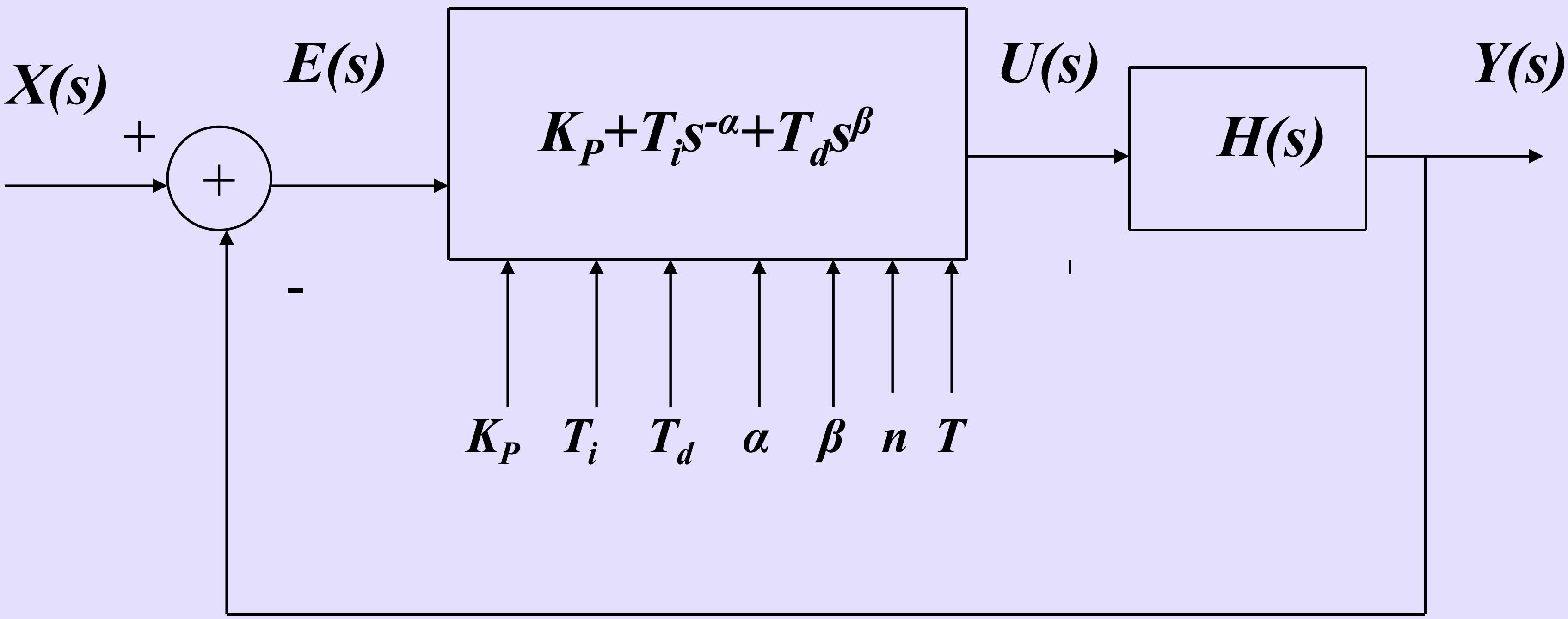
FRACTIONAL ORDER PID



BASIC CONTROLLER ACTION: **WHY NOT EXTEND IN A CONTINUOUS WAY?**

.α & β PROVIDE EXTRA DEGREE OF FREEDOM TO CONTROL ENGINEER.

$PI^{\alpha}D^{\beta}$



K_P PROPORTIONAL GAIN , T_i INTEGRAL RESET RATE
T_d DERIVATIVE TIME, α FRACTIONAL INTEGRAL ORDER
.β FRACTIONAL DERIVATIVE ORDER, n NUMBER OF SAMPLES, T SAMPLING INTERVAL
α, .β ARE EXTRA FREEDOM (KNOBS)

PIONEER APPLICATIONS IN CONTROLS

1.BODE (1945): FEEDBACK AMPLIFIER OPEN LOOP
TRANSFER FUNCTION (OVERSHOOT
INDEPENDENT OF GAIN)
FRACTIONAL INTEGRATOR

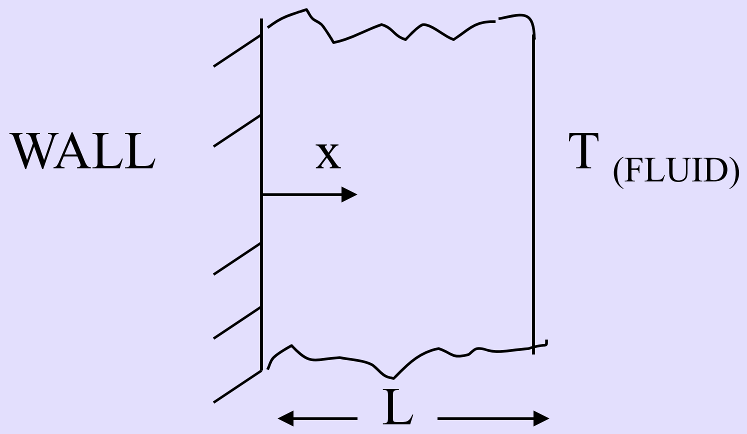
2. TUSTIN (1958): CONTROL OF MASSIVE OBJECTS, A
CONSTANT PHASE MARGIN OVER A RATHER
WIDE RANGE OF FREQUENCY.

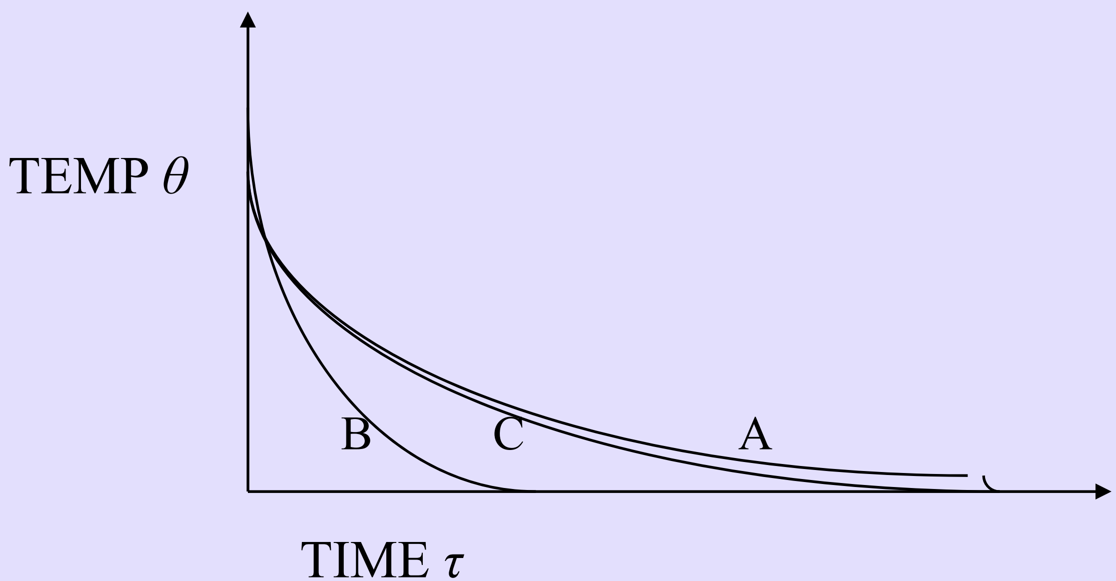
3. MANABE (1961): CONTROL WITH A FRACTIONAL INTEGRATOR.

4**. CARLSON HALIJACK (1961):** CONTROL OF SERVO WITH
FRACTIONAL INTEGRATOR

5. OUSTALOUP (1981): ROBUST CONTROL OF NON INTEGER
ORDER

HEAT FLOW TRANSIENT





A: EXACT
B: INTEGER ORDER
C: FRACTIONAL ORDER

$$B_i = \frac{hL}{k} (BIOT - NUMBER)\_SIZE - FACTOR = 10$$

$$h = CONVECTIVE - HEAT - COEFFICIENT\_(COMPLEX)$$

$$k = THERMAL - CONDUCTIVITY\_(COMPLEX)$$

$$L = WALL - THICKNESS$$

$$\frac{d\theta}{d\tau} + B_i = 0, -----------FOR - B$$

$$\frac{d^q\theta}{d\tau^q} + B_i = 0, ----q = 0.9 --FOR - C$$

$$FOR....B_i \rightarrow 0.1.....THEN...q \rightarrow 1.0$$

HEAT FLOW IS FRACTIONAL ORDER

REASONS FOR FRACTIONAL CONTROLS

IT IS THUS BEEN SHOWN THROUGH EXAMPLE THAT "TIME-DEPENDENT" TEMPERATURES IN A TRANSIENT THERMAL SYSTEMS CAN BE MODELED WITH FRACTIONAL DIFFERENTIAL EQUATIONS.

THE METHOD IS PARTICULARLY SUITABLE FOR COMPLEX SYSTEMS FOR WHICH A FIRST PRINCIPLE APPROACH IS TOO DIFFICULT OR COMPUTATIONALLY TIME CONSUMING.

SINCE THE BEST CONTROLLER IS ONE THAT USES IN SOME WAY THE ORDER OF SYSTEM BEING CONTROLLED-----A FRACTIONAL ORDER CONTROL SYSTEM---A SO CALLED $PI^{\alpha}D^{\beta}$

$$G(s) = K_p + K_I s^{-\alpha} + K_d s^{\beta}, \alpha . \& \beta > 0$$

CONTROLLERS OF THIS KIND CAN BE EASILY TUNED TO DIFFERENTIAL MODELS OF FRACTIONAL ORDER.

IN CONTRAST, WORKING WITH THE EXACT PDE WOULD NOT ONLY BE COMPUTATIONALLY INTENSIVE BUT IF FREQUENT ADJUSTMENTS OF THE SYSTEM PARAMETER IS DESIRED WILL BE MUCH MORE DIFFICULT

MOTIVATING EXAMPLE

CONSIDER DOUBLE INTEGRATOR PLANT $G(s)=A/s^2$, A IS OPEN LOOP GAIN.

PLANT BEING CONTROLLED BY FRACTIONAL ORDER CONTROLLER $H(s)=s^{\alpha}$ IN UNITY GAIN FEED BACK LOOP.

OPEN LOOP TF IS $F_0=G(s)H(s)=(A/s^2)s^{\alpha}=A/(s^{2-\alpha})$

THIS HAS (i) BODE AMPLITUDE PLOT CONSTANT SLOPE $-(2-\alpha)$

(ii) CROSS OVER FREQUENCY DEPENDS ON A

(iii) PHASE PLOT CONSTANT HORIZONTAL LINE $-(2-\alpha).\pi/2$

CLOSE LOOP TF IS $F_c=A/(s^{2-\alpha}+A)$

THIS HAS (i) GAIN MARGIN INFINITE

(ii) CONSTANT PHASE MARGIN $\alpha\,\pi/2$, $\phi_m = \pi\left(1-\frac{2-\alpha}{2}\right)$

ABOVE DISCUSSION OF USING FRACTIONAL ZERO AS *H(s)* GIVES STABILITY.

THE CLOSE LOOP STEP RESPONSE IS EXHIBITING " AN OVER-SHOOT INDEPENDENT OF GAIN *A* DEPENDS ON α"

$$y(t) = At^{2-\alpha}E_{2-\alpha,2-\alpha+1}(-At^{2-\alpha})$$

DC MOTOR CONTROL

SYSTEM TRANSFER FUNCTION: $G(s) = \frac{k}{Js(Ts+1)}$ **J : PAYLOAD INERTIA**

PHASE MARGIN OF CONTROLLED SYSTEM $\Phi_m = \arg\left[H(j\omega)G(j\omega)\right] + \pi$

CONTROLLER $H(s) = k_1 \frac{k_2 s + 1}{s^{\alpha}}, k_2 = T$

GIVING CONSTANT PHASE MERGIN

$$\Phi_m = \arg\left[H(j\omega)G(j\omega)\right] + \pi$$

$$= \arg\left[\frac{k_1 k / J}{(j\omega)^{1+\alpha}}\right] + \pi$$

$$= \arg\left[(j\omega)^{-(1+\alpha)}\right] + \pi$$

$$= \pi - (1+\alpha)\frac{\pi}{2} = \frac{\pi}{2}(1-\alpha)$$

STEP RESPONSE

$$y(t) = L^{-1}\left[\frac{k_1 k / J}{s\left(s^{1+\alpha} + \frac{k_1 k}{J}\right)}\right]$$

$$= \left(\frac{k_1 k}{J}\right) t^{1+\alpha} E_{1+\alpha, 2+\alpha}\left(-\frac{k_1 k}{J} t^{1+\alpha}\right)$$

ISODAMPING

$$H(s) = K + T_i s^{-\alpha} + T_d s^{\beta}$$

$$u(t) = K e(t) + T_i D^{-\alpha} e(t) + T_d D^{\beta} e(t)$$

$$G(s) = \frac{1}{a_n s^{\mu n} + \ldots\ldots + a_1 s^{\mu 1} + a_0 s^{\mu 0}}$$

$$G(s) H(s) + 1 = 0$$

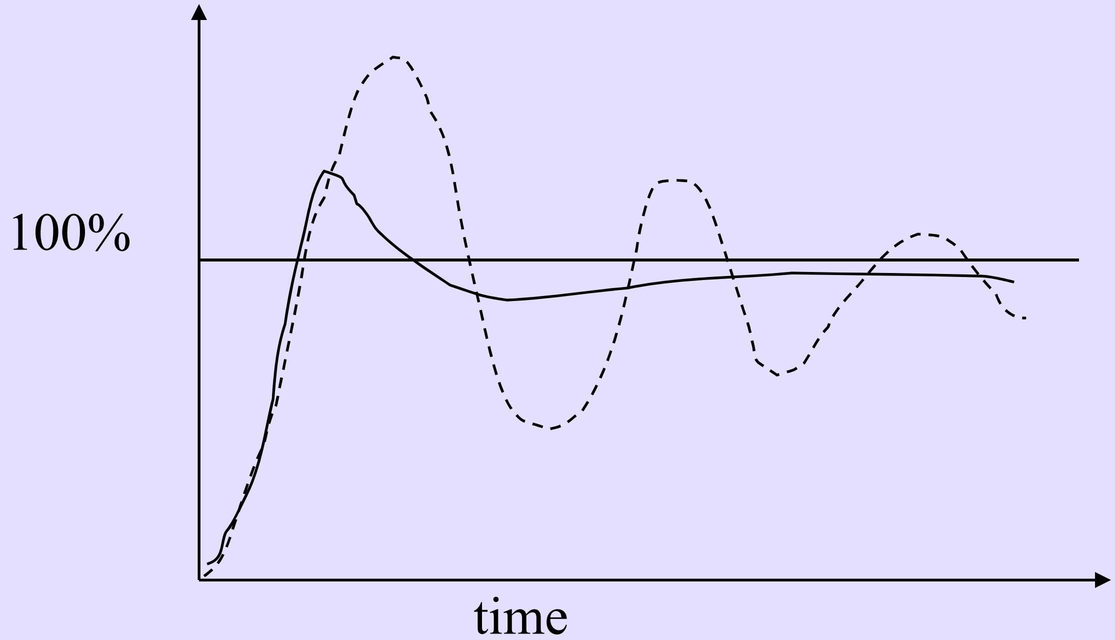
$$\sum_{k=0}^{n} a_k s^{\mu k} + (K + T_i s^{-\alpha} + T_d s^{\beta}) = 0$$

K PROPORTIONAL GAIN INFLUENCES PROPORTIONAL VALUE OF STATIC DEVIATION $\boldsymbol{E_t}$ **[%]**, $\boldsymbol{T_r}$ **[s]** SETTLING TIME.

GENERALLY AS ***K*** INCREASES SETTLING TIME COMES DOWN WITH STATIC DEVIATION.

$\boldsymbol{K > (100/E_t) - a_0}$

ABOVE STABILITY EQUATION BE SOLVED FOR REQUIRED STABILITY MEASURE & DAMPING MEASURE.



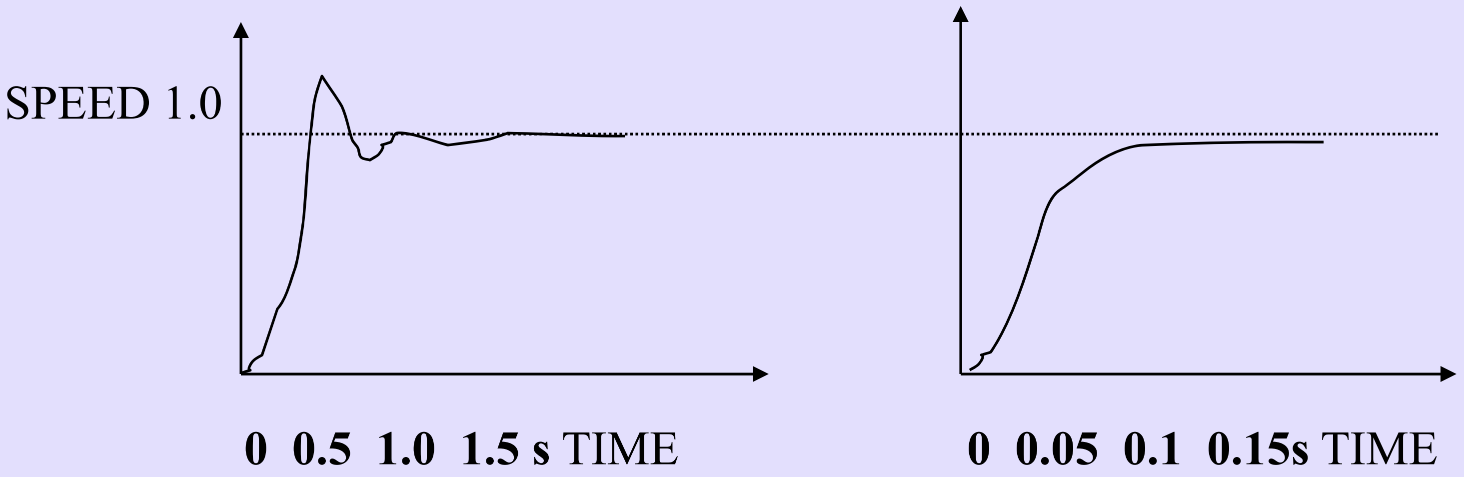
$$G(s)=\frac{1}{1+0.5s^{0.9}+0.8s^{2.2}} \sim G(s)=\frac{1}{1+0.2313s+0.7414s^{2}}$$

$$H_{IO}(s)=20.5+2.7343s, K=20.5, T_d=2.7343, \beta=1, T_i=0$$

$$H_{FO}(s)=20.5+5.79s^{0.95}, K=20.5, T_d=5.79, \beta=0.95, T_i=0$$

SOLID LINE SHOW FRACTIONAL ORDER PID CONTROLLER
DASHED LINE SHOW INTEGER ORDER PID CONTROLLER.

CONTROL *H(s)* IN FEED BACK LOOP CASE WITH OPTIMAL INTEGER ORDER *H(s)* AND FRACTIONAL ORDER H(s) FOR FRACTIONAL ORDER PLANT *G(s)*. GAS TURBINE MODEL.



CLOSE LOOP STEP RESPONCE

$$G(s)=\frac{107.44715}{0.0107s^{1.64345}+0.1587s^{0.7755}+1}$$

$$H_{IO}(s)=\frac{1.837X10^{10}s^{2}+1.313X10^{12}s+5.455X10^{12}}{s^{3}+6.344X10^{7}s^{2}+2.778X10^{13}s}$$

$$H_{FO}(s)=\frac{3.6682X10^{-5}s^{2.6435}+5.724X10^{-4}s^{1.7775}+5.8864X10^{-3}s^{1.6435}+3.6069X10^{-3}s+0.0916s^{0.775}+0.574}{5.086X10^{-8}s^{3}+5.0965X10^{-3}s^{2}+s}$$

OBSERVATIONS:

THE DYNAMIC PROPERTIES OF THE CLOSE LOOP WITH FRACTIONAL ORDER CONTROLLED SYSTEM & FRACTIONAL ORDER CONTROLLER ARE BETTER THAN DYNAMICAL PROPERTIES OF CLOSE LOOP WITH THE INTEGER ORDER CONTROLLER.
THE SYSTEM WITH INTEGER ORDER CONTROLLER STABILIZES SLOWER & HAS LARGER SURPLUS OSCILLATIONS.

WE CAN SEE THAT USE OF FRACTIONAL ORDER CONTROLLERS LEADS TO IMPROVEMENT OF THE CONTROL OF FRACTIONAL ORDER SYSTEM.

FRACTIONAL ORDER CONTROL SYSTEM **MORE ROBUST**,WHICH MEANS THEY ARE LESS SENSITIVE TO CHANGES OF THE SYSTEM PARAMETERS & CONTROL PARAMETER.

OVERSHOOT INDEPENDENT OF GAIN (LOADING) DEPENDENT ON FRACTIONAL ORDER ONLY GIVES ROBUSTNESS. (**ISODAMPING)**

COMMENTS:

A "REALITY" SYSTEM IS OF FRACTIONAL ORDER BE MODELLED AS INTEGER ORDER TRANSFER FUNCTION (TF) THEN AN OPTIMIZED INTEGER ORDER PID WITH PARAMETERS TUNED BE EMPLOYED GIVES STABLE RESPONSE TODAY. THE USE HAS CHANGED THE PARAMETER OF ACTUAL SYSTEM, AND WE AGAIN MODEL IT WITH INTEGER ORDER AND RETUNE WITH SAY T_d OF PID CHANGED AND SHOW IN SIMULATION ITS OVERALL CLOSE LOOP STABILITY. BUT WHEN EMPLOYED IN REALITY SYSTEM THE SAME RETUNED PID MAKES THE PLANT UNSTABLE. HERE SYSTEM IDENTIFICATION AND CONTROL IN FRACTIONAL ORDERED WAY IS WHAT IS REQUIRED.

THE FRACTIONAL ORDER DIFFERENTIAL EQUATION DEPLOYMENT FOR DATA FITTING IS MORE ROBUST AS COMPARED TO PRESENT PARABOLIC CUBIC EXPONENTIAL REGRESSION METHODS.

REAL SYSTEMS ARE FRACTIONAL ORDER SYSTEMS

SIZE FACTOR

SEVERAL MODES

PARAMETRIC SPREADS

DISTRIBUTED COUPLINGS

HISTORY

MEMORY

HERIDITY

NON LOCAL VARIATIONS (DIVERGENCE)

WHY NOT CONTROLLER BE OF FRACTIONAL ORDER

CONTROLLER CLOSE TO REALITY.REFLECTS ACTUAL ORDER

EFFICIENT EXPENDITURE OF LESS CONTROL EFFORT,FUEL EFFICIENT

ROBUST TAKES CARE OF PARAMETRIC SPREADS.

SHAPING OF FREQUENCY DOMAIN CHARACTERISTICS AT WILL.

TIME DOMAIN RESPONSE THEREFORE BE TUNED PRECISELY.

GAINS MORE DEGREE OF FREEDOM (EXTRA KNOBS)

TO ENHANCE SCIENTIFIC KNOWLEDGE-COMPLEX ORDER--

SOME IMPORTANT LEMMA

$$G(s) = \frac{e^{-s\tau} K \prod_{j=1}^{m_0} (s^{\mu_j} - z_j)}{s^{\alpha_0 - \beta_0} \prod_{i=1}^{n_0} (s^{\mu_i} - p_i)}$$

FRACTIONAL ORDER SYSTEM WILL BE OF ABOVE FORM

LEMMA-1

MAGNITUDE DECIBEL LOG FREQUENCY OF $\boldsymbol{s^{\mu}}$, A LINE WITH SLOPE $\boldsymbol{20\mu}$, WHEN $\omega>0$, CROSSES ZERO db AT $\omega=1$

THE PHASE LOG FREQUENCY DIAGRAM IS INDEPENDENT WITH FREQUENCY.UNDER CERTAIN FRACTIONAL ORDER PHASE IS $\boldsymbol{\mu\pi/2}$, $\omega>0$

LEMMA-2

MAGNITUDE DECIBEL LOG FREQUENCY OF $T_a s^{\mu}+1$ A LINE WITH SLOPE $20\,\mu$ WHEN $\omega > \omega_b$

THE PHASE DIAGRAM STARTS FROM ORIGIN AND APPROACHES $\mu\pi/2$. AT BREAK FREQUENCY ω_b, PHASE IS $\mu\pi/4$.

LEMMA-3

MAGNITUDE DECIBEL LOG FREQUENCY OF $1/(T_a s^{\mu} +1)$, A LINE OF SLOPE -20μ WHEN $\omega > \omega_b$

THE PHASE DIAGRAM STARTS FROM ORIGIN & APPROACHES $-\mu\pi/2$. AT BREAK FREQUENCY ω_b PHASE IS. $-\mu\pi/4$.

STABILITY AREA IN w PLANE FOR FRACTIONAL ORDER SYSTEM

PROPERTY OF $\quad {}_0d_t^q x(t) = -ax(t) + bu(t)$ **TO STUDY FOR STABILTY**

POLE LOCATION OF SYSTEM TRANSFER FUNCTION FOR INTEGER ORDER SYSTEM (*q*=1) AND STUDIED IN COMPLEX s-PLANE.
THE STABILITY BOUNDARY IN s-PLANE IS IMAGINARY AXIS AND ANY POLE SHOULD BE IN L.H.P. IN s-PLANE.
FOR FRACTIONAL ORDER SYSTEM CORRESPONDING STUDY WILL BE IN s^q-PLANE. (ALSO CALLED w-PLANE OR "WEDGE"-PLANE)

$$\frac{X(s)}{U(s)} = \frac{b}{s^q + a}, q > 0$$

$$s \to w$$

$$w = s^q = \rho e^{j\phi} = \alpha + j\beta$$

$s = re^{j\theta}$ **THEN** $w = s^q = \left(re^{j\theta}\right)^q = r^q e^{jq\theta} = \rho e^{j\phi}$

$$\rho = r^q, \phi = q\theta$$

s-w PLANE STABILTY BOUNDARY MAPPING

IMAGINARY AXIS OF s-PLANE MAPS ONTO PAIR OF LINES MAKING A WEDGE AT $\phi = \pm q\pi / 2$

$$s = re^{\pm j\pi/2} \rightarrow w = r^q e^{\pm jq\pi/2}$$

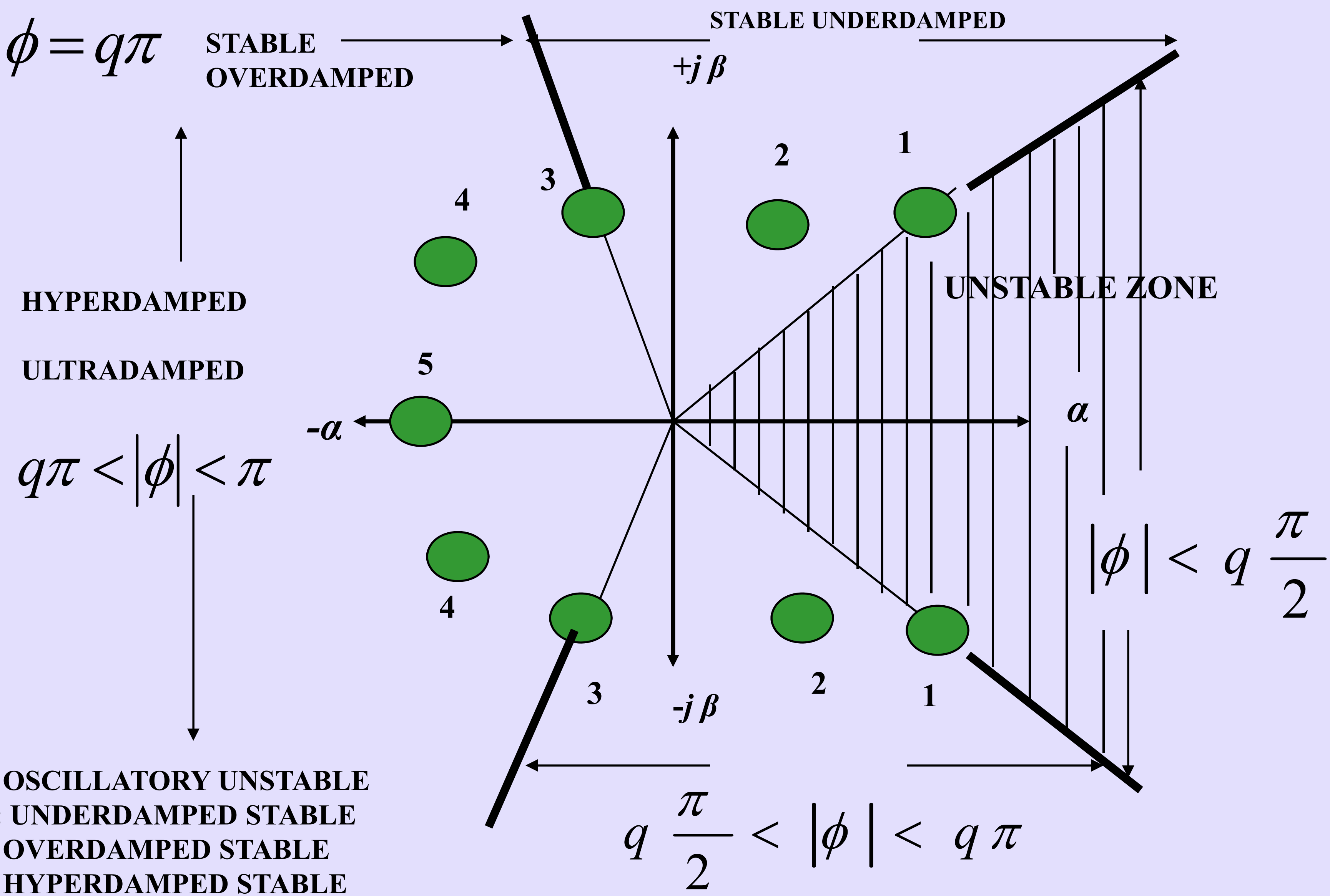
THE R.H.S OF s-PLANE MAPS INTO A WEDGE IN w-PLANE OF $|\phi| < q\dfrac{\pi}{2}$

NEGATIVE REAL AXIS OF s-PLANE $s = re^{\pm j\pi} \rightarrow w = r^q e^{\pm jq\pi}$

THUS ENTIRE PRIMARY SHEET OF s-PLANE MAPS TO w-PLANE OF WEDGE ANGLE <$^+_-$ *q*180^o. FOR *q*=1/2 THIS WEDGE ANGLE IS $^+_-$90^0. AND FOR THIS ORDER STABILTY BOUNDARY IN w-PLANE $^+_-$45^0.

THE PROPERTY OF FRACTIONAL ORDER CONTROLS IS DETERMINED BY POLE LOCATION IN w-PLANE.

POLE LOCATION IN w-PLANE & STABILITY BOUNDARY



1: OSCILLATORY UNSTABLE
2 : UNDERDAMPED STABLE
3: OVERDAMPED STABLE
4: HYPERDAMPED STABLE
5: ULTRADAMPED STABLE

POLES IN w-PLANE VIS-À-VIS POLES IN s-PLANE & PROPERTY

1	$\|\phi\| < q\frac{\pi}{2}$	s-RHP $\Re(s) > 0$	**UNSTABLE**
2	$q\frac{\pi}{2} < \|\phi\| < q\pi$	s-LHP $\Re(s) < 0$	**STABLE** **OSCILLATORY** **UNDERDAMPED**
3	$\phi = q\pi$	**S-NEGATIVE REAL AXIS** $s = re^{\pm j\pi}$	**STABLE** **OVERDAMPED**
4	$q\pi < \|\phi\| < \pi$	**SECONDARY REIMANN-SHEET**	**STABLE** **HYPERDAMPED**
5	$\phi = \pi$	**SECONDARY REIMANN-SHEET**	**STABLE** **ULTRADAMPED OVER-HYPER-DAMPED**

s-PLANE w-PLANE POLE PLACEMENT EXAMPLE

VALUE OF *a* IN THE T.F. DETERMINES THE POLE LOCATION IN s-PLANE & w-PLANE.

$$H(s)=\frac{1}{s+as^{0.5}+1}\leftrightarrow\frac{1}{w^2+aw+1}$$

$$w=\rho e^{j\phi}=s^{0.5}=\left(re^{j\theta}\right)^{0.5}$$

VALUE OF *a*	$w=\rho e^{j\phi}$	$s=re^{j\theta}$	**PROPERTY**
$a<-\sqrt{2}$	$w<e^{\pm j\frac{\pi}{4}}$	$s<e^{\pm j\frac{\pi}{2}}$	UNSTABLE
$0>a>-\sqrt{2}$	$e^{\pm j\frac{\pi}{2}}<w<e^{\pm j\frac{\pi}{4}}$	$e^{\pm j\pi}<s<e^{\pm j\frac{\pi}{2}}$	UNDERDAMPED
$2>a>0$	$e^{\pm j\pi}<w<e^{\pm j\frac{\pi}{2}}$	$e^{\pm j2\pi}<s<e^{\pm j\pi}$	HYPERDAMPED (SECONDARY REIMANN-SHEET)
$a>2$	$w>e^{\pm j\pi}$	$s>e^{\pm j2\pi}$	ULTRADAMPED (SECONDARY REIMANN-SHEET)

FRACTIONAL RESONANCE

$$H(s) = \frac{1}{s + as^{0.5} + 1}, 0 > a > -\sqrt{2}$$

w-PLANE POLE OF THE TRANSFER FUNCTION *H* (s) REMAIN IN THE UNDERDAMPED REGION. FOR THIS VALUE OF NEGATIVE *a*. THE SYSTEM IS ALSO STABLE FOR THIS VALUE OF *a*.

THE FACT THAT THE UNDERDAPMED POLES IMPLY RESONANCE WHILE THE TRANFER FUNCTION *H* (s) INDICATES AS HIGHEST POWER OF s BEING A FIRST ORDER SYSTEM.

FIRST ORDER SYSTEM CAN STILL GO TO RESONANCE DUE TO PRESENCE OF FRACTIONAL ORDER ELEMENTS.

ADDING MORE FRACTIONAL ORDER TERMS WILL LEAD TO SEVERAL RESONANCES.

THIS IS FRACTIONAL ORDER RESONANCE.

CONSEQUENTLY IT APPEARS THAT

"<u>HIGHEST</u> <u>POWER</u>"

OF THE LAPLACE VARIABLE IN THE CHARACTERISTIC EQUATION OF TRANFER FUNCTION IS

<u>NO LONGER AN INDICATOR OF THE EFFECTIVE ORDER OF THE SYSTEM OF A FRACTIONAL ORDER</u>

OR NUMBER OF RESONANCES TO EXPECT IN ITS FREQUENCY RESPONSE.

REALIZATION OF FRACTIONAL ORDER CONTROLLER

DIGITAL REALIZATION

a) GRUNWALD-LETNIKOV (GL) METHOD
b) POWER SERIES EXPANTION (PSE) METHOD
c) CONTINUED FRACTION EXPANTION (CFE) METHOD

ANALOG REALIZATION

a) OPERATIONAL AMPLIFIER CIRCUIT WITH FRACTANCE
b) CIRCUIT SYNTHESIS MULTIPLE LOOPS OF IMPEDANCE & ADMITTANCE (CFE)

CONCEPT OF GENERATING FUNCTION

BACKWARD DIFFERENCE

$$G(z^{-1}) = \frac{1 - z^{-1}}{T}$$

TRAPEZOIDAL (TUSTIN)

$$G_T(z^{-1}) = \left(\frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}\right) \qquad H_T(z) = \frac{T}{2}\frac{z+1}{z-1}$$

SIMPSON

$$H_S(z) = \frac{T(z^2 + 4z + 1)}{3(z^2 - 1)}$$

MIXED TUSTIN & SIMPSON (AL-ALOUNI)

$$H(z) = aH_S(z) + (1 - a)H_T(z), a \in [0,1]$$

$$H(z) = \frac{T(3-a)\left\{z^2 + \left[2(3+a)/(3-a)\right]z + 1\right\}}{6(z^2 - 1)}$$

USE GENERATING FUNCTIONS TO REALIZE FRACTIONAL ORDER DIFFERENTIAL OPERATIONS BY

POWER SERIES EXPANSION (PSE)

CONTINUED FRACTION EXPANSION (CFE)

PSE GIVES FIR FILTER REALIZATION CFE GIVES IIR FILTER REALIZATION

MIXED GENERATING FUNCTIONS OF TUSTIN AND SIMPSON BY AL-ALOUNI (1997) GIVES ONE MORE FREEDOM KNOB AS TUNER.

GL METHOD & SHORT MEMORY PRINCIPLE

$$
{}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & \Re(\alpha) > 0 \\ 1 & \Re(\alpha) = 0 \\ \displaystyle\int_0^t (d\tau)^{-\alpha} & \Re(\alpha) < 0 \end{cases}
$$

$$
{}_{(t-L)}D_t^{\alpha} f(t) \approx h^{-\alpha} \sum_{j=0}^{N(t)} \omega_j^{(\alpha)} f(t - jh)
$$

$$
N(t) = \min\left\{ \left[\frac{t}{h}\right], \left[\frac{L}{h}\right] \right\}
$$

$$
\omega_0^{(\alpha)} = 1, \omega_j^{(\alpha)} = \left(1 - \frac{1+\alpha}{j}\right) \omega_{j-1}^{(\alpha)}
$$

$$
j = 1, 2, 3 \ldots . N(t)
$$

DIGITAL REALIZATION OF $PI^{\alpha}D^{\beta}$ BY G-L METHOD

$$H(s) = \frac{U(s)}{E(s)} = K + T_i s^{-\alpha} + T_d s^{\beta}$$

T_i:INTEGRAL RESET RATE IS PER UNIT SECOND,
T_d:DERIVATIVE TIME IN SEC.

$$u(t) = Ke(t) + T_i . D_t^{-\alpha} e(t) + T_d . D^{\beta} e(t)$$

$$u(k) = Ke(k) + \frac{T_i}{T^{-\alpha}} \sum_{j=v}^{k} q_j e(k - jT) + \frac{T_d}{T^{\beta}} \sum_{j=v}^{k} d_j e(k - jT)$$

$$q_j \,\&\, d_j, b_0 = 1, b_j = (1 - \frac{1+\alpha}{j}) b_{j-1}$$

BINOMIAL COEFICIENTS

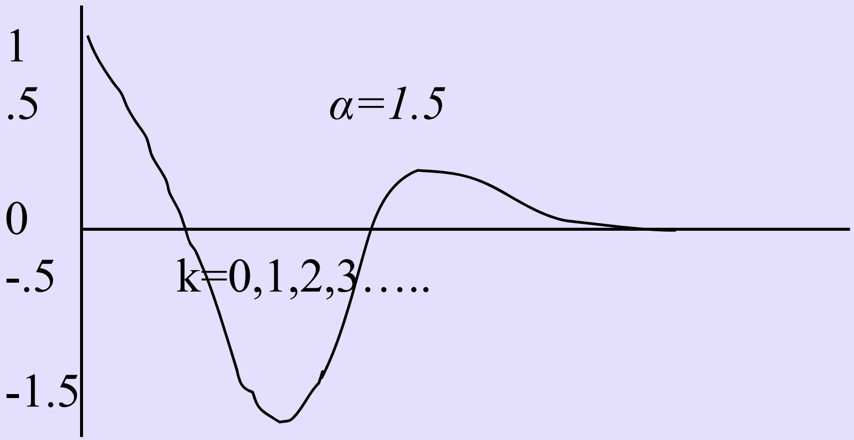
FOR DISCRETE STEP SIZE *(k=1,2,....)*, *T* is LENGTH OF TIME-STEP (SAMPLING TIME).

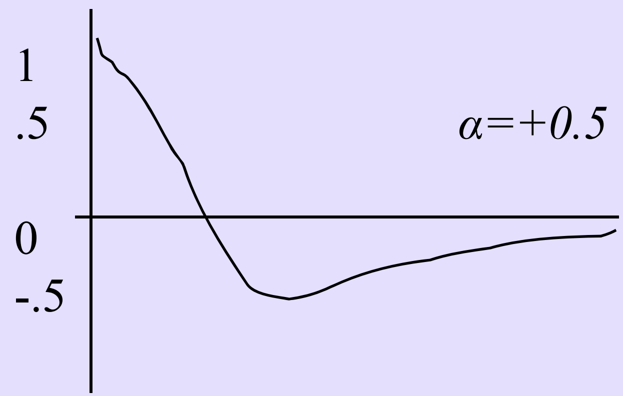
G-L METHOD REQUIRES ENTIRE HISTORY *v=0*, SO HAVE **SHORT MEMORY PRINCIPLE**. *L*=MEMORY LENGTH

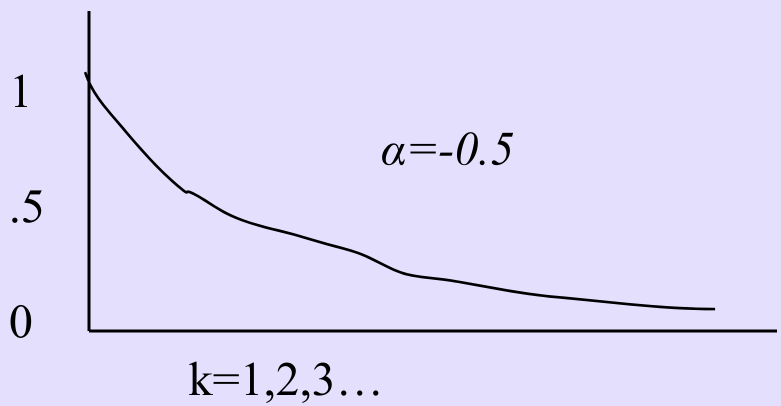
$$v = 0 .FOR .k < \frac{L}{T}, OR .v = k - (\frac{L}{T}), FOR .k > \frac{L}{T}$$

BINOMIAL COEFFICIENTS IN G-L METHOD

$$ {}_aD^{\alpha}_{t}e(t) \cong \frac{1}{h^{\alpha}} \sum_{j=0}^{k} b_j e(k - jh) $$







<u>OBSERVATION</u> b_j TENDS TO *0* AS *j* TENDS TO INFINITY.

G-L SERIES CAN BE SUITABLY TRUNCATED.

DIGITAL REALIZATION OF PI^aD^b BY POWER SERIES EXPANTION (PSE)

PI^aD^b ITSELF AN INFINITE DIMENSIONAL LINEAR FILTER

TRUNCATION OF PSE PROVIDES FINITE DIMMENSION,

IN PSE s^r IS EXPANDED.

$$s = \left(\frac{2}{T}\right)\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$$\left\{\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)\right\}^r \approx 1 - (2r)z^{-1} + (2r^2)z^{-2} - \left(\frac{2}{3}r + \frac{4}{3}r^3\right)z^{-3}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots + \left(\frac{4}{3}r^2 + \frac{2}{3}r^4\right)z^{-4} - \left(\frac{2}{5}r + \frac{4}{3}r^3 + \frac{4}{15}r^5\right)z^{-5}$$

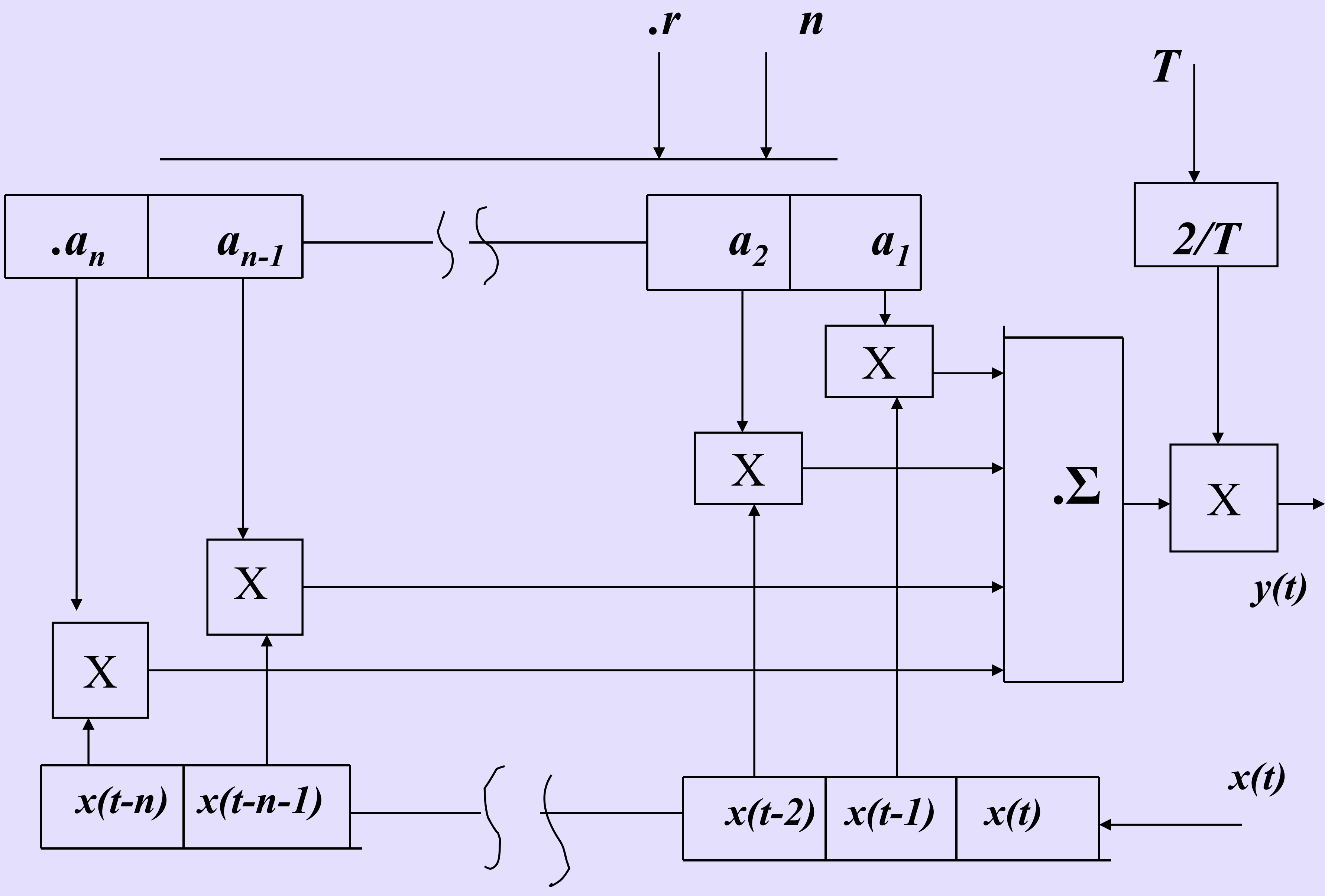
$$\ldots\ldots\ldots\ldots\ldots\ldots + \left(\frac{46}{45}r^2 + \frac{8}{9}r^4 + \frac{4}{45}r^6\right)z^{-6}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots + \left(\frac{2}{7}r + \frac{56}{45}r^3 + \frac{4}{9}r^5 + \frac{8}{316}r^7\right)z^{-7}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots + \left(\frac{88}{105}r^2 + \frac{44}{45}r^4 + \frac{8}{45}r^6 + \frac{2}{315}r^8\right)z^{-8}$$

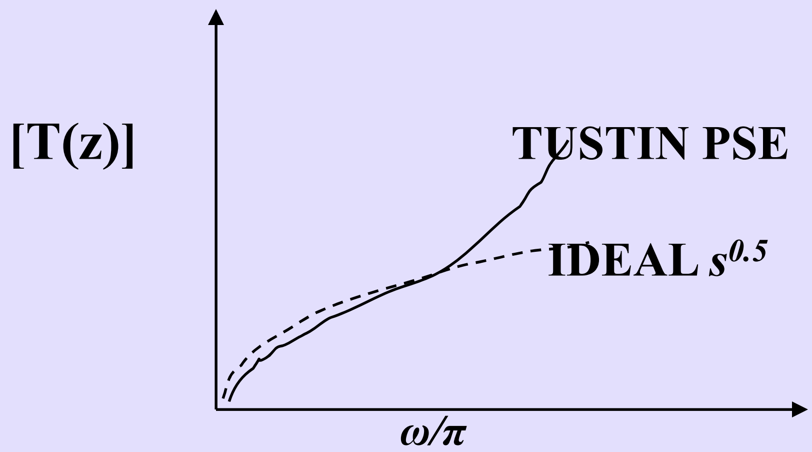
$$\ldots\ldots\ldots\ldots\ldots\ldots + \left(\frac{2}{9}r + \frac{3272}{2835}r^3 + \frac{8}{135}r^7 + \frac{76}{135}r^5 + \frac{4}{2835}r^9\right)z^{-9}$$

PSE DIGITAL REALIZATION s^r FIR FILTER STRUCTURE



$$Z(s^r) = T(z) = \left(\frac{2}{T}\right)^r \sum_{k=0}^{M} c(k) z^{-k}$$

PSE OF TUSTIN FOR s^r



ERRORS IN HIGH FREQUENCY:INTRODUCE **FRACTIONAL DELAY**

$$Q(z) = \left(\frac{2}{T}\right)^r \sum_{k=0}^{M} \beta(k) z^{-(N+kd)}$$

REDUCTION IN HIGH FREQUENCY ERRORS

$$F(z) = \frac{2}{Td}\frac{1 - z^{-d}}{1 + z^{-d}}$$

$$T(z) \rightarrow Q(z) = z^{-N}\left[\frac{2}{Td}\frac{1 - z^{-d}}{1 + z^{-d}}\right]^{r}$$

$$Q(z) = \sum_{k=0}^{M} \left\{\frac{2}{T}\right\}^{r} \frac{c(k)}{d^{r}} z^{-(N + kd)}$$

$$\ldots\ldots\ldots\ldots = \left(\frac{2}{T}\right)^{r} \sum_{k=0}^{M} \beta(k) z^{-(N + kd)}$$

FRACTIONAL DELAY OF USING UNIFORM SAMPLING INTERVAL & OBTAIN NON-INTEGER ORDER MULTIPLE OF SAMPLING INTERVAL.

FRACTIONAL DELAY FACILITATES THE USE OF TRADITIONALLY WELL KNOWN METHODS FOR UNIFORM SAMPLING, YET THE OBSERVATIONS OF SIGNAL VALUE AT ARBITARY LOCATIONS BETWEEN THE SAMPLES (INTERPOLATION)

DIGITAL IMPLEMENTATION s^r CONTINUED FRACTION EXPANSION (CFE)

$$D^{\pm r}(z)=\frac{Y(z)}{X(z)}=\left(\frac{2}{T}\right)^{\pm r}\ CFE\left[\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^{\pm r}\right]_{p,q}=\left(\frac{2}{T}\right)^{\pm r}\frac{P_p(z^{-1})}{Q_q(z^{-1})}$$

$$D^{r}(z)=1+\cfrac{z^{-1}}{\cfrac{-1}{2}.\cfrac{1}{r}+\cfrac{z^{-1}}{-2+\cfrac{z^{-1}}{\cfrac{3}{2}\cfrac{r}{r^2-1}}+\cfrac{z^{-1}}{2+\cfrac{z^{-1}}{-\cfrac{5}{2}\cfrac{r^2-1}{r(-4+r^r)}+}}}}$$

$$p = q = 1, r = 0.5, T = 1mS$$

$$G_1(z) = 44.72 \frac{z - 0.5}{z + 0.5}$$

$$p = q = 3, r = 0.5, T = 1mS$$

$$G_3(z) = 44.72 \frac{z^3 - 0.5z^2 - 0.5z + 0.125}{z^3 + 0.5z^2 - 0.5z - 0.125}$$

$$G_9(D^{\pm r}\{z\}) = K \frac{a_9 z^9 - a_8 z^8 - a_7 z^7 + a_6 z^6 + a_5 z^5 - a_4 z^4 - a_3 z^3 + a_2 z^2 + a_1 z^1 - a_0}{a_9 z^9 + a_8 z^8 - a_7 z^7 - a_6 z^6 + a_5 z^5 + a_4 z^4 - a_3 z^3 - a_2 z^2 + a_1 z^1 + a_0}$$

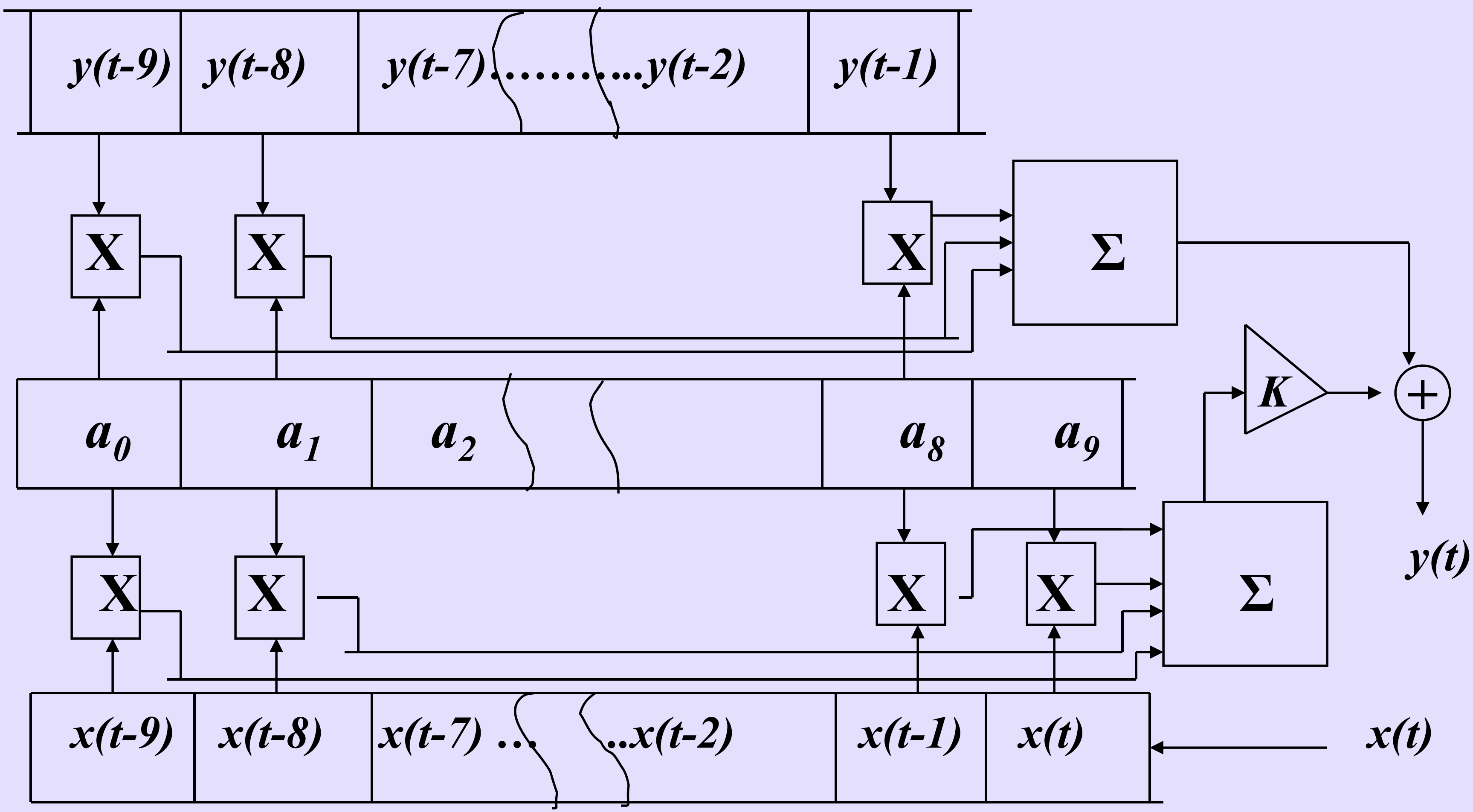
$$K = \left(\frac{2}{T}\right)^{\pm r}, r = 0.5, T = 1mS, K = 44.72$$

IIR TYPE IMPLEMENTATION EXPONENTIAL AVERAGING

$$p = q = 1, r = 0.5, T = 1mS, K = 44.72$$

$$y(t) = Kx(t) - \frac{K}{2} x(t-1) - 0.5 y(t-1)$$

CFE DIGITAL REALIZATION IIR STRUCTURE



ODD ORDER POLYNOMIAL EXPANSION IN CFE

FOR ODD CFE EXPANSION THE POLE-ZERO MAPS ARE NICELY BEHAVED, THAT IS ALL THE POLE-ZERO LIE INSIDE THE UNIT CIRCLE & THE POLES AND ZEROS ARE INTERLACED ALONG SEGMENT OF THE REAL AXIS CORRESPONDING TO z (-1,1). HOWEVER WHEN EVEN EXPANSION IS USED THERE MAY BE ONE CANCELLING POLE-ZERO PAIR, WHICH SOMETIMES MAY NOT BE DESIRABLE.

THEREFORE SUGGESTION IS FOR CFE EXPANSION USE ODD POLYNOMIAL EXPANSION

ANALOG REALIZATION BY FRACTANCE

IMPEDANCE OF GOOD (NON-LEAKY) CAPACITOR ***$Z_c=1/(j\omega C)$***
DIELECTRIC CONSTANT IS ***$\varepsilon=\varepsilon'-j0$***
DUE TO DI-ELECTRIC ABSORPTION ***$Z_c=1/(j\omega C)^{0.999}$,***
$\varepsilon=\varepsilon'-j(1X10^{-3})$
IF CAPACITOR IS LOSSY ***$\varepsilon=\varepsilon'-j\,\varepsilon''$, $\varepsilon'=\varepsilon''=10^6$***, FOR WIDE FREQUENCY RANGE & TEMPERATURE THEN **FRACTANCE** IS REALIZED WITH TRANSFER FUNCTION

MADE BY DI-ELECTRIC LITHIUM HYDRAZIUM SULPHATE $Li\ N_2H_2SO_4$,HERE ORDER $\alpha=-0.4$ TO -0.6

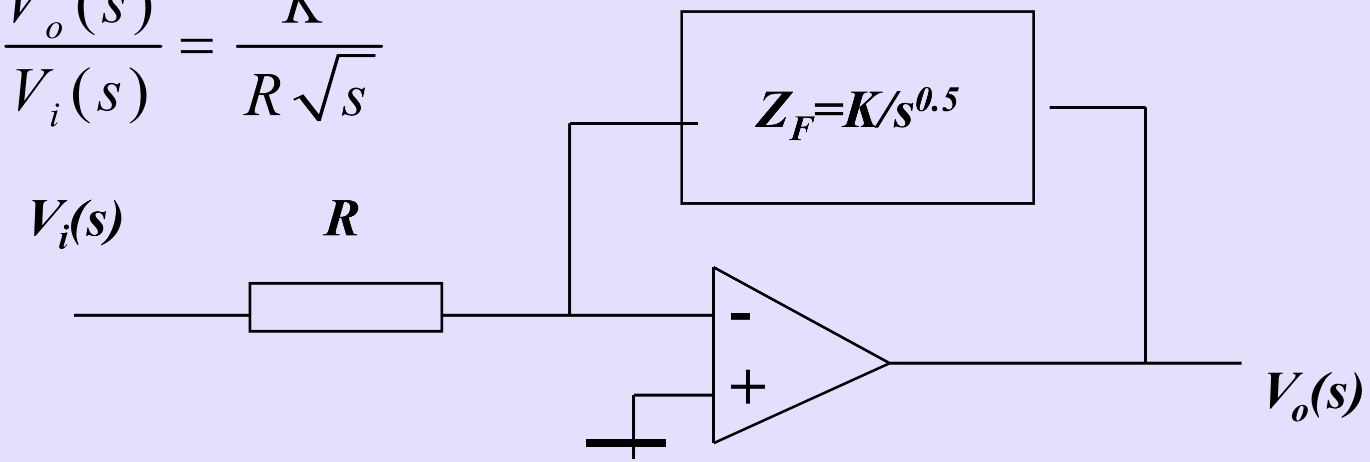
$$\varepsilon = \varepsilon_r \omega^{-1/2}(1-j) = \varepsilon_r \sqrt{2}(j\omega)^{-1/2}$$

$$Z_F = \frac{1}{j\omega C\varepsilon} = \frac{1}{j\omega C\varepsilon_r\sqrt{2}(j\omega)^{-1/2}} = \frac{1}{C\varepsilon_r\sqrt{2}(j\omega)^{1/2}}$$

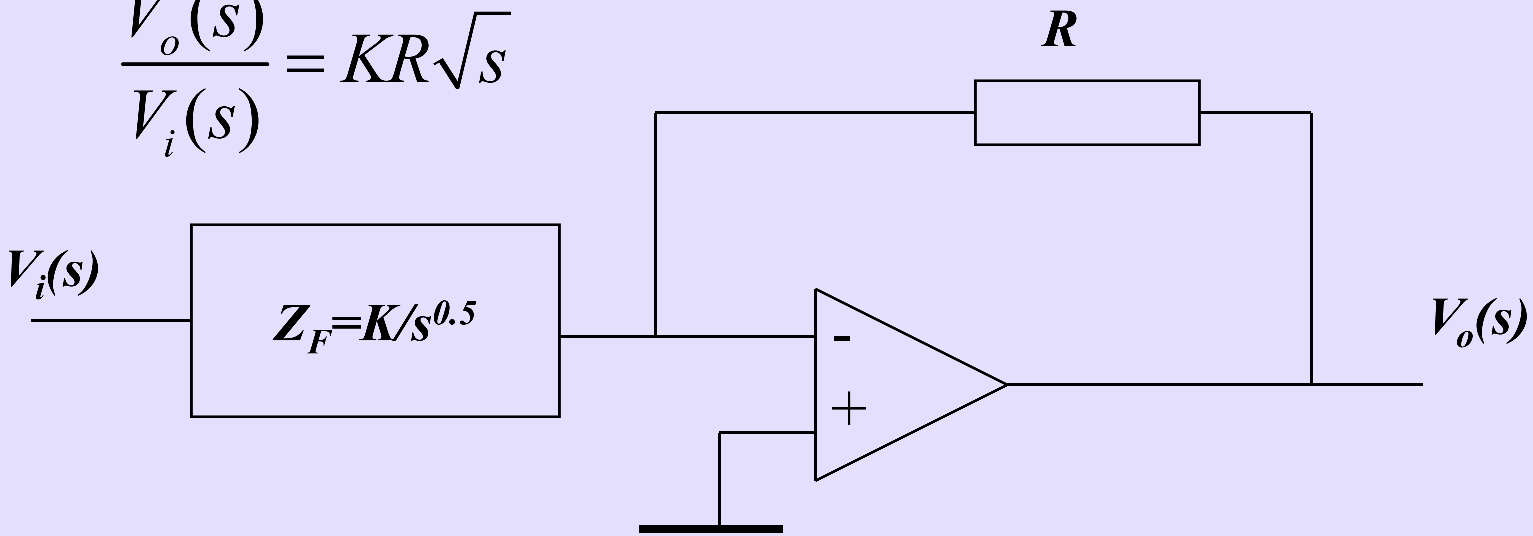
$$Z_F = \frac{K}{s^{0.5}}$$

FRACTANCE CIRCUIT DIAGRAM

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{R\sqrt{s}}$$



$$\frac{V_o(s)}{V_i(s)} = KR\sqrt{s}$$



CIRCUIT SYNTHESIS BY IMPEDANCE ADMITTANCE MULTIPLE LOOP (CFE)

$$Z(s)=Z_1(s)+\cfrac{1}{Y_2(s)+\cfrac{1}{Z_3(s)+\cfrac{1}{+.....+Z_{2n-1}(s)+\cfrac{1}{Y_{2n}(s)}}}}$$

$$x=a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\cfrac{1}{a_3+..}}}$$

$$x = \pi$$

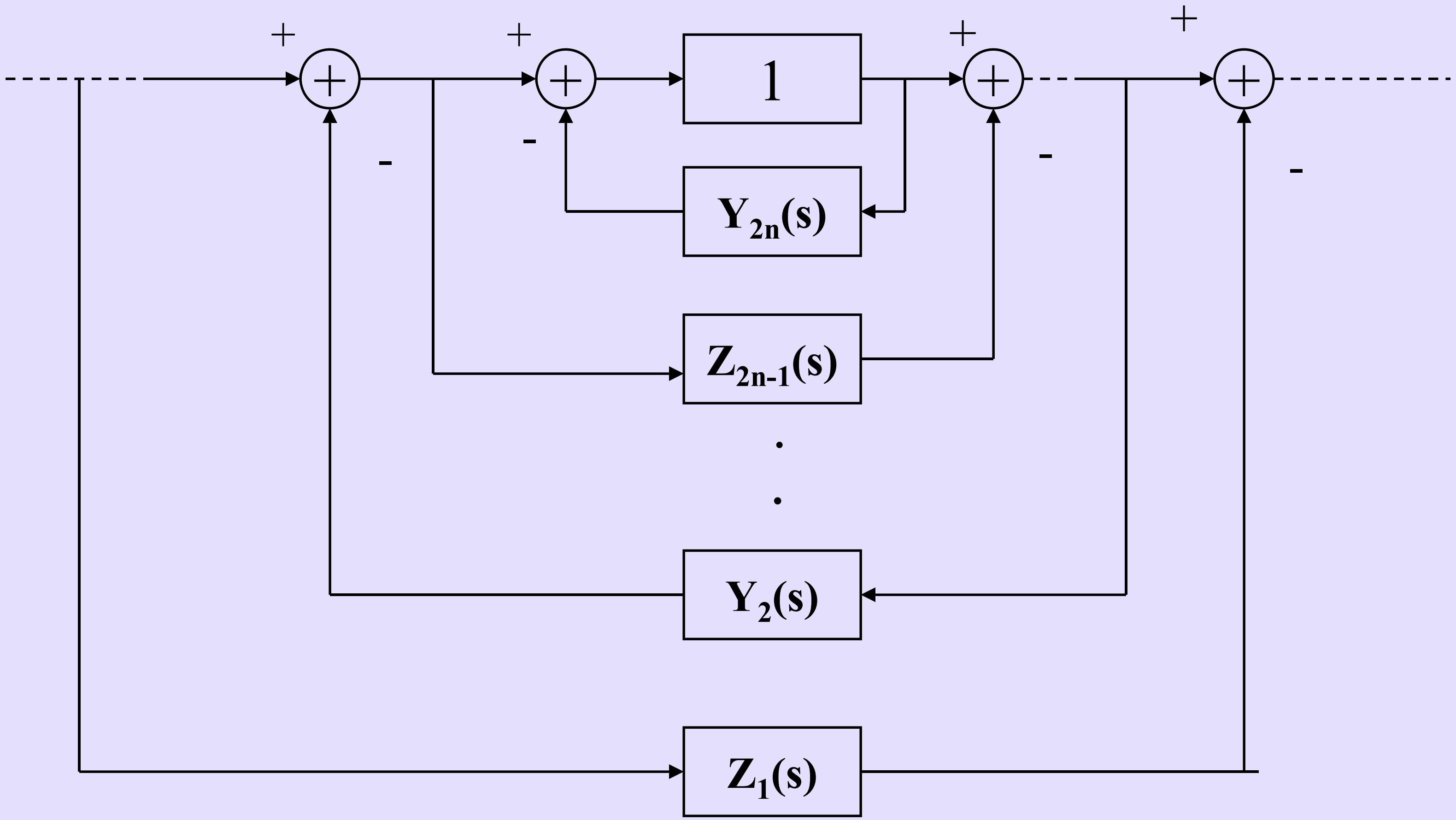
$$a_0 = [\pi] = 3$$

$$a_1 = \left[\frac{1}{\pi - 3}\right] = 7$$

$$a_2 = \left[\frac{1}{\frac{1}{\pi - 3} - 7}\right] = 15$$

CFE IN NUMBER THEORY IS GENERAL REPRESENTATION OF REAL NUMBERS.

MULTIPLE LOOP DIAGRAM OF Z(s) Y(s) OF CFE



GENERAL ADVANTAGE OF CIRCUIT SYNTHESIS WITH CFE

CFE CAN BE USED FOR REALIZATION OF ARBITARY TRANSFER FUNCTION EVEN TRANSCENDENTAL & FRACTANCE. CFE CAN PROVIDE A GENERAL TOOL FOR VARIOUS REALIZATIONS OF FRACTIONAL ORDER CONTROLS (BOTH ANALOG & DIGITAL).

THE CASE OF NEGATIVE COEFFICIENTS IN CFE IS SPECIAL INTEREST OF REALIZATION OF NEGATIVE IMPEDANCE/ADMITTANCE (NEGATIVE R, NEGATIVE L,NEGATIVE C) BY OP-AMP CIRCUITS.

FRACTIONAL ORDER STATE SPACE MODEL

$$G(s): \_\_\ a_2 y^{(\alpha)}(t) + a_1 y^{(\beta)}(t) + a_0 y(t) = u(t)$$

$$H(s): PI^{\lambda} D^{\delta}\ : \_\_\_\ u(t) = Ke(t) + T_i D_t^{-\lambda} e(t) + T_d D_t^{\delta} e(t)$$

$$y(t) = x(t) = x_1(t), \_\_\_\_\_\ x^{(\beta)}(t) = x_2(t)$$

$$\begin{bmatrix} x_1^{(\beta)}(t) \\ x_2^{(\alpha-\beta)}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{a_0}{a_2} & -\dfrac{a_1}{a_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{a_2} \end{bmatrix} \begin{bmatrix} 0 \\ u(t) \end{bmatrix}$$

$$y(t) = x_1(t)$$

STATE-VARIABLE:

$$GL\ldots {}_aD_t^{\pm r} f(t) = \lim_{T \to 0} \frac{1}{T^{\pm r}} \sum_{j=0}^{\left[\frac{t-a}{T}\right]} b_j^{\pm r} f(t - jT) = \lim_{T \to 0} \frac{1}{T^{\pm r}} \Delta_T^{\pm r} f(t)$$

$$x_{1,k+1} = -\sum_{j=1}^{k+1} b_j^{\beta} x_{1,k+1-j} + T^{\beta} x_{2,k}$$

$$x_{2,k+1} = -\sum_{j=1}^{k+1} c_j^{(\alpha-\beta)} x_{2,k+1-j} + T^{(\alpha-\beta)} \left[-\frac{a_0}{a_2} x_{1,k} - \frac{a_1}{a_2} x_{2,k} + \frac{1}{a_2} u_k \right]$$

$$y_k = x_{1,k}$$

STATE SPACE FRACTIONAL CONTROLS

$$D^{\alpha} \mathrm{X} = \mathrm{A}\,\mathrm{X} + \mathrm{B}\,U$$

$$y = C\,\mathrm{X} + D\,U$$

$$\begin{bmatrix} D^{\alpha}x_1 \\ D^{\alpha}x_2 \\ * \\ D^{\alpha}x_k \end{bmatrix} = \begin{bmatrix} -b_{n-1} & -b_{n-2} & * & * & -b_0 \\ 1 & 0 & * & * & 0 \\ * & * & * & * & * \\ * & * & * & * & * \\ 0 & 0 & * & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ * \\ x_k \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ * \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} a_m & a_{m-1} & * & a_1 & a_0 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & * & x_{k-1} & x_k \end{bmatrix}^T$$

STATE VECTOR FEED-BACK CONTROLLER POLE PLACEMENT

$${}_{0}D_{t}^{q}\,\bar{x}(t) = \mathrm{A}\,\bar{x}(t) + \mathrm{B}\,\bar{u}(t) = {}_{0}d_{t}^{q}\,\bar{x}(t) + \bar{\psi}(q, \bar{x}, a, 0, t)$$

$$\bar{u}(t) = -\mathrm{K}\,\bar{x}(t)$$

$$\bar{y}(t) = \bar{C}\,\bar{x}(t) + \bar{D}\,\bar{u}(t)$$

POLE PLACE IN *w*-PLANE AS:

$$w = -2 \pm j4, \& \; w = -6$$

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -6 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \bar{C} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \bar{D} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\rho(A_c) = \begin{bmatrix} \mathrm{B} & \mathrm{AB} & \mathrm{A}^2\mathrm{B} \end{bmatrix} = 3$$

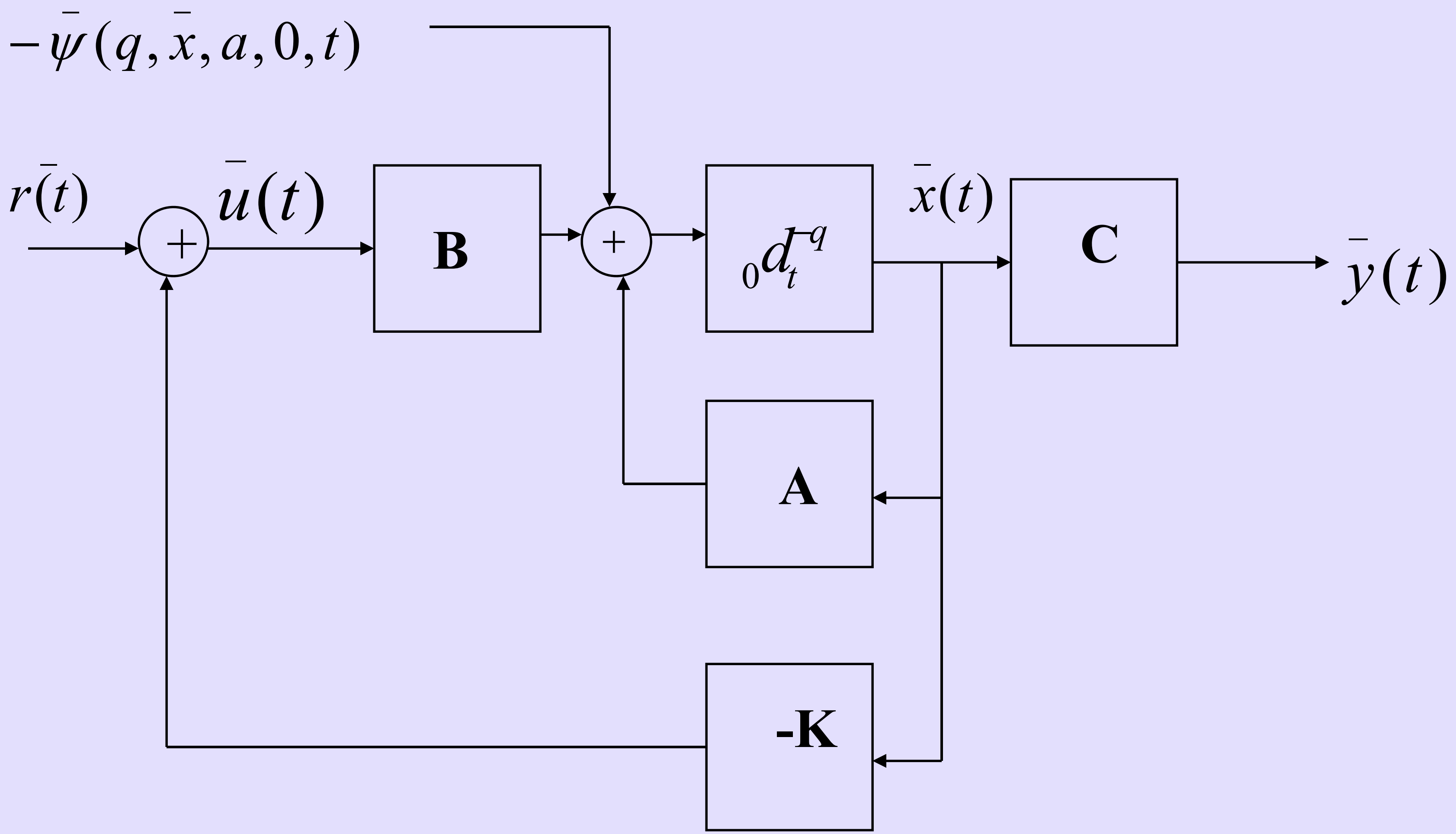
CONTROLLABLE SYSTEM

$$\alpha(w) = (w + 2 - j4)(w + 2 + j4)(w + 6) = w^3 + 10w^2 + 44w + 120$$

$$\det\left[w\mathrm{I} - \mathrm{A} + \mathrm{BK}\right] = w^3 + (6 + k_3)w^2 + (4 + k_2)w + (2 + k_1)$$

$$\mathrm{K} = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} = \begin{bmatrix} 4 & 40 & 118 \end{bmatrix}$$

STATE FEED BACK VECTOR CONTROLLER BLOCK



OBSERVER IN FRACTIONAL ORDER VECTOR SPACE

$$ {}_0D_t^q \bar{x}(t) = {}_0d_t^q \bar{x}(t) + \bar{\psi}(q, \bar{x}, a, 0, t) = \mathrm{A}\bar{x}(t) + \mathrm{B}\bar{u}(t) \qquad \bar{y}(t) = \bar{C}\,\bar{x}(t) $$

$$ \mathrm{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \bar{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} $$

$$ w = -2 \pm j3.464, \& \, w = -5 $$

POLE LOCATION IN *w*-PLANE

$$ \rho\left[\mathrm{A}_o\right] = \begin{bmatrix} \bar{C} \\ \bar{C}\,\mathrm{A} \\ \bar{C}\,\mathrm{A}^2 \end{bmatrix} = 3 $$

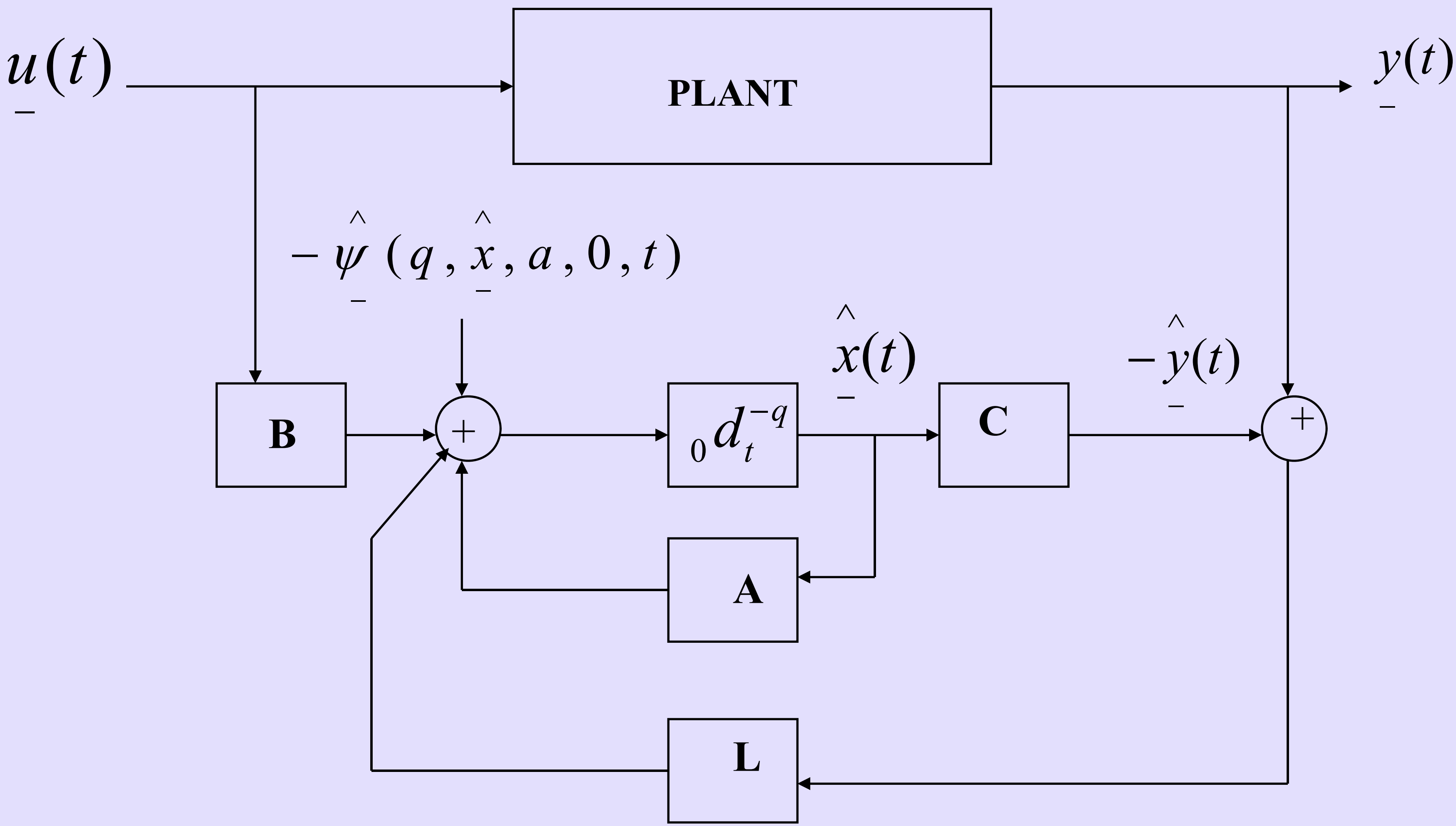
$$ \alpha(w) = (w + 2 - j3.464)(w + 2 + j3.464)(w + 5) $$
$$ = w^3 + 9w^2 + 36w + 80 $$

$$ \det\left[w\mathrm{I} - \mathrm{A} + \bar{L}\,\bar{C} \right] \equiv \alpha(w) $$

OBSERVER GAINS

$$ \bar{L} = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 1 \end{bmatrix} $$

OBSERVER OF FRACTIONAL VECTOR



STATE TRANSITION MATRIX:

$$X(t) = L^{-1}\left\{X(s)\right\} = L^{-1}\left\{(s^{\alpha}I - A)^{-1}BU(s) + (s^{\alpha}I - A)^{-1}X(0)\right\}$$

BY DEFINING MATRIX FOR *t*>0 $\Phi(t) \equiv L^{-1}\left\{(s^{\alpha}I - A)^{-1}\right\}$

$$X(t) = \Phi(t)X(0) + \Phi(t) * \left[BU(t)\right] = \Phi(t)X(0) + \int_0^t \Phi(t-\tau)BU(\tau)d\tau$$

$$X(t) = \left[1 + \frac{AX(0)}{\Gamma(1+\alpha)}t^{\alpha} + \frac{A^2X(0)}{\Gamma(1+2\alpha)}t^{2\alpha} + \dots\dots\dots\right]$$

$$X(t) = \left[\sum_{k=0}^{\infty} \frac{A^k t^{k\alpha}}{\Gamma(1+k\alpha)}\right]X(0) = E_{\alpha}(At^{\alpha})X(0)$$

DISCRETIZE STATE SPACE MODEL FORMALISM

TRADITIONAL INTEGER MODEL

$$x_{k+1} = \mathrm{A}\,x_k + \mathrm{B}\,u_k + w_k$$

$$y_k = C\,x_k + v_k$$

FIRST ORDER FINITE DIFFERENCE FOR SAMPLE x_k IS

$$\Delta^1 x_k$$

STATE VARIABLE: x_k
SYSTEM INPUT: u_k
SYSTEM OUTPUT: y_k
SYSTEM NOISE: w_k
OUTPUT NOISE: v_k

$$\Delta^1 x_{k+1} = x_{k+1} - x_k$$

$$\mathrm{A}_d = \left(\mathrm{A} - \mathrm{I}\right)$$

$$\Delta^1 x_{k+1} = \mathrm{A}_d\,x_k + \mathrm{B}\,u_k + w_k$$

$$x_{k+1} = \Delta x_{k+1} + x_k$$

$$y_k = C\,x_k + v_k$$

LINEAR FRACTIONAL: STOCHASTIC DISCRETE STATE SYSTEM:

USE FRACTIONAL DIFFERENCE AS:

$$\Delta^n x_k = \frac{1}{h^n} \sum_{j=0}^{k} (-1)^j \binom{n}{j} x_{k-j}$$

$$\Delta^n x_{k+1} = \mathrm{A}_d x_k + \mathrm{B} u_k + w_k$$

$$x_{k+1} = \Delta^n x_{k+1} - \sum_{j=1}^{k+1} (-1)^j \binom{n}{j} x_{k+1-j}$$

$$y_k = C x_k + v_k$$

SYSTEM WITH SEVERAL ORDERS REPRESENTATION AS:

$$\Delta^\gamma x_{k+1} = \mathrm{A}_d x_k + \mathrm{B} u_k + w_k$$

WHERE $\gamma_k = diag\left[\binom{n_1}{k}, \binom{n_2}{k} \ldots \binom{n_N}{k}\right]$

$$x_{k+1} = \Delta^\gamma x_{k+1} - \sum_{j=1}^{k} \gamma_j x_{k+1-j}$$

$$\Delta^\gamma x_{k+1} = \begin{bmatrix} \Delta^{n_1} x_{1,k+1} \\ \Delta^{n_2} x_{2,k+1} \\ * \\ \Delta^{n_N} x_{N,k+1} \end{bmatrix}$$

$$y_k = C x_k + v_k$$

COMMENTS ON VECTOR SPACE FRACTIONAL CONTROL

FOR VECTOR STATE SPACE REPRESENTATIONS

USE THE NORMAL CONTROLLABILITY OBSERVABILTY TESTS

USE VECTOR FEED BACK GAIN VALUES BY PLACEMENT OF POLES IN *w*-PLANE

USE A,B,C,D STATE,INPUT,OUTPUT,FEED-THROUGH MATRICES AS FOR INTEGER ORDER CONTROLLER, WITHOUT BOTHERING ABOUT INITIALIZATION VECTOR.

LQR OPTIMAL REGULATOR YET TO BE DESCRIBED SO USE INTEGER ORDER APPROACHES.

USE KALMAN DECOMPOSITION RULES AND MAP FRACTIONAL ORDER KALMAN FILTER FOR STATE ESTIMATION

DISADVANTAGE OF FRACTIONAL CONTROL METHOD:

1. FRACTIONAL ORDER DIFFERENTIAL EQUATION ACCUMULATE THE WHOLE INFORMATION OF THE FUNCTION IN A WEIGHTED FORM.
2. FRACTIONALLY DIFFERENTIATED STATE VARIABLES MUST BE KNOWN AS LONG AS SYSTEM HAS BEEN OPERATED.
3. THIS IS KNOWN AS INITIALIZATION FUNCTION.
4. FOR INTEGER ORDER IT IS CONSTANT & FOR FRACTIONAL ORDER IT IS TIME VARYING.
5. INTEGER ORDER SYSTEM SET OF STATES ALONG WITH SYSTEM EQUATIONS ARE SUFFICIENT TO PREDICT THE RESPONSE.
6. THE FRACTIONAL DYNAMICS VARIABLES DO NOT REPRESENT THE STATE OF THE SYSTEM.
7. FRACTIONAL DYNAMICS REQUIRE HISTORY OF STATES (OR SUFFICIENT NUMBER BY SHORT MEMORY PRINCIPLE) FOR INITIALIZATION FUNCTION COMPUTATION.
8. THE MEMORY EFFECT REQUIRES LARGE SIZE MEMORY.
9. FOR GL MINIMUM 100 POINTS.
10. FOR PSE/CFE TEN TIMES LESS THAN GL.

OFFERS OF CONTROLLERS IN FRACTIONAL ORDER UNIVERSE

1. **INTEGRAL:** FRACTIONAL INTEGRAL $H(s)=K s^{-q}$ FOR COMPENSATOR HAS INTERESTING FEATURE OF FRACTIONAL INTEGRAL AS THEY STILL ALLOW TRACKING OF STEP REFERENCE WHILE ALLOWING FREEDOM TO TUNE THE LOW & HIGH FREQUENCY BEHAVIOR BY TUNING q.
2. **DERIVATIVE:** ALTHOUGH PURE DERIVATIVE CONTROL IS SELDOM USED, DERIVATIVE OF FRACTIONAL ORDER $H(s)=K s^{q}$ WILL HAVE LESS NOISE AMPLIFICATION AT HIGH FREQUENCY.
3. **PI,PD,PID:** ALLOWS ANY VALUE OF q AND HAVE CONTINUUM ORDER VARIATION OF I & D: $H(s)=K_p+K_i s^{-q1}+K_d s^{q2}$.

   LEADS & LAGS: LEAD COMPENSATORS ARE OFTEN USED TO HELP STABILIZE MARGINALLY UNSTABLE SYSTEMS. LAGS ARE USED REDUCE THE MAGNITUDE OF THE H.F. LOOP GAINS OF THE SYSTEM. USING FRACTIONAL LEAD LAGS BENEFITS THE EASIER SHAPING OF THE OPEN-LOOP AND CLOSE-LOOP FREQUENCY RESPONSE.

$$H(s) = \frac{K(s^{q_1}+a)}{(s^{q_2}+b)}$$

4. **START POINT SINGULARITY:**

$$g(0)=\lim_{s\to\infty} sG(s)=\pm\infty \rightarrow g(t)=L^{-1}\left\{G(s)\right\}_{t\to 0}=\pm\infty \qquad H(s)=ks^{q-1}$$

SYSTEM ORDER IDENTIFICATION

CLASSICALLY FOR INTEGER ORDER:

1. THE HIGHEST DERIVATIVE IN ORDINARY DIFFERENTIAL EQUATION.
2. THE HIGHEST POWER OF LAPLACE VARIABLE s IN CHARACTERISTIC EQUATION.
3. THE LENGTH OF THE STATE VECTOR.
4. THE NUMBER OF SINGULARITIES IN THE CHARACTERISTIC EQUATION.
5. THE NUMBER OF ENERGY STORAGE ELEMENTS.
6. THE NUMBER OF INDEPENDENT SPATIAL DIRECTIONS IN WHICH TRAJECTORY CAN MOVE.
7. THE NUMBER OF INITIALIZATION CONSTANTS REQUIRED FOR DIFFERENTIAL EQUATION.
8. THE NUMBER OF DEVICES THAT CAN ADD 90^{0} SINOSOIDAL STEADY STATE PHASE LAG.
9. THE NUMBER OF DEVICES THAT RETAIN SOME MEMORY OF PAST.

SYSTEM ORDER IDENTIFICATION FOR FRACTIONAL SYSTEM

UNFORTUNATELY FOR FRACTIONAL ORDER SYSTEMS THE ORDER OF HIGHEST DERIVATIVE DOES NOT INFER AT ALL OF THE PROCESS OF SYSTEM ORDER IDENTIFICATION.INDEED THE MOST IMPORTANT CHARACTERISTICS OF ORDER IN INTEGER SYSTEMS IS THROUGH THE NUMBER OF INITIALIZATION CONSTANTS, WHICH TOGETHER WITH THE DIFFERENTIAL EQUATION ALLOW PREDICTION OF FUTURE BEHAVIOR. IN SYSTEM TERMINOLOGY THIS INFORMATION PROVIDES “STATES” OF THE SYSTEM.

CLEARLY THE ORDER OF HIGHEST DERIVATIVE IN FRACTIONAL SYSTEM DOES NOT HAVE THIS PROPERTY NOR DOES IT PREDICT THE ASSOCIATED NUMBER OF ENERGY STORAGE ELEMENTS. THUS THE ISSUE OF ORDER AND INFORMATION REQUIRED TOGETHER WITH FRACTIONAL DIFFERENTIAL EQUATION TO PREDICT FUTURE IS FUNDAMENTAL AND SHOULD BE TREATED DIFFERENTLY.

FROM SIMPLE EXAMPLES OF FRACTIONAL SYSTEMS A TIME DEPENDENT TERM RESULTING FROM THE INITIAL SPATIAL DISTRIBUTED CONDITIONS SHOULD BE ADDED TO THE FORCED RESPONSE.

SYSTEM ORDER IDENTIFICATION

CONCEPT OF CONTINUOUS ORDER DISTRIBUTION IN MASS-SPRING-DAMPER SYSTEM

$$m_0 d_t^2 x(t) + b_0 d_t^1 x(t) + kx(t) = f(t)$$

$$\left(ms^2 + bs + k\right) X(s) = F(s)$$

$$\left(ms^2 + bs + k_q s^q + K\right) X(s) = F(s)$$

$$\left(ms^2 + bs + k_{q2} s^{q2} + k_{q1} s^{q1} + k\right) X(s) = F(s)$$

$$\left(ms^2 + k_{q4} s^{q4} + k_{q3} s^{q3} + bs + k_{q2} s^{q2} + k_{q1} s^{q1} + k\right) X(s) = F(s)$$

$$\left(\sum_{n=0}^{N} k_n s^{qn}\right) X(s) = F(s) \qquad 0 \le q\,n \le 2$$

REMOVING RESTRICTION ON *qn*

$$\left(\int_0^\infty k(q) s^q dq\right) X(s) = F(s)$$

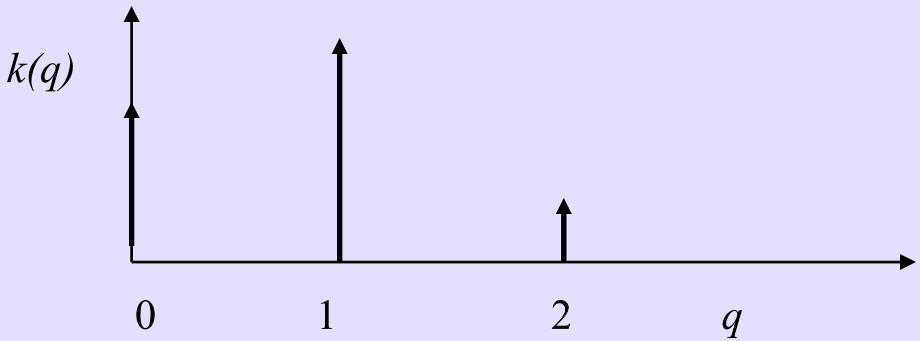
k(q)* IS CONTINUOUS FUNCTION OF ORDER *q

CONTINUOUS ORDER DISTRIBUTION:

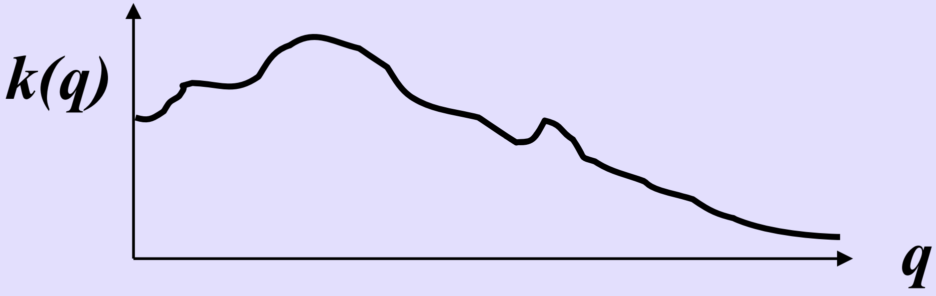
MASS SPRING DAMPER SYSTEM REPRESENTATION IN *k(q)* FUCTION

$$\left(\int_0^2 \left[m\delta(q-2)+b\delta(q-1)+k\delta(q)\right]s^q dq\right)X(s)=F(s)$$

DISCRETE ORDER AT 0,1,2



CONTINUOUS ORDER FUNCTION



SYSTEM IDENTIFICATION FROM FREQUENCY DOMAIN DATA

$$\left(\int_0^\infty k(q) s^q \, dq \right) X(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{\int_0^\infty k(q) s^q \, dq} = G(s)$$

$$\sum_{n=0}^{N} k_n (j\omega_j)^{nQ} Q = \frac{1}{G(j\omega_j)}$$ **FOR ANY *j***

$$Qk_0 + Qk_1(j\omega_1)^Q + Qk_2(j\omega_1)^{2Q} + \ldots + Qk_N(j\omega_1)^{NQ} = \frac{1}{G(j\omega_1)}$$

$$Qk_0 + Qk_1(j\omega_2)^Q + Qk_2(j\omega_2)^{2Q} + \ldots + Qk_N(j\omega_2)^{NQ} = \frac{1}{G(j\omega_2)}$$

....

....

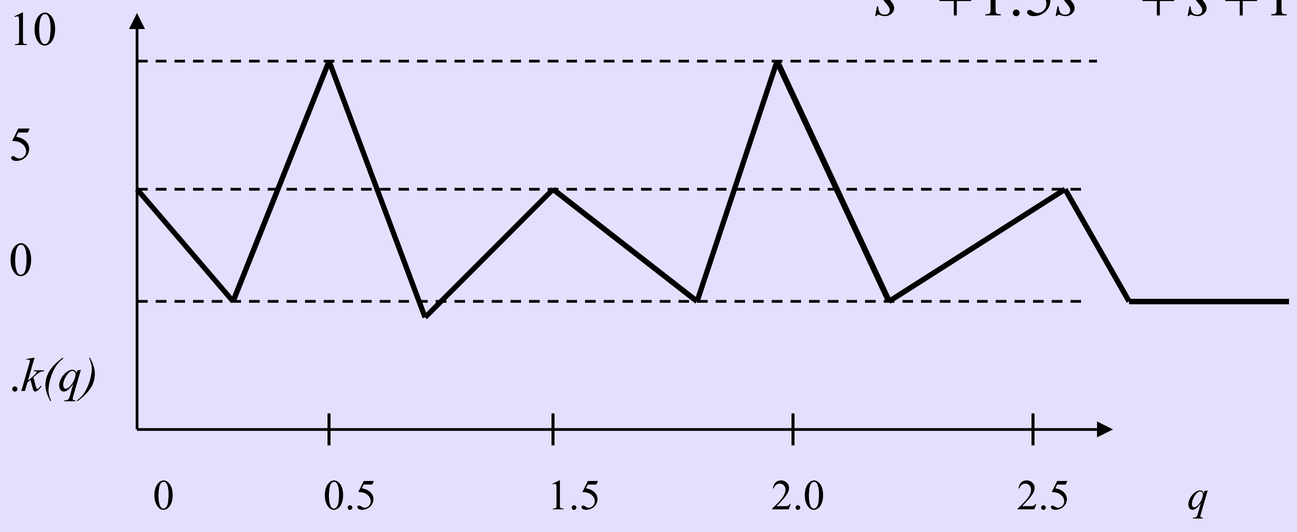
$$Qk_0 + Qk_1(j\omega_M)^Q + Qk_2(j\omega_M)^{2Q} + \ldots + Qk_N(j\omega_M)^{NQ} = \frac{1}{G(j\omega_M)}$$

ADVANCE RESEARCH IN SYSTEM IDENTIFICATION APPROACH

DISADVANTAGE IS

AS *Q* IS MADE SMALLER THE MATRIX TENDS TO BECOME SINGULAR.
SO IS THE CASE AS WHEN *N* IS MADE LARGER.
DESIGN OF PROPER FILTER AND INACCURACIES DUE TO SMEARING OF ORDER AT THE DESIGNATED ORDER POINT BE ACTIVE AREA OF NUMERICAL RESEARCH.
HAVE TIME DOMAIN APPROACH FRACTIONAL KALMAN FILTER.

$$G(s) = \frac{1}{s^2 + 1.5s^{1.5} + s + 1.5s^{0.5} + 1}$$



CONTINUUM FEED-BACK ORDER CONTROLLER

PLANT *G* (s) AS UNDAMPED OSCILLATOR

$$G(s) = \frac{1}{s^2 + 1}$$

CONTROLLED BY A POSSIBLE COMPENSATOR *H* (s) AS

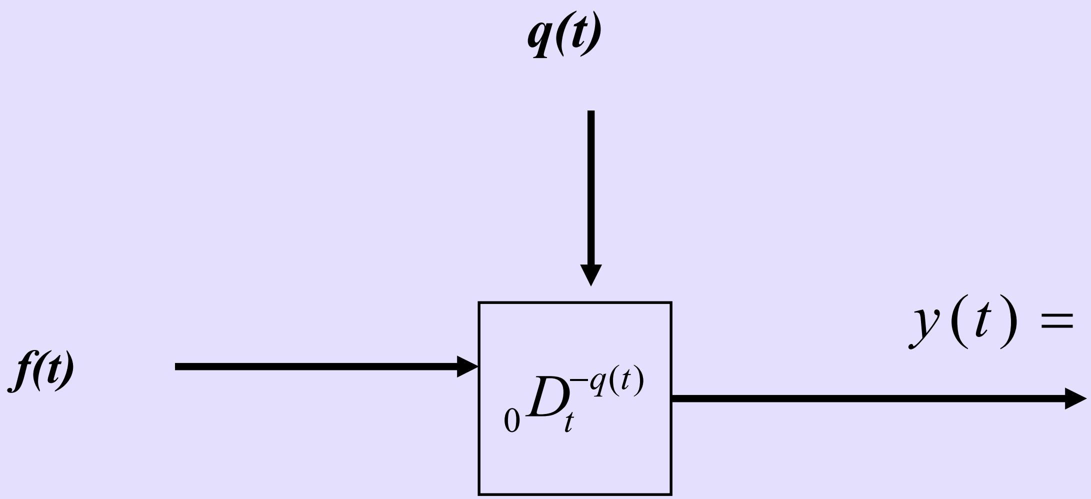
$$H(s) = \int_0^2 k(q) s^q dq$$

CLOSE LOOP FREQUENCY RESPONSE GIVEN BY CLOSE LOOP TF

$$G_{CL}(s) = \frac{\int_0^2 k(q) s^q dq}{s^2 + 1 + \int_0^2 k(q) s^q dq}$$

THIS COMPENSATOR ALLOWS INFINITE NUMBER OF FREQUENCIES IN CLOSE LOOP SYSTEM AND THUS ALLOWS CONSIDERABLE DESIGN FREEDOMS TO DESIGN *k* (*q*). CHAALANGE IN R&D TO CHOSE SUITABLE *k* (*q*). THE CONTROLLER ORDER DISTRIBUTION.

ADVANCE RESEARCH VARIABLE ORDER SYSTEM



$$y(t) = {}_0D_t^{-q(t)} f(t)$$

$$_0D_t^{q(t,y)} y(t) = f(t)$$

IS EVOLVING FIELD FOR RL & GL DEFINITIONS AND

EVOLVING FORMAL APPROACH OF COMPOSITION…

RESEARCH & DEVELOPMENT AREA IN FRACTIONAL ORDER CONTROL SYSTEM

FRACTIONAL ORDER INDUSTRIAL PID
PARALLEL EMBEDDED COMPUTING
PARALLEL DSP USAGE
CODE DEVELOPMENT FOR PARALLEL EMBEDDED COMPUTING
FPGA REALIZATION WITH TANDEEM EMBEDDED M.A.C
PACKAGE DEVELOPMENT FOR SYSTEM IDENTIFICATION WITH FRACTIONAL CALCULUS
TRANFORMATION OF INTEGER ORDER CONTROL THEORY TO FRACTIONAL ORDER CONTROLS.
DEVELOPMENT OF ELECTRICAL CIRCUIT ELEMENTS & CIRCUIT SYNTHESIS FOR FRACTANCE.
DEVELOPMENT OF BETTER DIGITAL ALGORITHMS
DEVELOPMENT OF TUNING RULES FOR FRACTIONAL PID.
BOOK ON FRACTIONAL ORDER SYSTEMS & CONTROLS.

A REAL REALITY OF NATURE

ALL SYSTEMS NEEDS A FRACTIONAL TIME DERIVATIVE IN THE EQUATION DESCRIBING THEM. SYSTEMS HAVE MEMORY OF ALL EARLIER EVENTS IS THUS NECESSARY TO INCLUDE THIS RECORD OF EARLIER EVENTS TO PREDICT THE FUTURE. THE CONLUSION IS OBVIOUS & UNAVOIDABLE

DEAD MATTER HAS MEMORY

EXPRESSED DIFFERENTLY WE MAY SAY NATURE WORKS WITH
FRACTIONAL DERIVATIVES

EPILOGUE

WHAT SEEMED AS PARADOX 300 YEARS AGO TO LEIBNIZ AN USEFUL CONSEQUENCE HAS TURNED TODAY AS ENVISAGED BY HIM ON SEPTEMBER 1695.

WE HAVE FRACTIONAL ORDER CONTROLLER AND A LANGUAGE WHAT NATURE UNDERSTAND

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CONCLUSION

SO LET US TALK WHAT NATURE UNDERSTANDS
AND
LOOK BEYOND PID

FOR EFFICIENT & ROBUST CONTROLS

AND

MILES MILES MILES MILES MILES

TO GO BEFORE I SLEEP

THANK YOU